# Towards Understanding Learning Representations: To What Extent Do Different Neural Networks Learn the Same Representation

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John Hopcroft

NeurIPS 2018 Spotlight

# Motivation

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- However, there is a lack of theory on what these representations really are.
- One fundamental question: are the representations learned by deep nets robust? In other words, are the learned representations commonly shared across multiple deep nets trained on the same task?

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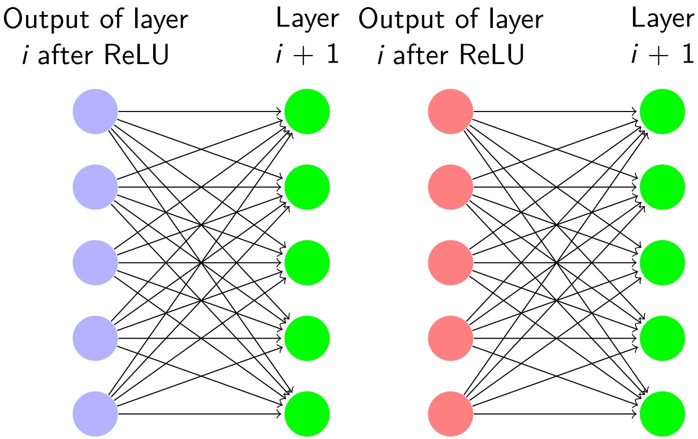
- How similar are intermediate layers?
- Do some groups of neurons in an intermediate layer learn *features/representations* that both deep nets share in common? How large are these groups?

# Two Groups of Neurons Learning the Same Representation: Exact Matches



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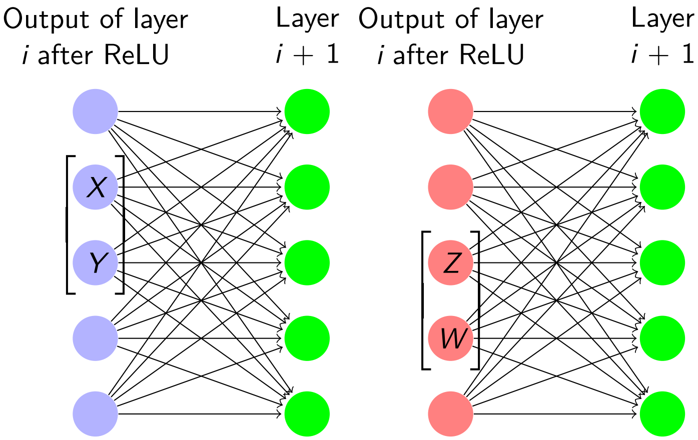

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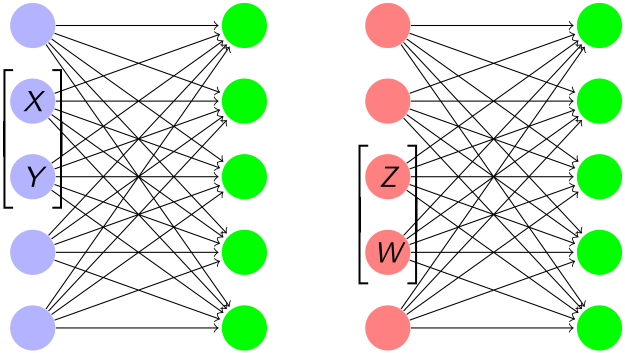
Output of layer $i$ after ReLU | Layer $i+1$ | Output of layer $i$ after ReLU | Layer $i+1$




For test examples $\mathbf{a}_1, \cdots, \mathbf{a}_d$, there exist $A$ and $B$ such that for all $i$,

