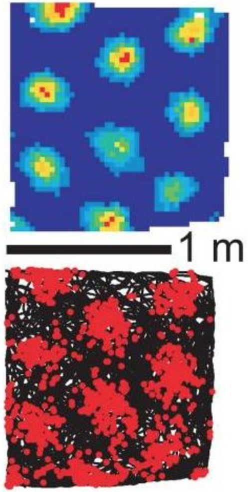
A unified theory for the origin of grid cells through the lens of pattern formation

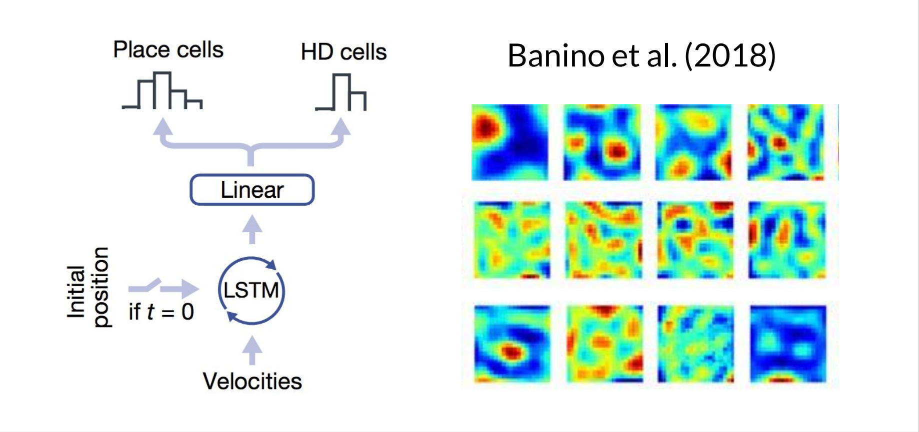
Ben Sorscher*, Gabriel C. Mel*, Surya Ganguli, Sam Ocko

