

# Every Parameter Matters: Ensuring the Convergence of Federated Learning with Dynamic Heterogeneous Models Reduction

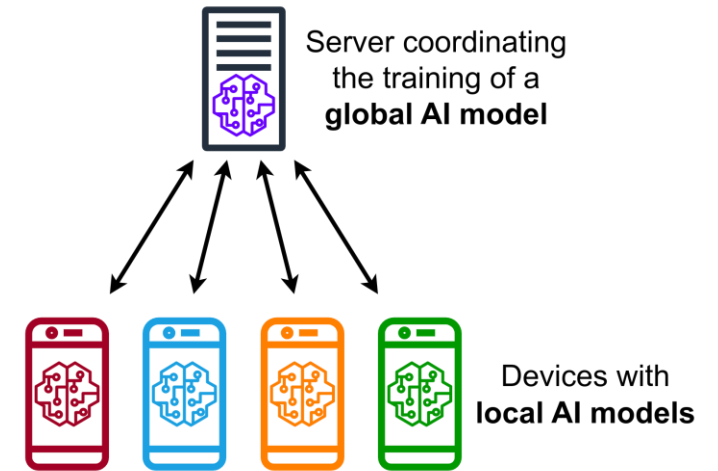
**Hanhan Zhou**<sup>1</sup>, Tian Lan<sup>1</sup>, Guru Venkataramani<sup>1</sup>, Wenbo Ding<sup>2</sup>

<sup>1</sup> The George Washington University

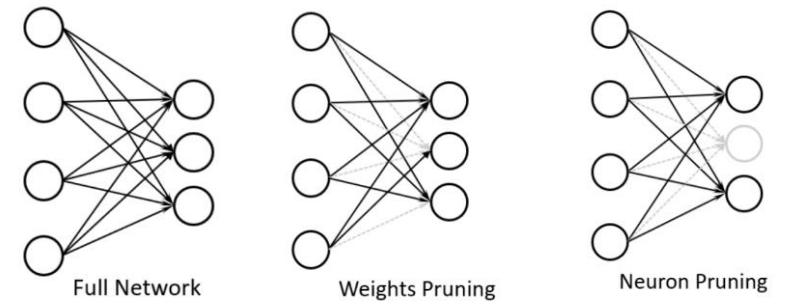
<sup>2</sup> Tsinghua-Berkeley Shenzhen Institute

# Background

- Federated Learning



- Heterogeneous Federated Learning



# Main Problem

- **Given a heterogeneous FL algorithm** that trains a shared global model through a sequence of time-varying and client-dependent local models, what conditions can guarantee its convergence?
- How do the trained models compare to that of standard FL?

# Contributions

$$\theta_{q,n,T}$$

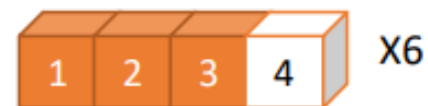
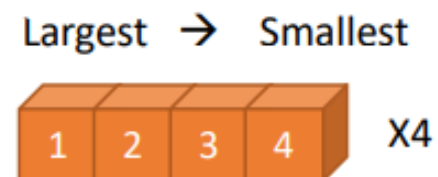
- We establish sufficient conditions for FL algorithms with heterogeneous local models to converge to a neighborhood of a stationary point of standard FL.
- We formulate the problem to allow any model reduction strategy and identify two key factors that impact the convergence:
  - pruning-induced noise
  - minimum coverage index
- The results are numerically validated.

# Convergence Analysis

- Key Notion: minimum covering index

$$\Gamma_{\min} = \min_{q,i} |\mathcal{N}_q^{(i)}|$$

Since  $|\mathcal{N}_q^{(i)}|$  is the number of heterogeneous local models containing the  $i$ th parameter,  $\Gamma_{\min}$  measures the minimum occurrence of the parameter in the local models in all rounds.



$$\Gamma_{\min} = 4$$

(a)

# Convergence Analysis

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**Theorem 1.** *Under Assumptions 1-4 and for arbitrary masks satisfying  $\Gamma_{\min} \geq 1$ , heterogeneous FL converges to a small neighborhood of a stationary point of standard FL as follows:*

$$\frac{1}{Q} \sum_{q=1}^Q \mathbb{E} \|\nabla F(\theta_q)\|^2 \leq \frac{G_0}{\sqrt{TQ}} + \frac{V_0}{Q} + \frac{I_0}{\Gamma_{\min}} \cdot \frac{\delta^2}{Q} \sum_{q=1}^Q \mathbb{E} \|\theta_q\|^2$$

where  $V_0 = 3L^2NG/\Gamma_{\min}$ ,  $I_0 = 3L^2N$ , and  $G_0 = 4\mathbb{E}[F(\theta_0)] + 6LN\sigma^2/\Gamma_{\min}^2$ , are constants depending on the initial model parameters and the gradient noise.

