Low-rank lottery tickets: finding efficient low-rank neural networks via matrix differential equations

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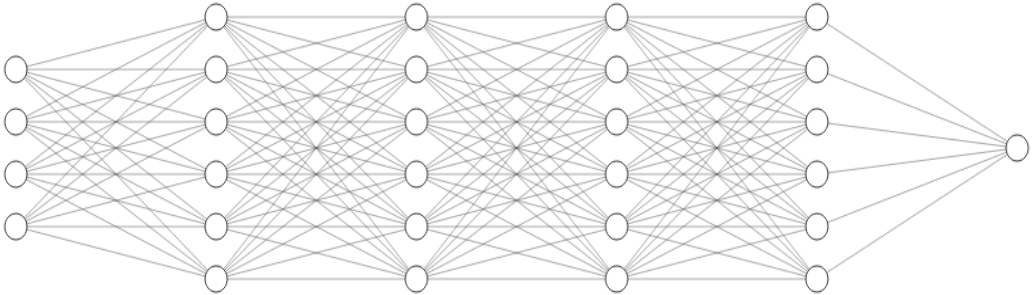
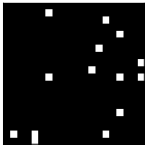
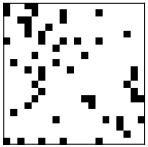


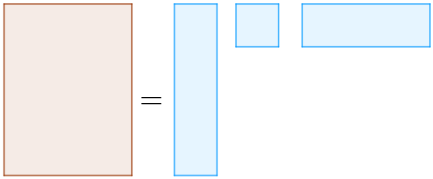
Model compression - an overview



(a) full-rank representation



(b) sparse representation



(c) low-rank representation

We want to compress neural networks **during** training using low-rank factorization.
Goals of **D**ynamical **L**ow-**R**ank **T**raining (DLRT):

- Low training and inference memory footprint
- Fast training and inference
- High accuracy of the compressed network

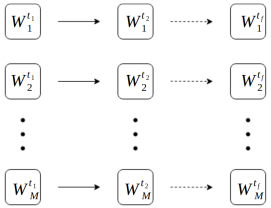
DLRT: Idea

Interpret the gradient update

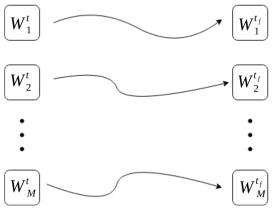
$$W^{t+1} = W^t - \eta \nabla_W \mathcal{L}, \qquad t = 1, 2, \dots \tag{1}$$

as a continuous time gradient flow. I.e.,

$$\dot{W}(t) = -\nabla_W \mathcal{L}, \qquad W(t_0) = W_0 \tag{2}$$



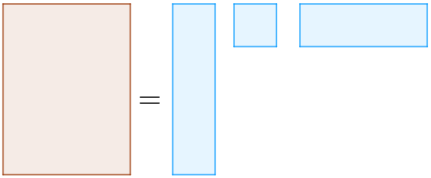
(a) discrete gradient update



(b) time-continous gradient flow

The continous time gradient flow system is **low-rank** for neural networks!

Dynamical Low-Rank Updates



$$W = U \quad S \quad V^\top$$

Find update rules for the low-rank factorization.

- Update the low-rank basis
- Expand the low-rank manifold
- Update the low-rank coefficients
- Reduce the rank of the low-rank manifold

Three low-rank updates instead of one full-rank update!

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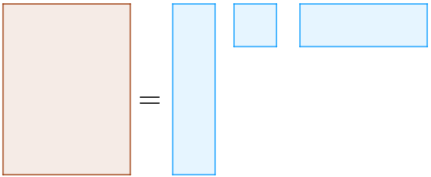

$$W = U \quad S \quad V^\top$$

$$U^t \to U^{t+1} \qquad V^{t,\top} \to V^{t+1,\top}$$

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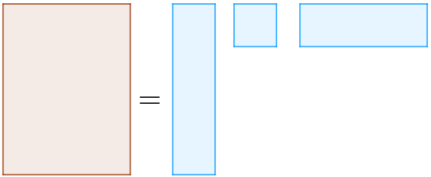

$$W = U \quad S \quad V^\top$$

$$S^t \to \widetilde{S}^t$$

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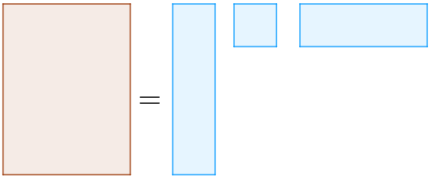

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$$\widetilde{S}^t \to \widetilde{S}^{t+1}$$