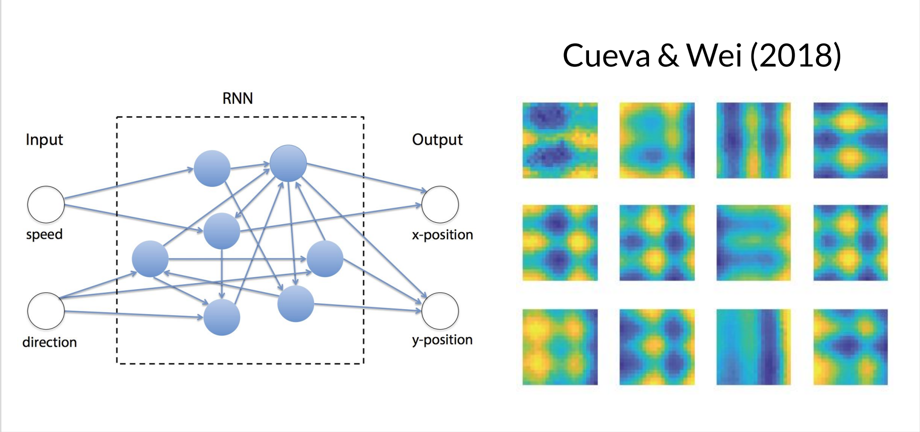
Grid cells

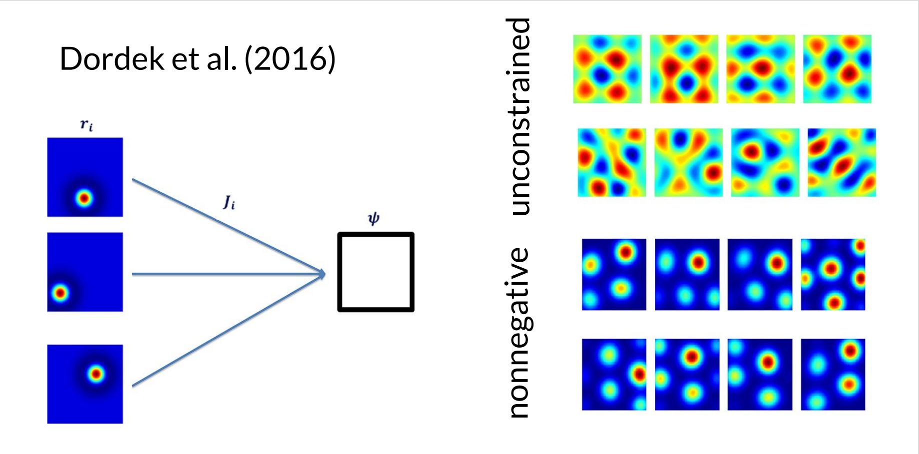


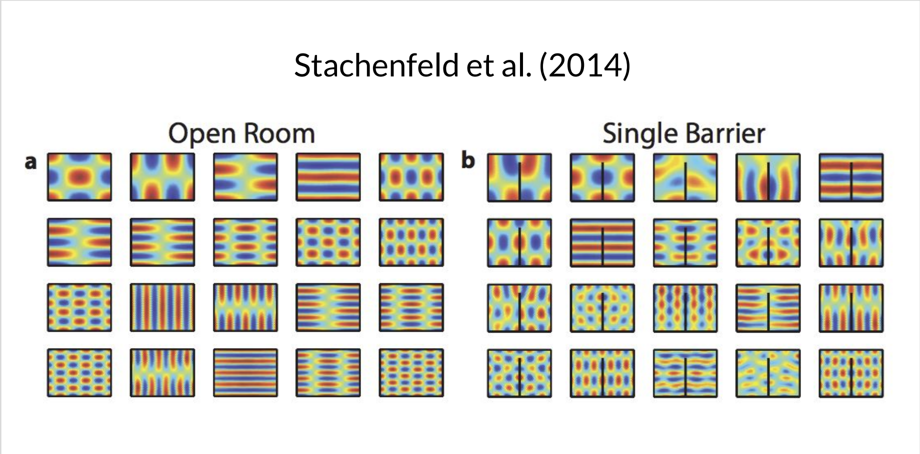
Krupic et al. (2012), Derdikman (2014)

Trained neural networks learn grid patterns









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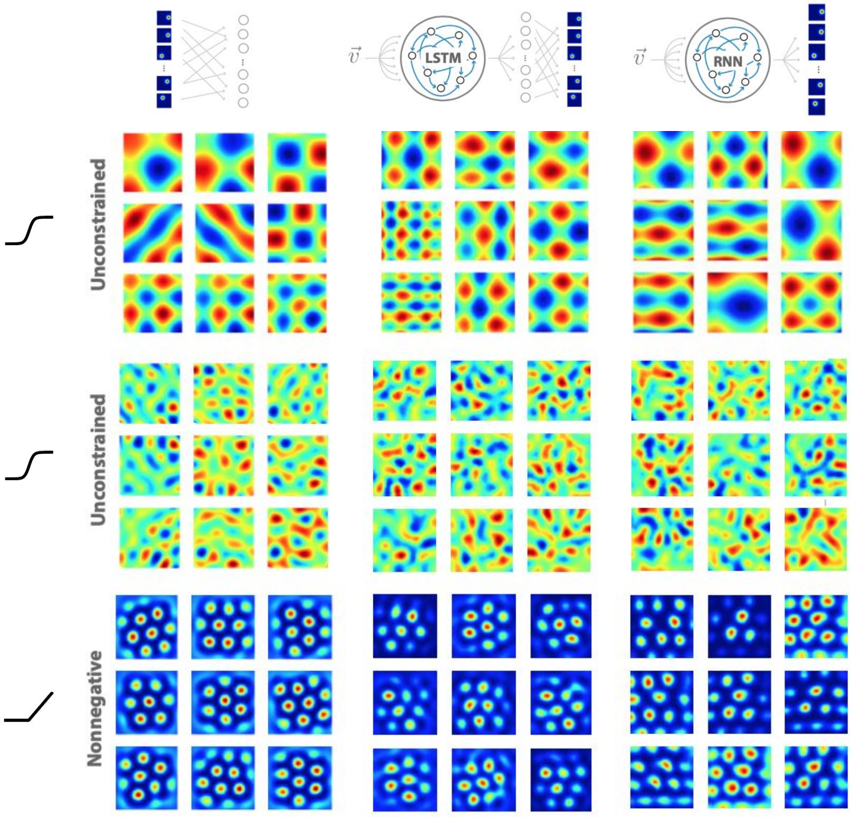
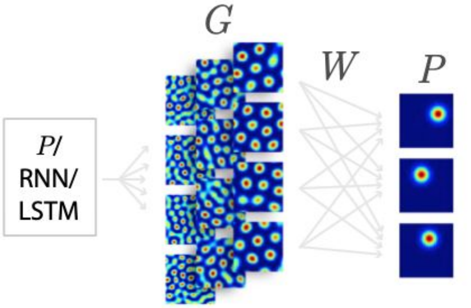


