Probabilistic Watershed
Sampling all spanning forests for seeded segmentation and semi-supervised learning

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What do we do?
We count forests!
Framework

Graph

Seeds (labeled nodes)

Edge-Costs

\( \sim \) affinity between nodes

Forest
Framework

- Graph
- Seeds (labeled nodes)
Graph
Seeds (labeled nodes)
Edge-Costs $\sim$ affinity between nodes
Graph
Seeds (labeled nodes)
Edge-Costs $\sim$ affinity between nodes
Forest
Forests

Watershed forest / minimum cost Spanning Forest (mSF)

Enrique Fita Sanmartin
Probabilistic Watershed
Counting Forests

\[
\Pr(\overline{q} \sim s_2) = \frac{\mathcal{F}_{s_2,q}^{s_1} \cdot q + \mathcal{F}_{s_1,q}^{s_2}}{\mathcal{F}_{s_1}^{s_2}}
\]
Probabilistic Watershed

\[ \Pr(f) = \frac{\exp(-\mu c(f))}{\sum_{f' \in F_{s_2}^{s_1}} \exp(-\mu c(f'))} = \frac{w(f)}{\sum_{f' \in F_{s_2}^{s_1}} w(f')} \]

\[ \Pr(q \sim s_2) := \sum_{f \in F_{s_2}^{s_1}} \Pr(f) = \frac{w(F_{s_2}^{s_1}, q)}{w(F_{s_2}^{s_1})} = \frac{\sum_{f \in F_{s_2}^{s_1}, q} w(f)}{\sum_{f \in F_{s_2}^{s_1}} w(f)} \]
How do we count the forests?

Matrix Tree Theorem [Kirchhoff, 1847]

Let $G = (V, E, w)$ an edge-weighted multigraph, $w(T)$, the sum of the weights of the spanning trees of $G$, $\mathbb{1}$ is a column vector of 1’s, $L$ is the Laplacian matrix and $L^{[v]}$ is the Laplacian matrix after removing an arbitrary row and column $v$, then

$$w(T) := \sum_{t \in T} w(t) = \sum_{t \in T} \prod_{e \in E_t} w(e) = \frac{1}{|V|} \det \left( L + \frac{1}{|V|} \mathbb{1} \mathbb{1}^T \right) = \det(L^{[v]}).$$
How do we count the forests?

Modification Matrix Tree Theorem

Let $G = (V, E, w)$ be an undirected edge-weighted connected graph, $r_{uv}^{\text{eff}}$ the effective resistance distance between $u, v \in V$ arbitrary vertices and $w(F_v^u)$ the sum of the weights of the 2-trees spanning forests separating $u$ and $v$, then

$$w(F_v^u) = w(T)r_{uv}^{\text{eff}}.$$
Probabilistic Watershed = Random Walker [Grady, 2006]

\[ q + q = q \]

Probabilistic Watershed = Random Walker [Grady, 2006]
Power Watershed new interpretation

Power Watershed [Couprie et al., 2011]

Random Walker [Grady, 2006]

minimize entropy

mSF/Watershed Forest

Counts mSFs

If #mSF = 1

If #mSF > 1
Summary

- Probabilistic Watershed = Random Walker [Grady, 2006].
- New interpretations of the Power Watershed [Couprie et al., 2011].
- Technique to count forests.

\[
\Pr(f) = \frac{\exp(-\mu c(f))}{\sum_{f' \in F_{s_1}^s} \exp(-\mu c(f'))} = \frac{w(f)}{\sum_{f' \in F_{s_1}^s} w(f')}
\]

Poster
Room East Exhibition Hall B + C #81
10:45 AM – 12:45 PM
References