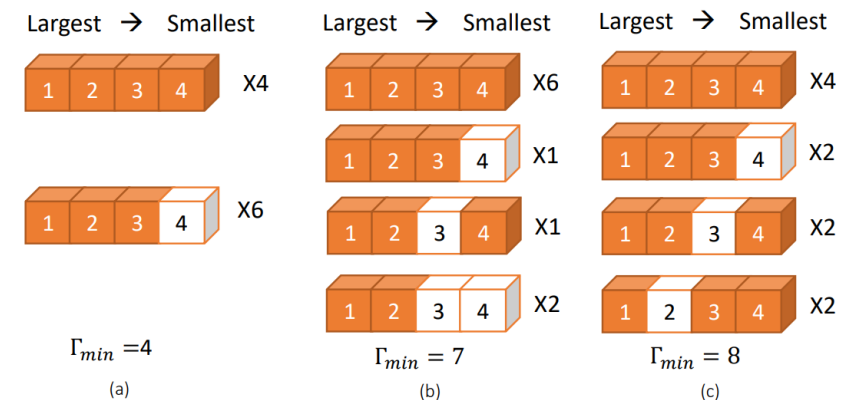
# Insights

- **Every Parameter Matters**

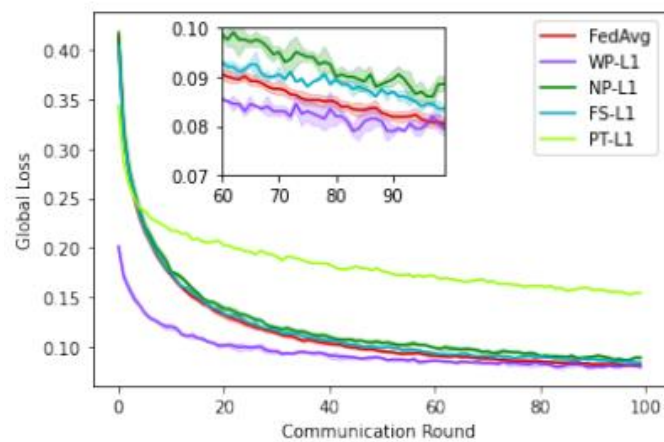
- The analysis shows that as long as each global parameter appears in at least one local model per communication round, heterogeneous federated learning can converge.

- **More coverage leads to faster convergence**

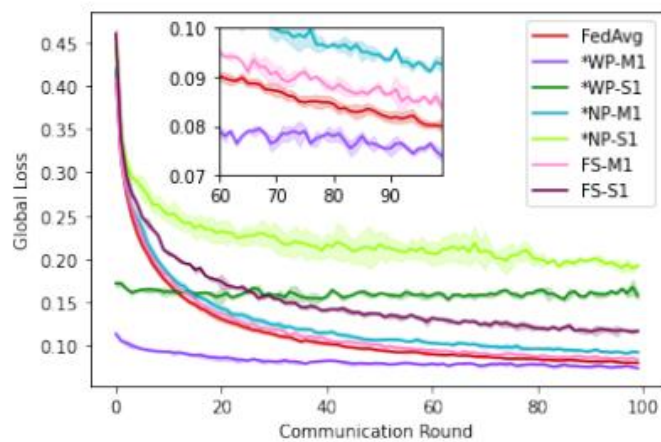
- Instead of pruning greedily for local heterogeneous models, the minimum coverage index should also be considered