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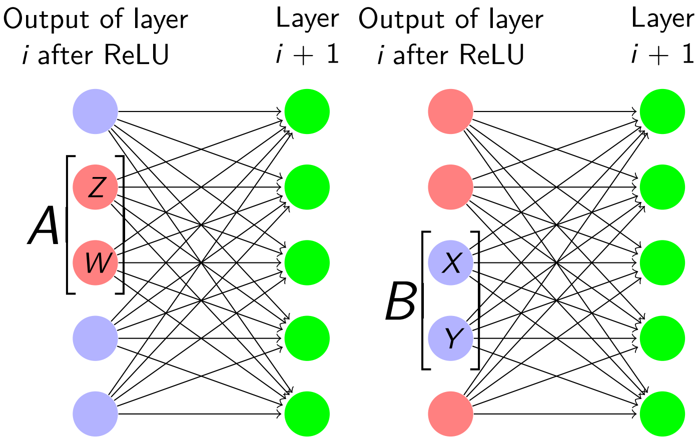


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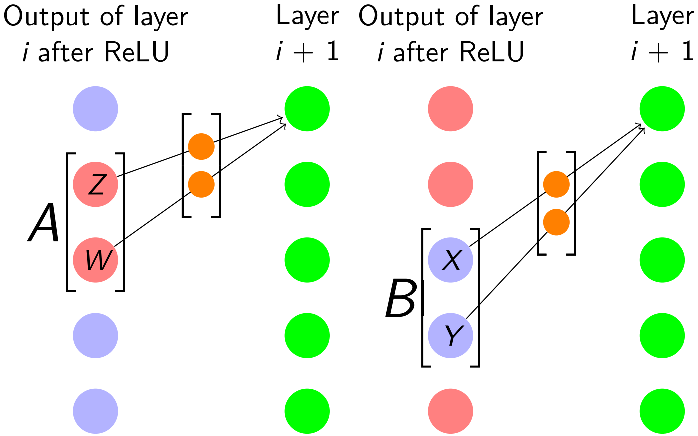


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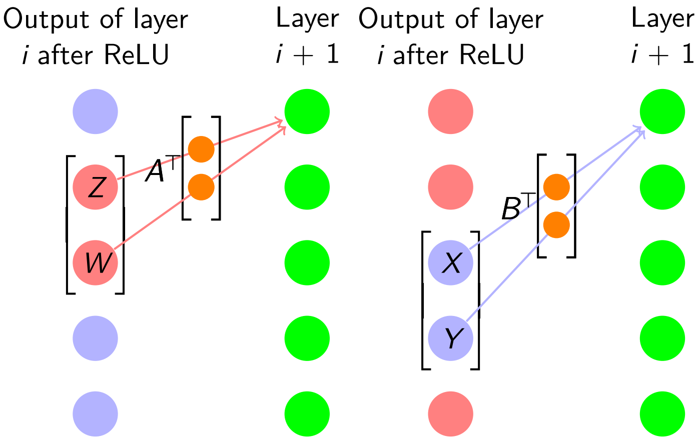


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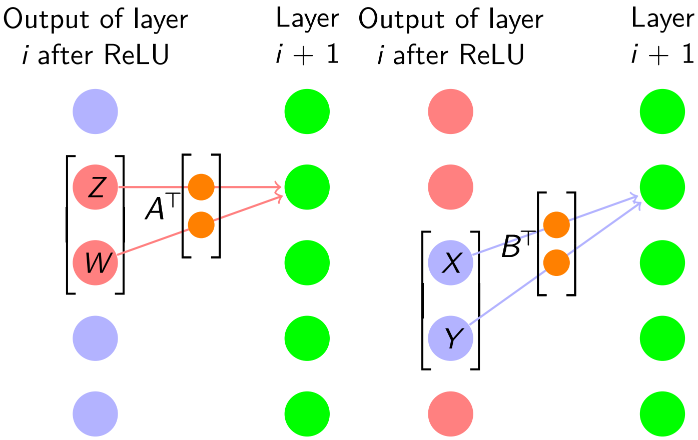


