Maximizing Revenue Under Market Shrinkage and Market Uncertainty

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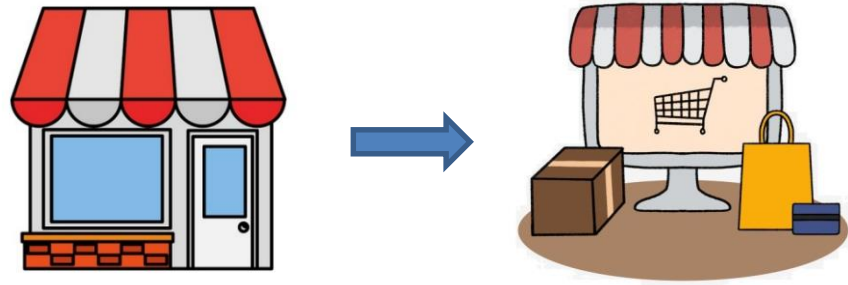
Examples of shrinking markets

Cord cutters

Retail stores

Labor markets among a shrinking population

Modeling a shrinking market

- Fixed set $S = \{v_1, \ldots, v_n\}$ of bidder valuations
- Seller knows S
- Each bidder in S shows up independently with probability p

$$S \qquad\qquad S_0$$
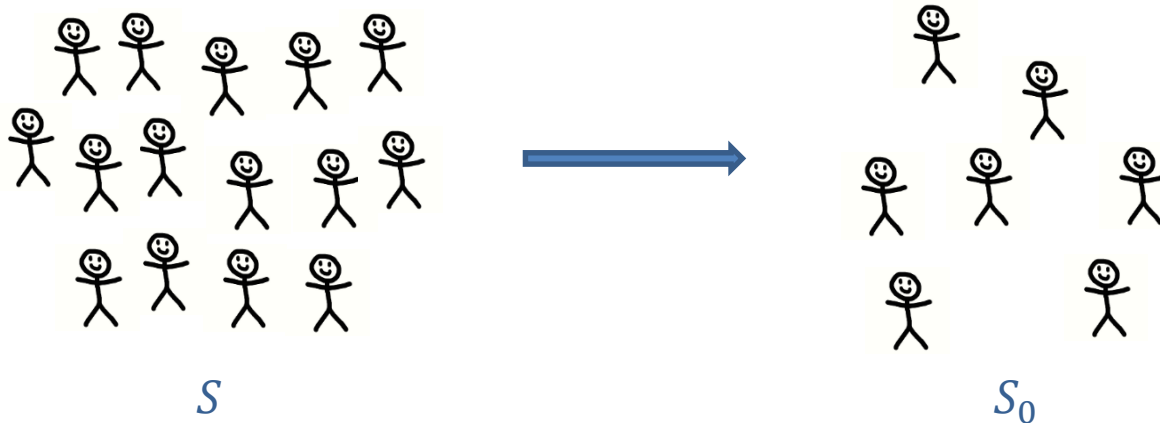
What fraction of revenue can the seller guarantee?

$$\sup_M \mathbf{E}[\text{Rev}_M(S_0)] \geq (???) \cdot W(S)$$

Revenue loss can be drastic

- At first glance answer might appear to be p (or even higher, if revenue thought to have diminishing returns in number of buyers)

- Example 1: $\mathbf{E}[\text{Rev}_{VCG}(S_0)] = p^2\text{Rev}_{VCG}(S) = p^2(W(S) - \varepsilon)$
  - Due to reduced competition among buyers

c	0	0
$c - \varepsilon/m$	0	0
0	c	0
0	$c - \varepsilon/m$	0
0	0	c
0	0	$c - \varepsilon/m$

VCG gets payment of $c - \varepsilon/m$ for each item so $\text{Rev}_{VCG}(S) = mc - \varepsilon = W(S) - \varepsilon$

But

$$\mathbf{E}[\text{Rev}_{VCG}(S_0)] = \sum_{\text{item } i} \mathbf{E}[\text{Rev from item } i]$$

$$= p^2(mc - \varepsilon)$$

Revenue loss can be drastic

If valuations can depend on what other bidders receive, things are even worse

Theorem (Balcan, **Prasad**, Sandholm NeurIPS'22). For any $\varepsilon > 0$ there exists a set S of bidders with allocational valuations such that

$$\sup \mathbf{E}[\mathrm{Rev}_M(S_0)] \leq p^{m/2} \cdot (\mathrm{Rev}_{VCG}(S) + 2\varepsilon) + \varepsilon$$

where the supremum is over all possible auctions M.