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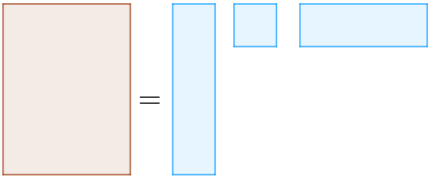
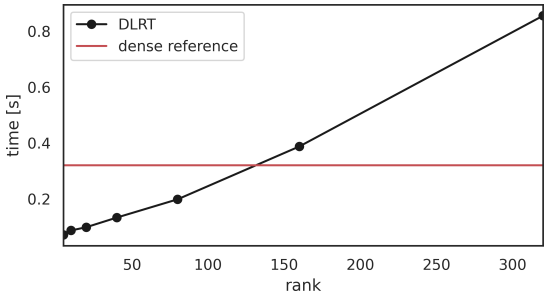


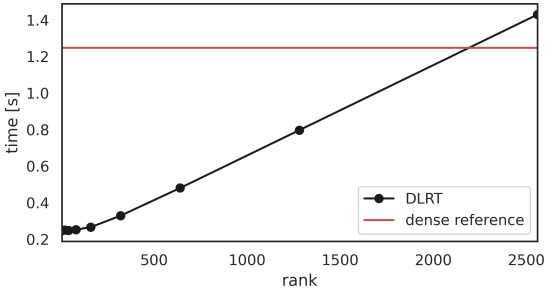
$$W = U \quad S \quad V^\top$$

$$\widetilde{S}^{t+1} \to S^{t+1}$$

DLRT efficiency



(a) Training timings

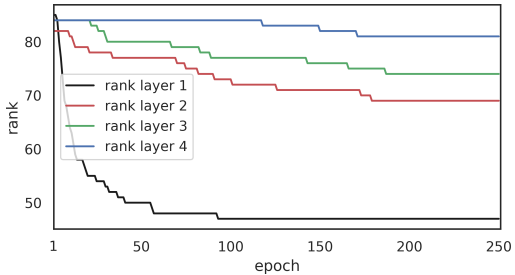


(b) Prediction timings

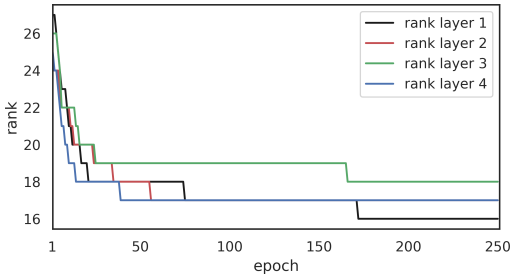
- DLRT **training is one order of magnitude faster** than the dense training for $r \ll M, N$
- DLRT **inference timing is one order of magnitude faster**

DLRT rank evolution

DLRT compression of a 5-layer, rank 500 network on MNIST.



(a) $\tau = 0.05$



(b) $\tau = 0.15$

- The only hyperparameter of DLRT is the compression rate τ
- τ determines, how aggressive we cut off singular values of S
- DLRT enables **fast initial compression** and convergence of the solution rank

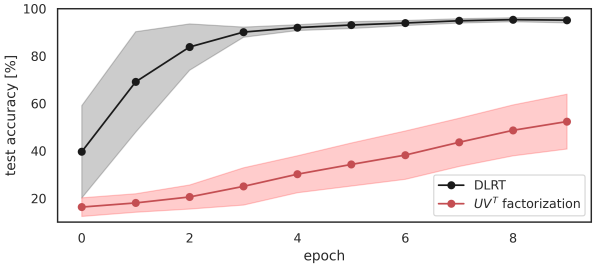
Compression results

- DLRT achieves SOTA compression during inference
- DLRT is able to compress networks during training

	ImageNet1k			
		test acc.[%]	compression rate	
	method	(to baseline)	eval[%]	train[%]
ResNet-50	**DLRT**	**−0.56**	**54.1**	**14.2**
	PP-2[SKVRN19]	−0.8	52.2	< 0
	PP-1[SKVRN19]	−0.2	44.2	< 0
	CP[HZS17]	−1.4	50.0	< 0
	SFP[HKD$^+$18]	−0.2	41.8	< 0
	ThiNet[LWL17]	−1.5	36.9	< 0
VGG16	**DLRT**	**−2.19**	**86**	**78.4**
	PP-1[SKVRN19]	−0.19	80.2	< 0
	CP[HZS17]	−1.80	80.0	< 0
	ThiNet[LWL17]	−0.47	69.04	< 0
	RNP(3X)[LRLZ17]	−2.43	66.67	< 0

DLRT stability

Training on low-rank manifolds is a non-trivial task.



(a) Training performance, using $W(t_0)$ with exponentially decaying singular values

- **DLRT is robust** with respect to weight initializations
- Vanilla factorization $W = UV^\top$ with direct gradient descent is **not robust** with respect to weight initializations

Thank you for your attention!

We welcome you to our poster session!

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References II

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