

# *Personalized Federated Learning towards Communication Efficiency, Robustness and Fairness*

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  - Expensive communication costs
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- The answer is **YES!**

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Can we balance different constraints of interest (i.e., communication efficiency, robustness and fairness) simultaneously?

- The answer is **YES!**
- We propose a personalized FL method named as `lp-proj` based on  **$L^P$ -regularization** and **low-dimensional random projection**. Multiple benefits of the proposed objective are explored from both theoretical and empirical perspectives.

- Conventional federated learning:

$$\min_{\mathbf{w} \in \mathbb{R}^d} F(\mathbf{w}) := G \{F_1(\mathbf{w}), \dots, F_N(\mathbf{w})\}. \quad (1)$$

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- We introduce a local model  $\mathbf{x}_k$  for each client  $k$  to fit the local data.
- **Infimal Convolution**: bridges local models and global model.

$$F_k(\mathbf{w}) = \{f_k \otimes \lambda g\}(\mathbf{w}) := \min_{\mathbf{x}_k \in \mathbb{R}^d} f_k(\mathbf{x}_k) + \lambda g(\mathbf{w} - \mathbf{x}_k), \quad (2)$$
$$f_k(\mathbf{x}_k) = \mathbb{E}_{\xi_k} \left[ \tilde{f}_k(\mathbf{x}_k; \xi_k) \right].$$

- $g$  is the smoothing kernel, which is designed to characterize the relationship between local models and global model.



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- **Random projection** would preserve similarity of data vectors.
- We propose to **regularize the projection of local models in a shared low-dimensional space**.

$$g(\mathbf{w} - \mathbf{x}_k) = \frac{1}{p} \|\mathbf{P}(\mathbf{w} - \mathbf{x}_k)\|_p^p = \frac{1}{p} \|\tilde{\mathbf{w}} - \mathbf{P}\mathbf{x}_k\|. \quad (3)$$

- $\mathbf{P}$  is a  $d_{\text{sub}} \times d$  random matrix generated initially and fixed during training.
- $d_{\text{sub}} \ll d$  is the dimension of the shared-and-fixed random subspace.

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- $d_{\text{sub}} \ll d$  is the dimension of the shared-and-fixed random subspace.
- Combining (1), (2) and (3), our personalized FL method is formulated as a **bi-level problem**. We introduce the algorithm `1p-proj`, which alternatively minimizes the local and global objectives with gradient descent.

## *Benefits: Communication Efficiency, Robustness and Fairness*

(Intuitions are provided here. For formal theoretical analysis, please refer to Section 4 in our paper.)

- **Communication Efficiency:** The global model  $\tilde{\mathbf{w}}$  is restricted to lie in a fixed low-dimensional subspace.  $\implies$  Only  $\tilde{\mathbf{w}}$  of dimension  $d_{\text{sub}}$ , instead of the full model  $\mathbf{x}_k$  of dimension  $d$ , is communicated each round.

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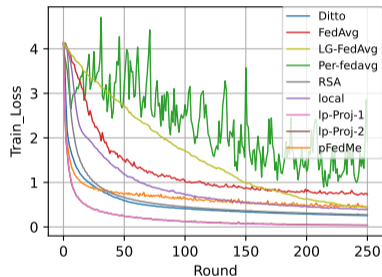
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- **Robustness and Fairness:**
  - (Near) consensus of model parameters in the low-dimensional subspace leaves flexibility towards personalization and better generalization to the local data distribution.
  - $L^p$ -norm regularization is equivalent to launching an uncertainty set to the model parameter.  $\implies$  We can enhance accuracy by searching for a model adaptive to the local data distribution in the uncertainty set.

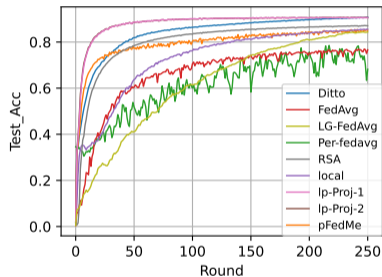
# Numerical Experiments - Personalization Accuracy Performance

- Personalization Accuracy Performance:

EMNIST, Train Loss



EMNIST, Test Acc



- lp-proj has comparable or even superior performance than other SOTA methods. Moreover, the training process is more stable as the loss and accuracy curves have less fluctuation.



- **Communication Efficiency:**

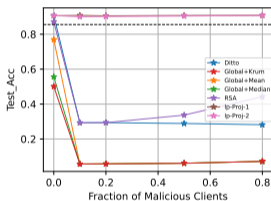
Method	EMNIST			
	Bytes Budget	Test Acc	Target Acc	Used Bytes
FedAvg	4236900	*	0.7	445851400
Sketch	4236900	*	0.7	*
lp-proj-1	4236900	<b>0.906</b>	0.7	<b>174720</b>
lp-proj-2	4236900	<b>0.906</b>	0.7	196560
LBGM	4236900	*	0.7	769902776
QSGD	4236900	*	0.7	673302175
DGC	4236900	*	0.7	*it
LG-FedAvg	4236900	0.071	0.7	230786010

- Given a communication budget of bytes, lp-proj obtains  $\sim 83.5\%$  test accuracy improvement on EMNIST.
- Given a target test accuracy, the communication cost is saved by 1320x on EMNIST compared with the best competing method.

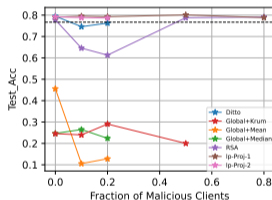
# Numerical Experiments - Robustness

- **Robustness:**

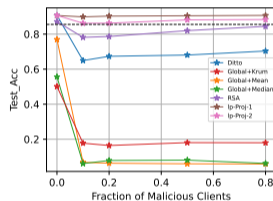
EMNIST,  
same-value



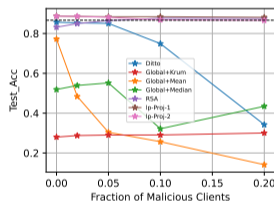
CIFAR,  
sign-flipping



EMNIST,  
Gaussian



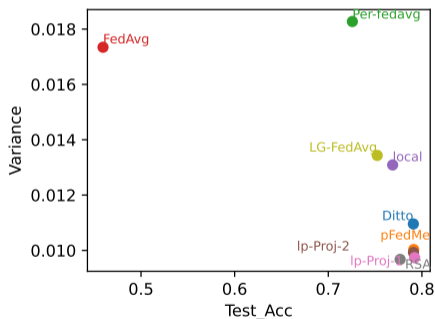
Synthetic(0,0),  
data-poisoning



- $l_p$ -proj is resistant to standard malicious attacks.
- Random projection helps alleviate the attacks applied in the original space, while the  $L^p$ -norm helps eliminate outliers further.

- **Fairness:**

CIFAR,  
accuracy-fairness trade-off



- lp-proj provides accurate and fair solutions that are comparable to other SOTA methods.
- On CIFAR, lp-proj-1 achieves the highest test accuracy of 79.22% with the lowest variance of 0.0097 among all the competitors.

# Conclusion

- We propose a simple yet powerful personalized FL approach based on **infimal convolution** and **subspace projection**.
- We present convergence results for smooth objectives with square regularizers.
- Theoretical analysis and numerical experiments show that our approach could promote **communication efficiency**, **robustness** and **performance fairness**.
- **Code Implementation:** `https://github.com/desternylin/perfed`

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# Thanks for listening!