Smoothing Structured Decomposable Circuits

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Probabilistic Circuits

Tractable computation graph, encoding a distribution.

SOTA for:

- Inference algorithms for PGMs / probabilistic programs
- Discrete density estimation

Exact likelihoods and partition function!

Gaining popularity:

**Tractable Probabilistic Models**: (UAI19 / AAAI20 tutorial)
Tractability

Different combination of properties leads to different families of circuits

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<td>Determinism</td>
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<td>Smoothness</td>
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<td>MPE</td>
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<td>Marginal MAP</td>
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...with different tractability properties.
Smoothness

Definition
A circuit is **smooth** if for every pair of children $c_1$ and $c_2$ of a $\oplus$-gate, $\text{vars}_{c_1} = \text{vars}_{c_2}$.

Figure: Two equivalent circuits computing $(x_0 \otimes x_1) \oplus x_2$. The left one is not smooth and the right one is smooth.
Smoothing a Circuit: Prior Work

- Go to each gate $O(m)$ and fill in each variable $O(n)$
- Quadratic Complexity $O(nm)$
- Problematic when $n \geq 1,000$ and $m \geq 1,000,000$

Our near-linear smoothing algorithm: $O(m \cdot \alpha(m, n))$
Smoothing a Circuit: Our Work

Key Insight: missing variables for each gate form two intervals.

![Diagram showing two intervals labeled A and B on a line labeled $\pi$]

**Figure:** $A \setminus B$ forms two intervals

We need to fill in $2m$ intervals.
Semigroup Range Sum

Theorem

Given \( n \) variables defined over a semigroup and \( m \) intervals, the sum of all intervals can be computed using \( O(m \cdot \alpha(m, n)) \) additions [Chazelle and Rosenberg 1989].

\( \alpha(m, n) \) is the inverse Ackermann function, which grows very slowly.

*The original theorem only bounds the number of additions. We bound the number of computations.*
Takeaways

▶ Probabilistic circuits can encode complex distributions.

▶ They can compute exact likelihoods, marginals, and more
  ▶ But only if they are smooth.

▶ Best smoothing algorithm was quadratic.

▶ We propose a near linear time smoothing algorithm.
Thanks!

Poster: East Exhibition Hall B+C #182, 10:45AM

Code: https://github.com/AndyShih12/SSDC

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