1. Why are the optimal maps grids?

2. What determines the optimal grid type - square, amorphous, or hexagonal?

Gradient descent as a *pattern forming dynamics*



$$\min_{W,G} \|P - WG\|^2$$

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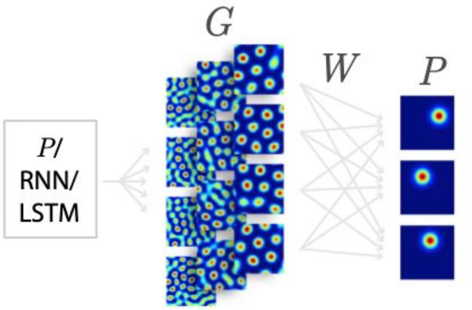

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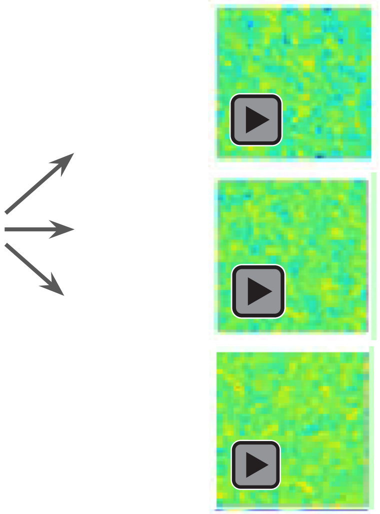

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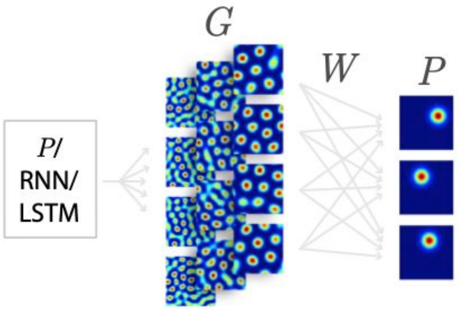

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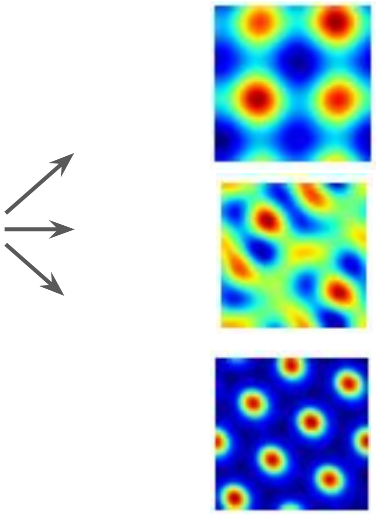

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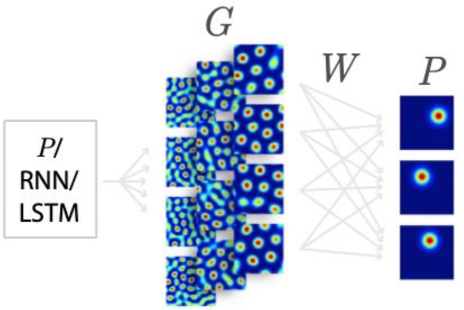

