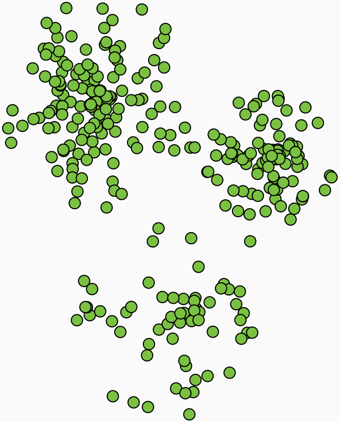
Improved Coresets for Euclidean k-Means

Vincent Cohen-Addad, Kasper Green Larsen, David Saulpic
Chris Schwiegelshohn, Omar Ali Sheikh-Omar

k-Means Clustering



Problem Definition

Let A be a set of n points in d dimensional Euclidean space and let k be a positive integer. The objective consists of finding a set of k centers S minimizing

$$\mathrm{cost}(A, S) := \sum_{p \in A} \min_{c \in S} \|p - c\|^2.$$

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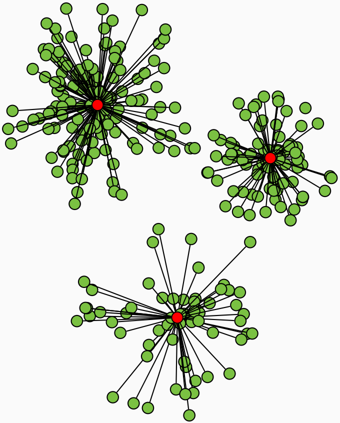

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Given a set of points A, a weighted subset $\Omega \subset A$ is a (k, ε)-coreset if for *all* sets S of k centers it holds

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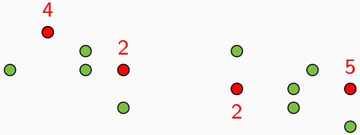

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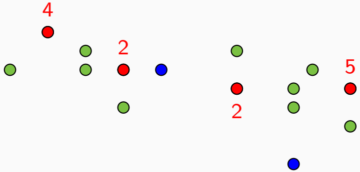

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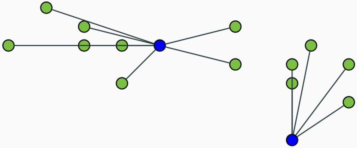

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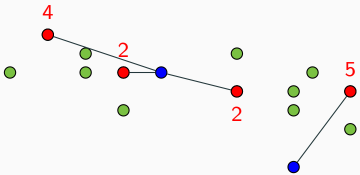

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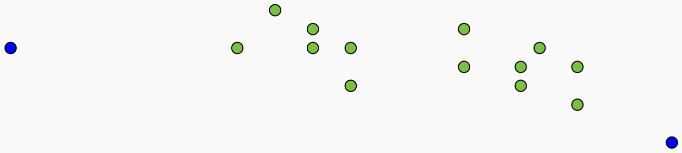

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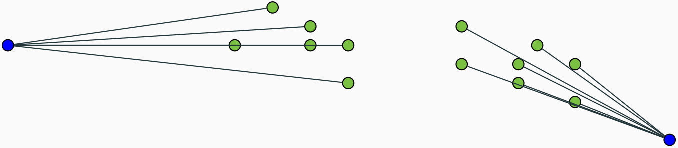

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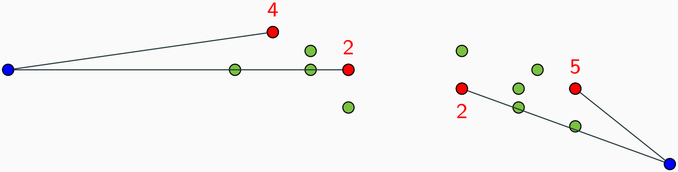

Theoretical Results on Coresets for Euclidean k-Means

Upper Bounds

Har-Peled, Mazumdar (STOC'04)	$O(k\epsilon^{-d+2}\log n)$
Chen (Sicomp'09)	$O(dk^2\epsilon^{-2}\log n)$
Langberg, Schulman (SODA'10)	$O(d^2k^3\epsilon^{-2})$
Feldman, Langberg (STOC'11)	$O(dk\epsilon^{-4})$
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Becchetti, Bury, Cohen-Addad, Grandoni, S. (STOC'19)	$O(k\epsilon^{-8})$
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Lower Bounds

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Cohen-Addad, Larsen, Saulpic, S., Sheikh-Omar (NeurIPS'22)	$O(k^{1.5}\epsilon^{-2})$

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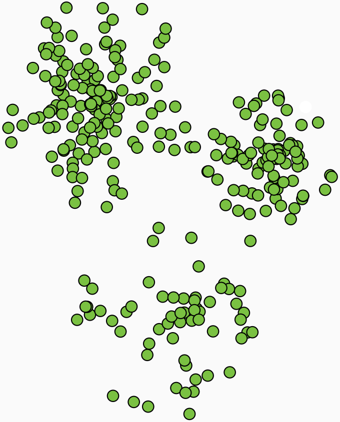
Coreset Algorithms

Algorithm

1. Sample S points (typically non unformly)
2. Weigh each point inversely proportionate to the sampling probability



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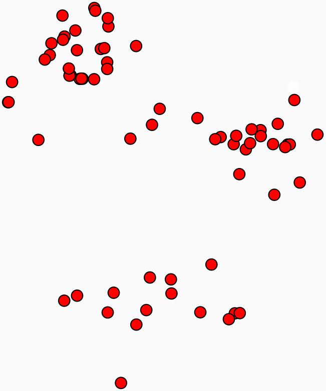

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$\Theta(\varepsilon^{-2}\min(k, \varepsilon^{-2}))$	$\Theta(k)$	$O(k\varepsilon^{-2}\min(k, \varepsilon^{-2}))$
$\Theta(\varepsilon^{-2})$	$O(k \cdot \sqrt{k})$	$O(k^{1.5}\varepsilon^{-2})$