Escaping large revenue loss

Enabled by two main assumptions:

- *Winner monotonicity*
  - if bidder i wins in VCG, and j leaves, i still wins in VCG

- *Welfare submodularity*
  - efficient welfare a submodular function

 e.g. bidders with gross-substitutes valuations

How much revenue can be preserved?

General possibility result: rich enough set of mechanisms always contains one robust to shrinkage

Theorem (Balcan, **Prasad**, Sandholm NeurIPS'22). Exists auction M s.t.

$$\mathbf{E}[\mathrm{Rev}_M(S_0)] \geq \Omega\left(\frac{p^2}{k^{1+\log_{1/\gamma}(4/p)}}\right) \cdot W(S)$$

γ a constant depending on S, $k \approx$ max number of winners in VCG

A shrinkage-robust auction can be computed by sampling simulated shrunken markets and maximizing empirical revenue

Techniques
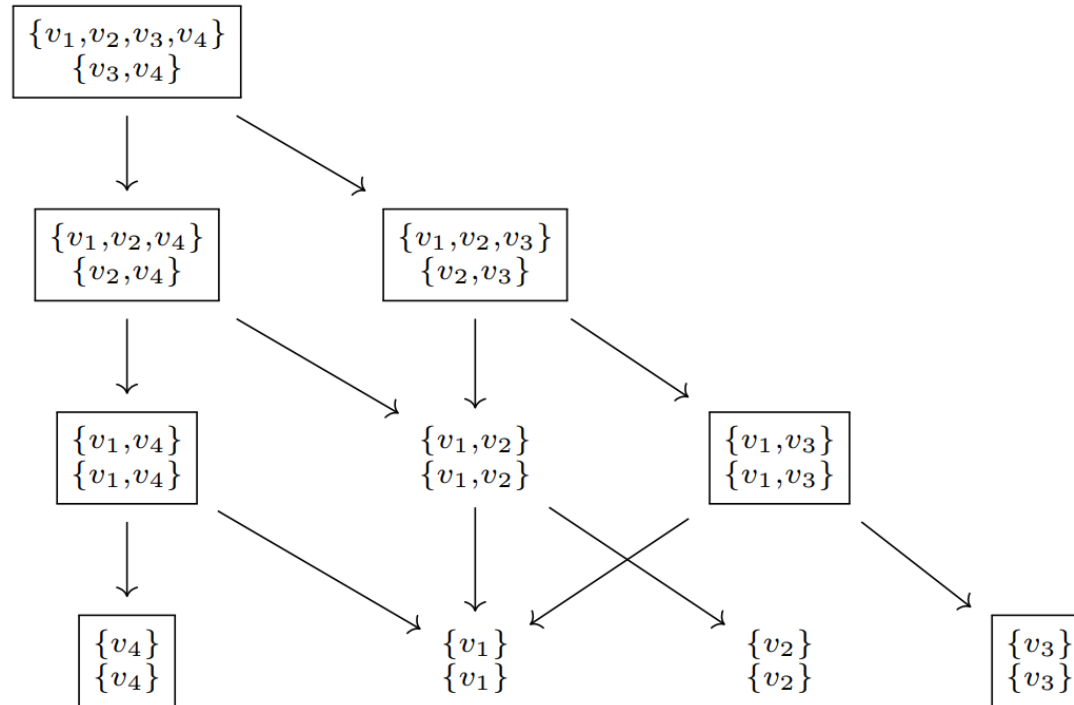
- *Winner diagram:* concise way of capturing all meaningful executions of an auction
- Randomize over a high-welfare subgraph of the winner diagram

Practically-motivated applications

- Our result yields refined guarantees when the mechanism designer:

  - Limits the number of winners

  - Places bundling constraints on the items

Conclusions

- First formal model of market shrinkage in combinatorial auctions

- Can serve as a testbed for many other mechanism design questions with market uncertainty