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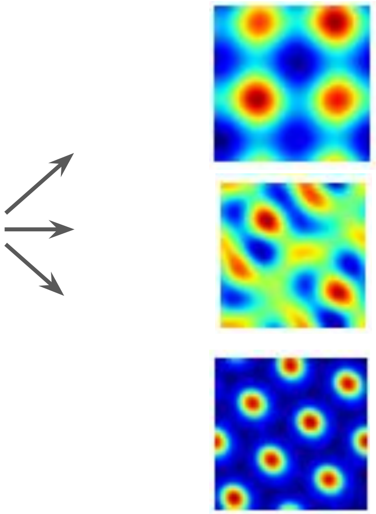

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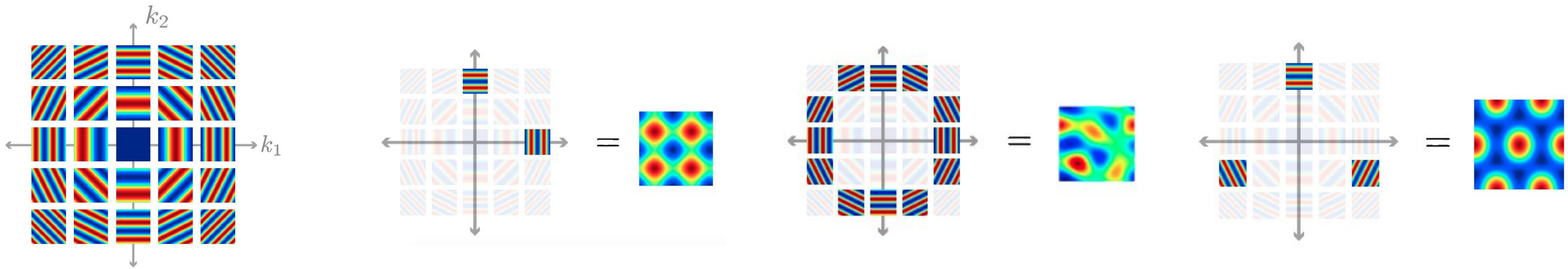
Translation invariance => Fourier modes



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Nonnegativity yields hexagonal grids

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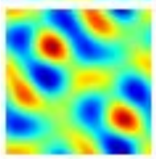
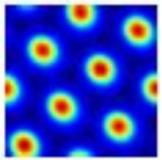


$$\min_{W,G} \|P - WG\|^2 + \sigma(G)$$



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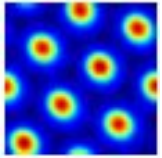

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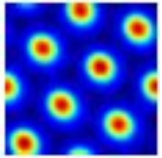

Taylor expand constraint σ

$$\sigma(G) \approx \sigma_0 + \sigma_1 G + \frac{1}{2}\sigma_2 G^2 + \frac{1}{6}\sigma_3 G^3 + \cdots$$

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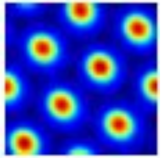

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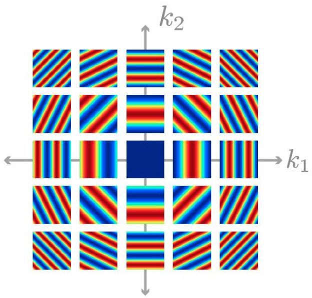

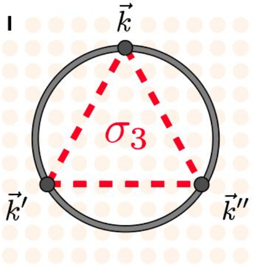

“Three-body interaction” between stripes 60° apart

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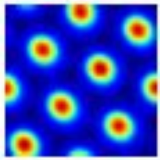