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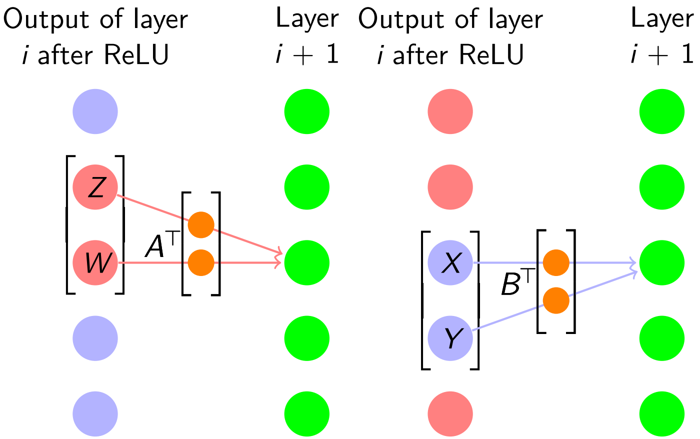


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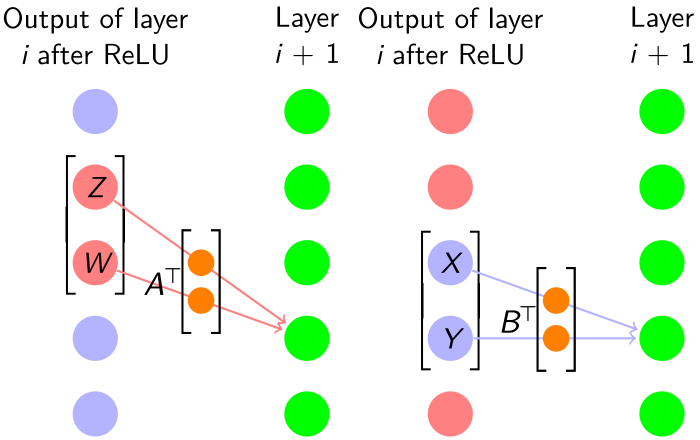


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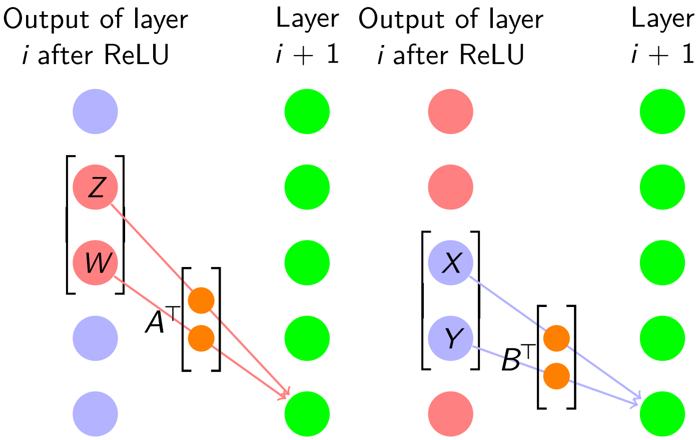


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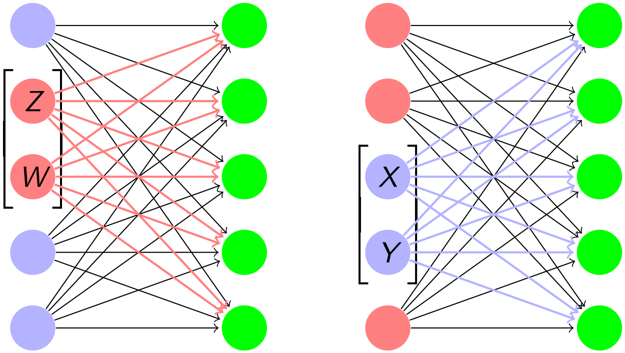


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$$\mathrm{span}(\underbrace{[X(\mathbf{a}_1), \cdots, X(\mathbf{a}_d)]}_{\text{activation vector of } X}, \underbrace{[Y(\mathbf{a}_1), \cdots, Y(\mathbf{a}_d)]}_{\text{activation vector of } Y})$$

$$= \mathrm{span}(\underbrace{[Z(\mathbf{a}_1), \cdots, Z(\mathbf{a}_d)]}_{\text{activation vector of } Z}, \underbrace{[W(\mathbf{a}_1), \cdots, W(\mathbf{a}_d)]}_{\text{activation vector of } W})$$

We say $(\{X, Y\}, \{Z, W\})$ form an exact match!

