Learning Bipartite Graphs: Heavy Tails and Multiple Components

a talk by

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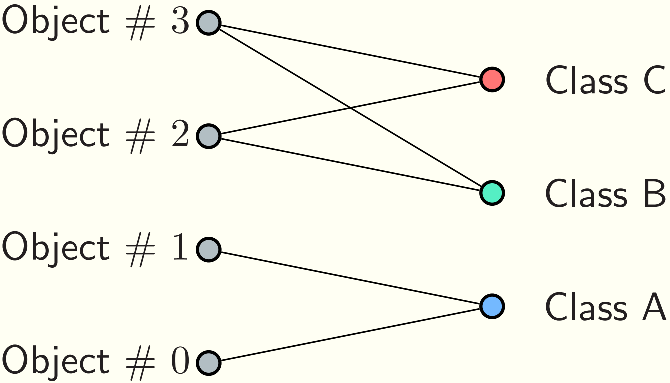
Undirected Weighted Bipartite Graphs



Goal: learn a Laplacian matrix of a bipartite graph from data

State-of-the-art Methods

- **Bipartite Structure**[1]

$$\begin{aligned} \underset{\boldsymbol{B}, \boldsymbol{V} \in \mathbb{R}^{p \times k}}{\text{minimize}} \quad & \|\boldsymbol{B} - \boldsymbol{A}\|_{\mathrm{F}}^2 + \eta \operatorname{tr}\left(\boldsymbol{V}^\top \boldsymbol{L} \boldsymbol{V}\right), \\ \text{subject to} \quad & \boldsymbol{B} \geq \boldsymbol{0},\ \boldsymbol{B}\boldsymbol{1}_q = \boldsymbol{1}_r,\ \boldsymbol{V}^\top \boldsymbol{V} = \boldsymbol{I}_k, \end{aligned}$$

- **Spectral Regularization**[2]

$$\begin{aligned} \underset{\boldsymbol{w} \geq \boldsymbol{0}, \boldsymbol{V}, \boldsymbol{U}, \boldsymbol{\psi}, \boldsymbol{\lambda}}{\text{minimize}} \quad & \operatorname{tr}\left(\mathcal{L}\boldsymbol{w}\boldsymbol{S}\right) - \log\det{}^*\left(\mathcal{L}\boldsymbol{w}\right) \\ & + \frac{\gamma}{2}\left\|\mathcal{A}\boldsymbol{w} - \boldsymbol{U}\mathrm{Diag}(\boldsymbol{\psi})\boldsymbol{U}^\top\right\|_{\mathrm{F}}^2 + \frac{\beta}{2}\left\|\mathcal{L}\boldsymbol{w} - \boldsymbol{V}\mathrm{Diag}(\boldsymbol{\lambda})\boldsymbol{V}^\top\right\|_{\mathrm{F}}^2, \\ \text{subject to} \quad & \boldsymbol{U}^\top\boldsymbol{U} = \boldsymbol{I},\ \boldsymbol{U} \in \mathbb{R}^{p \times p},\ \boldsymbol{\psi} \in C_{\boldsymbol{\psi}},\ \boldsymbol{V}^\top\boldsymbol{V} = \boldsymbol{I},\ \boldsymbol{V} \in \mathbb{R}^{p \times p},\ \boldsymbol{\lambda} \in C_{\boldsymbol{\lambda}}, \end{aligned}$$

where $C_{\boldsymbol{\lambda}}$ and $C_{\boldsymbol{\psi}}$ are convex sets that define the eigenvalues of the Laplacian and adjacency matrices of k-component bipartite graphs

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[1] Nie, F. *et al*. Learning a Structured Optimal Bipartite Graph for Co-Clustering, NeurIPS 2017.

[2] Kumar, S. *et al*. A Unified Framework for Structured Graph Learning via Spectral Constraints, JMLR 2020.

Proposed Formulation

- k-component, bipartite, student-t graph learning:

$$\begin{array}{ll} \underset{\boldsymbol{L}\succeq\boldsymbol{0},\boldsymbol{B}}{\text{minimize}} & \frac{p+\nu}{n}\sum_{i=1}^{n}\log\left(1+\frac{h_i+\operatorname{tr}\left(\boldsymbol{B}\boldsymbol{G}_i\right)}{\nu}\right)-\log\det^{*}\left(\boldsymbol{L}\right), \\ \text{subject to} & \boldsymbol{L}=\begin{bmatrix}\boldsymbol{I}_r & -\boldsymbol{B}\\ -\boldsymbol{B}^\top & \text{Diag}\left(\boldsymbol{B}^\top\boldsymbol{1}_r\right)\end{bmatrix},\ \text{rank}(\boldsymbol{L})=p-k,\ \boldsymbol{B}\geq\boldsymbol{0},\boldsymbol{B}\boldsymbol{1}_q=\boldsymbol{1}_r. \end{array}$$

- we employ the alternating direction method of multipliers (ADMM) and majorization-minimization (MM) to find stationary points of the proposed optimization problems
- see our supplementary material for convergence proofs 😄

Datasets and Benchmark Algorithms

Datasets (Log-returns)

- US Stock Market ($r = 333$ S&P500 stocks $q = 8$ S&P Sector Indices, from Jan. 5th 2016 to Jan. 5th 2021, $n = 1291$ daily observations)

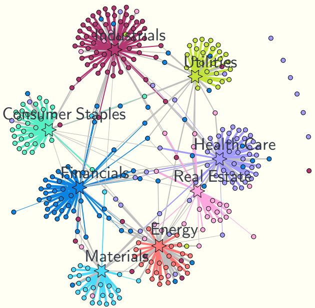
Benchmark Models

- Bipartite structure: SOBG[3]
- Spectral regularization method: SGLA[4]

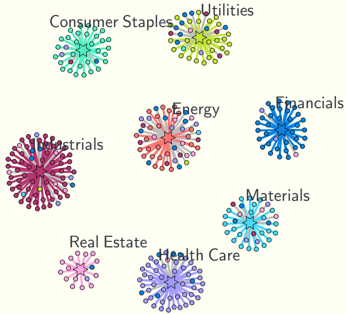
[3] Nie, F. *et al*. Learning a Structured Optimal Bipartite Graph for Co-Clustering, NeurIPS 2017.

[4] Kumar, S. *et al*. A Unified Framework for Structured Graph Learning via Spectral Constraints, JMLR 2020.

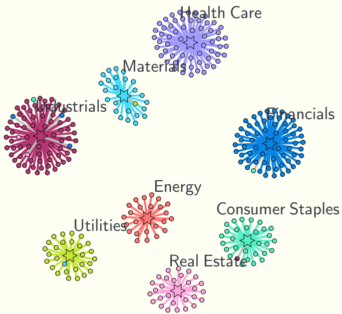
Learned Graphs



(a) SGLA, accuracy $= 0.77$, modularity $= 0.56$



(b) SOBG, accuracy $= 0.75$, modularity $= 0.61$



(c) proposed, accuracy $= 0.97$, modularity $= 0.82$

Thank You!!

- https://github.com/mirca/bipartite
- https://mirca.github.io