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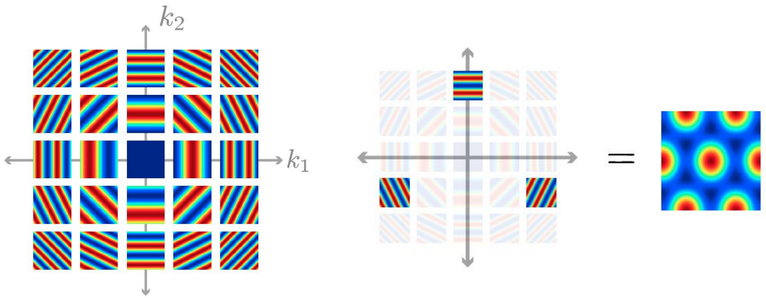
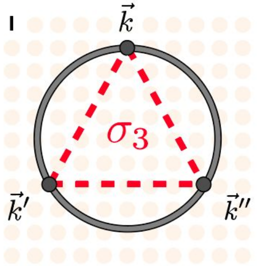
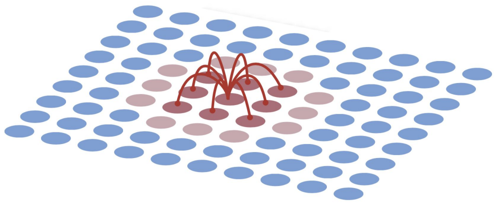




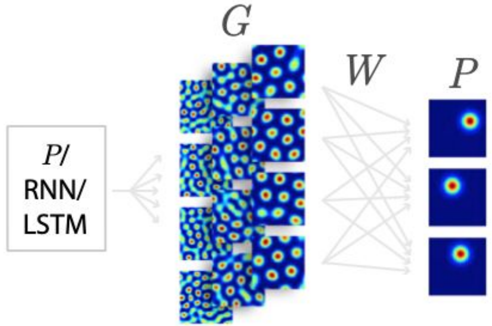
“Three-body interaction” between stripes 60º apart

Unifying mechanistic and normative models

Grid cell model RNNs



Normative encoding models



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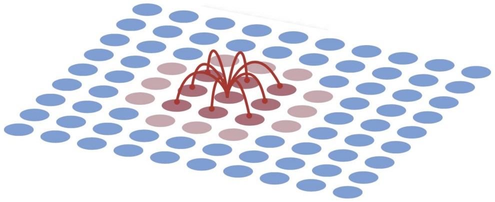

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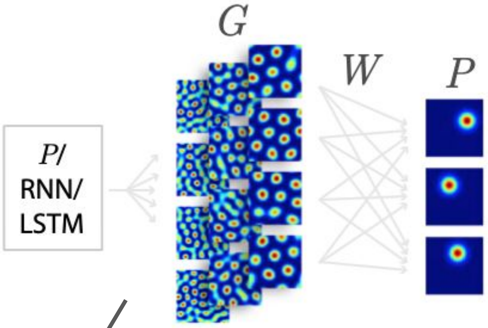
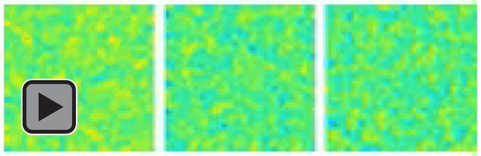


Activity dynamics

Gradient descent



Pattern forming dynamics

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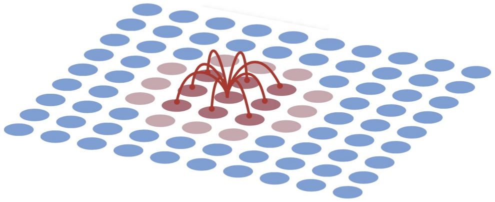

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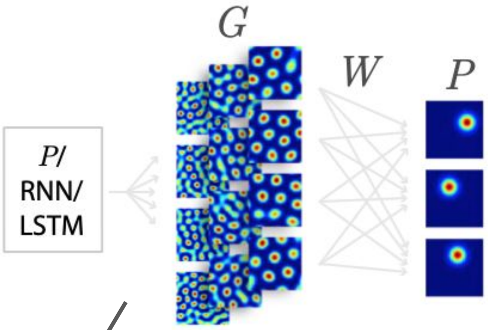
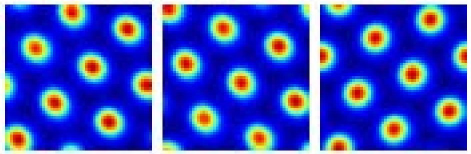


Activity dynamics = Gradient descent



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