Implicit Generation and Generalization with Energy Based Models

Yilun Du and Igor Mordatch
Energy-Based Model

• Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \quad Z(\theta) = \int \exp(-E_\theta(x)) \, dx \]

see [LeCun et al, 2006] for review
Energy-Based Model

• Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \]

• Train to maximize data likelihood

\[ \mathcal{L}_{\text{ML}}(\theta) = \mathbb{E}_{x \sim p_D} [- \log p_\theta(x)] \]
Energy-Based Model

• Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \]

• Train to maximize data likelihood

\[ \mathcal{L}_{ML}(\theta) = \mathbb{E}_{x \sim p_D} [-\log p_\theta(x)] \]

• gradient:

\[ \mathbb{E}_{x^+ \sim p_D} [\nabla_\theta E_\theta(x^+)] - \mathbb{E}_{x^- \sim p_\theta} [\nabla_\theta E_\theta(x^-)] \]

See [Turner, 2006] for derivation
Energy-Based Model

• Distribution defined by energy function

\[ p_\theta(x) = \frac{\exp(-E_\theta(x))}{Z(\theta)} \]

• Train to maximize data likelihood

• gradient:

\[ E_{x^+ \sim p_D} [\nabla_\theta E_\theta(x^+)] - E_{x^- \sim p_\theta} [\nabla_\theta E_\theta(x^-)] \]

• Generate model samples implicitly via stochastic optimization

\[ \tilde{x}^k = \tilde{x}^{k-1} - \frac{\lambda}{2} \nabla_x E_\theta(\tilde{x}^{k-1}) + \omega^k, \ \omega^k \sim \mathcal{N}(0, \lambda) \]

Langevin Dynamics

[Welling and Teh, 2011]
Why Energy-Based Generative Models?

1. Implicit Generation
   - Flexibility
   - One Object to Learn
   - Compositionality
   - Generic Initialization and Computation Time

2. Intriguing Properties
   - Robustness
   - Online Learning
Why Do EBMs Work Now?

More compute and modern deep learning practices

Faster Sampling

• Continuous gradient based sampling using Langevin Dynamics
• Replay buffer of past samples (similar to persistent CD)

Stability improvements

• Constrain Lipschitz constant of energy function (spectral norm)
• Smoother activations (swish)
• And others ...
Comparison to Other Generative Models

- Training Cost
  - SNGAN
  - Glow
  - PixelCNN++
  - EBM

- Sampling Speed
  - SNGAN
  - Glow
  - PixelCNN++
  - EBM
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ImageNet 128x128
Cross Class Mapping
Cross Class Mapping
Surprising Benefits of Energy-Based Models

- Robustness
- Continual Learning
- Compositionality
- Trajectory Modeling
Surprising Benefits of Energy-Based Models

• Robustness

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Out-of-Distribution Relative Likelihoods

Also observed by [Hendrycks et al 2018] and [Nalisnick et al 2019]
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Out-of-Distribution Relative Likelihoods

Also observed by [Hendrycks et al 2018] and [Nalisnick et al 2019]
Out-of-Distribution Generalization

• Following [Hendrycks and Gimpel, 2016]
Robust Classification

(a) $L_\infty$ robustness

(b) $L_2$ Robustness
Robust Classification

(recent follow-up submission at ICLR 2020 improves baseline EBM performance)
Surprising Benefits of Energy-Based Models

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## Continual Learning: Split MNIST

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Evaluation by [Hsu et al, 2019]
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Evaluation by [Hsu at al, 2019]

EBM: 64.99 ± 4.27 (10 seeds)
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Evaluation by [Hsu at al, 2019]

EBM: 64.99 ± 4.27

Would any generative model work instead? Doesn’t look like it:
VAE: 40.04 ± 1.31
Surprising Benefits of Energy-Based Models

- Robustness
- Continual Learning
- Compositionality
- Trajectory Modeling
Compositionality via Sum of EBMs
[Hinton, 1999]

Specify a concept by successively adding constraints
Compositionality via Sum of Energies

Specify a concept by successively adding constraints

Compositional Visual Generation with EBMs [Du, Li, Mordatch, 2019]
Compositionality via Sum of Energies

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EBMs for Trajectory Modeling and Control

[Du, Lin, Mordatch, 2019]

- Train energy to model pairwise state transitions $s_t$, $s_{t+1}$
- Trajectory probability:

$$p_\theta(\tau) = p_\theta(s_1, s_2, \ldots, s_T) = \prod_{t=1}^{T-1} p_\theta(s_t, s_{t+1})$$

$$\propto \exp(-\sum_{t=1}^{T} E(s_t, s_{t+1}))$$
EBMs for Trajectory Modeling and Control

[Du, Lin, Mordatch, 2019]

• Train energy to model pairwise state transitions $s_t, s_{t+1}$
• Generate trajectories that achieve specific tasks:

$$p_\theta(s_2, \ldots, s_T|s_1, R) \propto \exp(- \sum_{t=1}^{T-1} E(s_t, s_{t+1}) - \sum_{t=1}^{T} R(s_t))$$

EBM

Task

(similar to direct trajectory optimization)
# EBMs for Control

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<th>Model</th>
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<th>Reacher</th>
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<td>Online</td>
<td>EBM</td>
<td>-20.38</td>
<td>-162.97</td>
<td>-29.87</td>
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<td>Action FF</td>
<td>-850.67</td>
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<td>-42.37</td>
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Source Code

- Images
  - https://github.com/openai/ebm_code_release
- Trajectories
  - https://github.com/yilundu/model_based_planning_ebm
- Compositionality
  - https://drive.google.com/file/d/138w7Oj8rQI_e40_RfZJq2WKWb41NgKn3
- Interactive Notebook
  - https://drive.google.com/file/d/1fCFRw_YtqQPSNoqznIh2b1L2baFgLz4W/view