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# *Exact/Approximate* Matches between Two Groups of Neurons

- Suppose $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_d$ are the test examples. The output of neuron $X$ on these test examples form a vector $(X(\mathbf{a}_1), X(\mathbf{a}_2), \cdots, X(\mathbf{a}_d))$ called the activation vector [Raghu et al., 2017].

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- If the activation vectors of two groups of neurons span the same linear subspace, we say the two groups of neurons form an exact match.
- If the activation vector of every neuron in each group is $\varepsilon$-close to the linear subspace spanned by the other group, we say the two groups form an $\varepsilon$-approximate match.
  - Vector $\mathbf{u}$ is $\varepsilon$-close to linear subspace $S$ if the sine of the angle between $\mathbf{u}$ and $S$ is at most $\varepsilon$, or equivalently, $\min_{\mathbf{v}\in S} \|\mathbf{u}-\mathbf{v}\|_2 \le \varepsilon\|\mathbf{u}\|_2$.

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- We define simple matches to be matches that are not the union of smaller matches.
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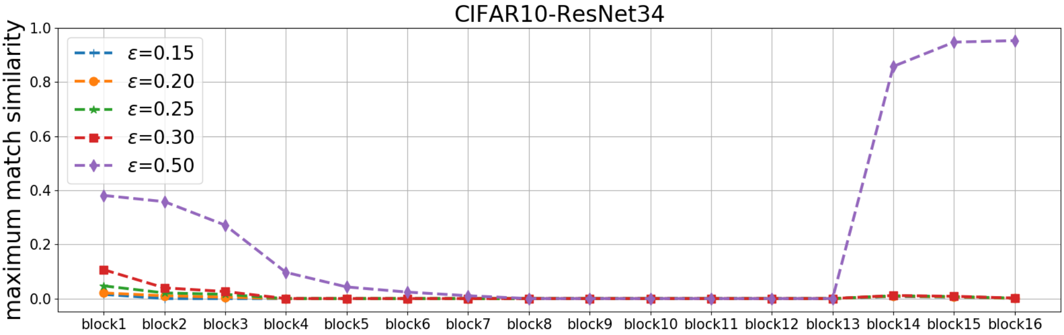
- We designed algorithms for finding the maximum match and the simple matches, and we implemented the algorithms to conduct experiments.

# Experimental Findings: Few Matches in Intermediate Layers

Figure: Size of maximum match / number of neurons across layers



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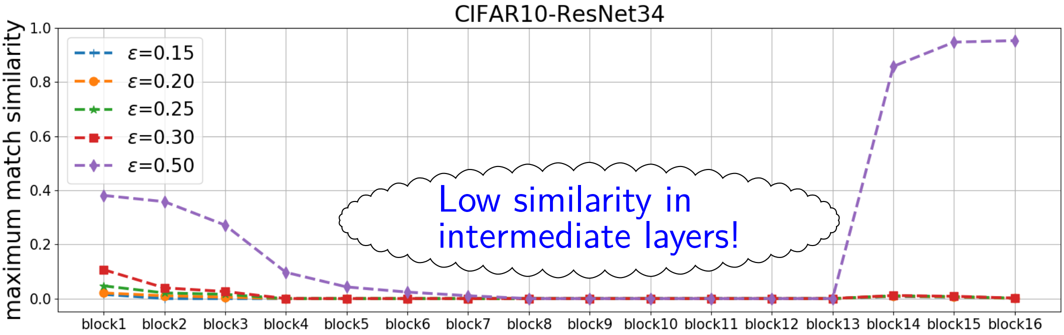

# Thank you!

Come to the poster for more details!

05:00 – 07:00 PM @ Room 210 & 230 AB **#26**