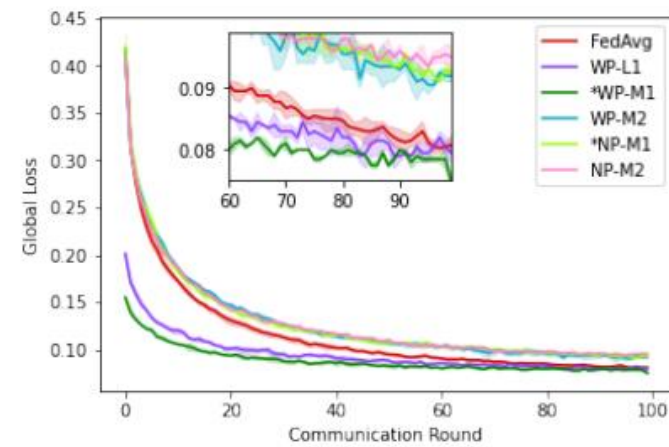
# Experiments



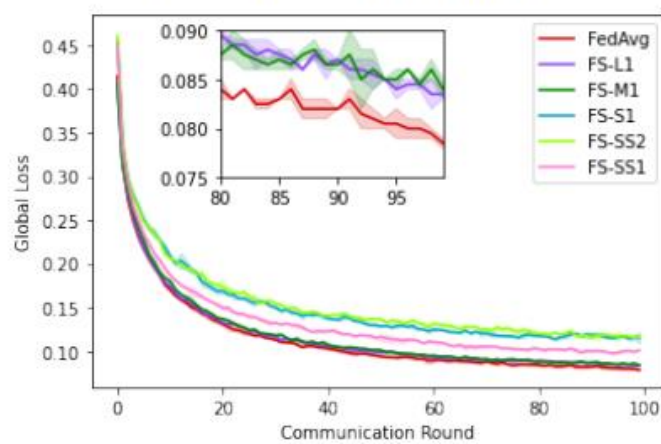
(a) Different Pruning Techniques



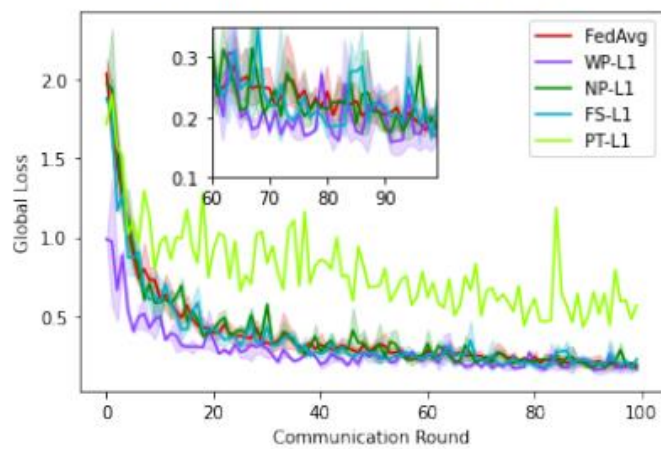
(b) Impact of Pruning Level



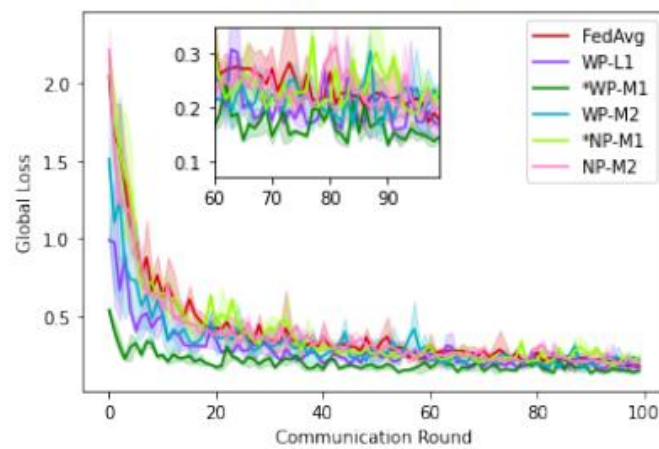
(c) Impact of  $\Gamma_{min}$



(d) Pruning-induced Noise



(e) Pruning Techniques w/ non-IID



(f) Impact of  $\Gamma_{min}$  w/ non-IID



# Conclusion

- We have provided convergence guarantees for heterogeneous federated learning algorithms employing arbitrary, dynamic local models under sufficient conditions.
- The analysis identifies model reduction noise and minimum coverage index as two key factors that impact the convergence gap.
- These insights can guide the design of optimized model reduction strategies to improve convergence.
- Experiments on image classification validate the theory and show optimized strategies guided by the analysis can improve accuracy.