

Limits of Transformer Language Models on Learning to Compose Algorithms

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We investigate how well transformer language models can learn algorithmic compositional tasks

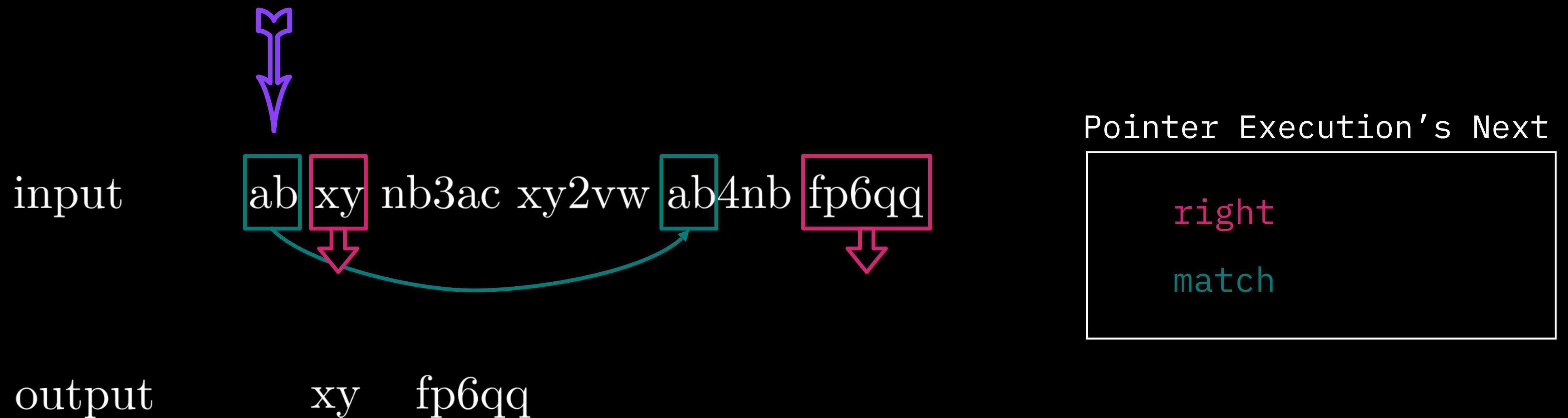
$$\begin{array}{r} \times \quad \boxed{8049} \\ \quad \quad \boxed{11} \\ \hline \boxed{8049} \\ \boxed{8049} \\ \hline \boxed{88539} \end{array}$$

Step-by-step multiplication

digit multiplication

addition

We design new tasks based on pointer execution^[1, 2] to benchmark compositional learning



[1] Abnar et al. Adaptivity and Modularity for Efficient Generalization Over Task Complexity. ArXiv, 2023

[2] Zhang et al. Pointer value retrieval: A new benchmark for understanding the limits of neural network generalization. ArXiv, 2021.

We outline four possible hypotheses to characterise more formally the sample efficiency of the learning process

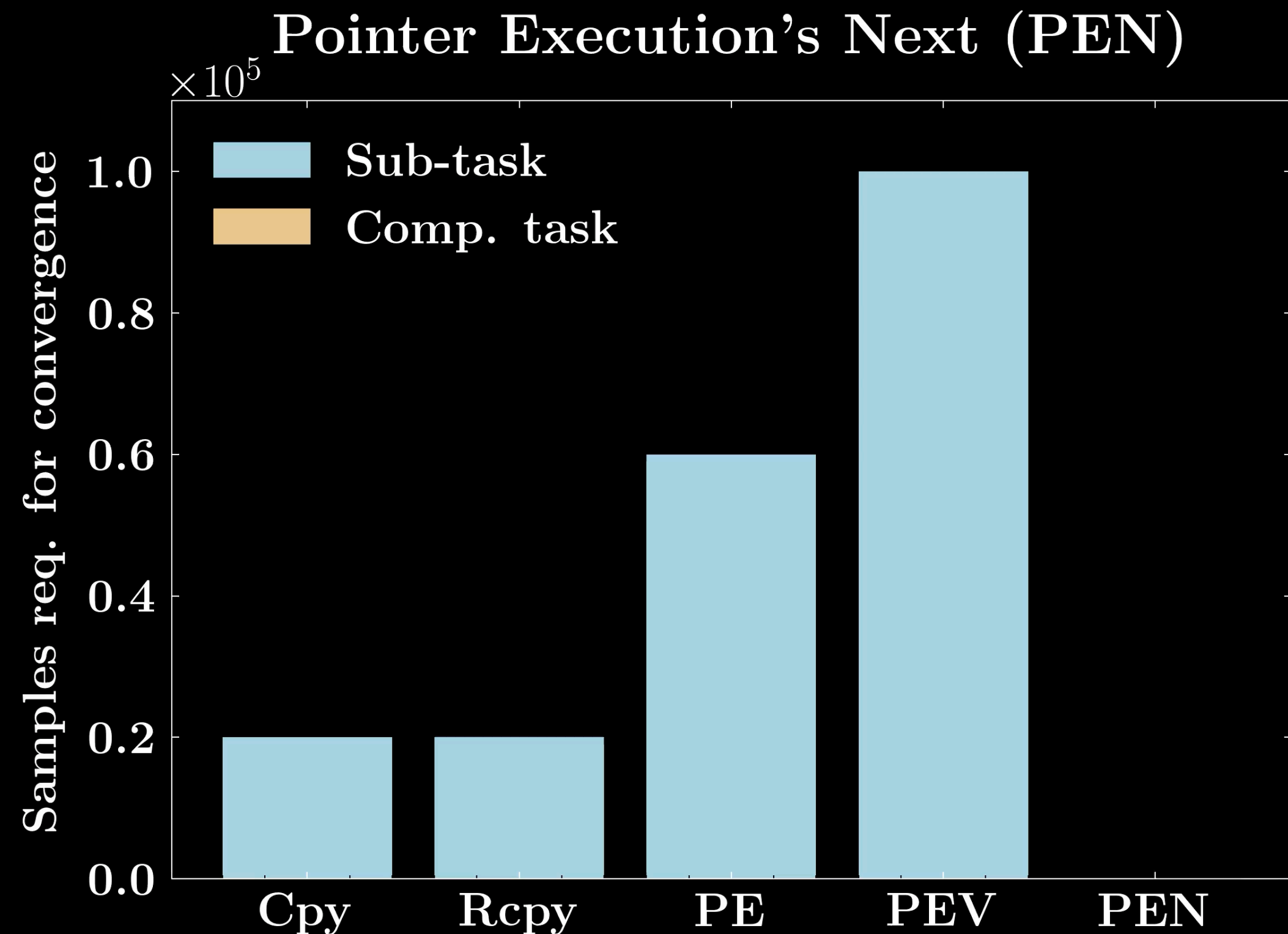
\mathcal{H}_1 constant number of samples

\mathcal{H}_2 fewer samples than those required to learn the most difficult sub-task

\mathcal{H}_3 fewer samples than the sum of samples needed to learn every sub-task

\mathcal{H}_4 more samples than \mathcal{H}_3

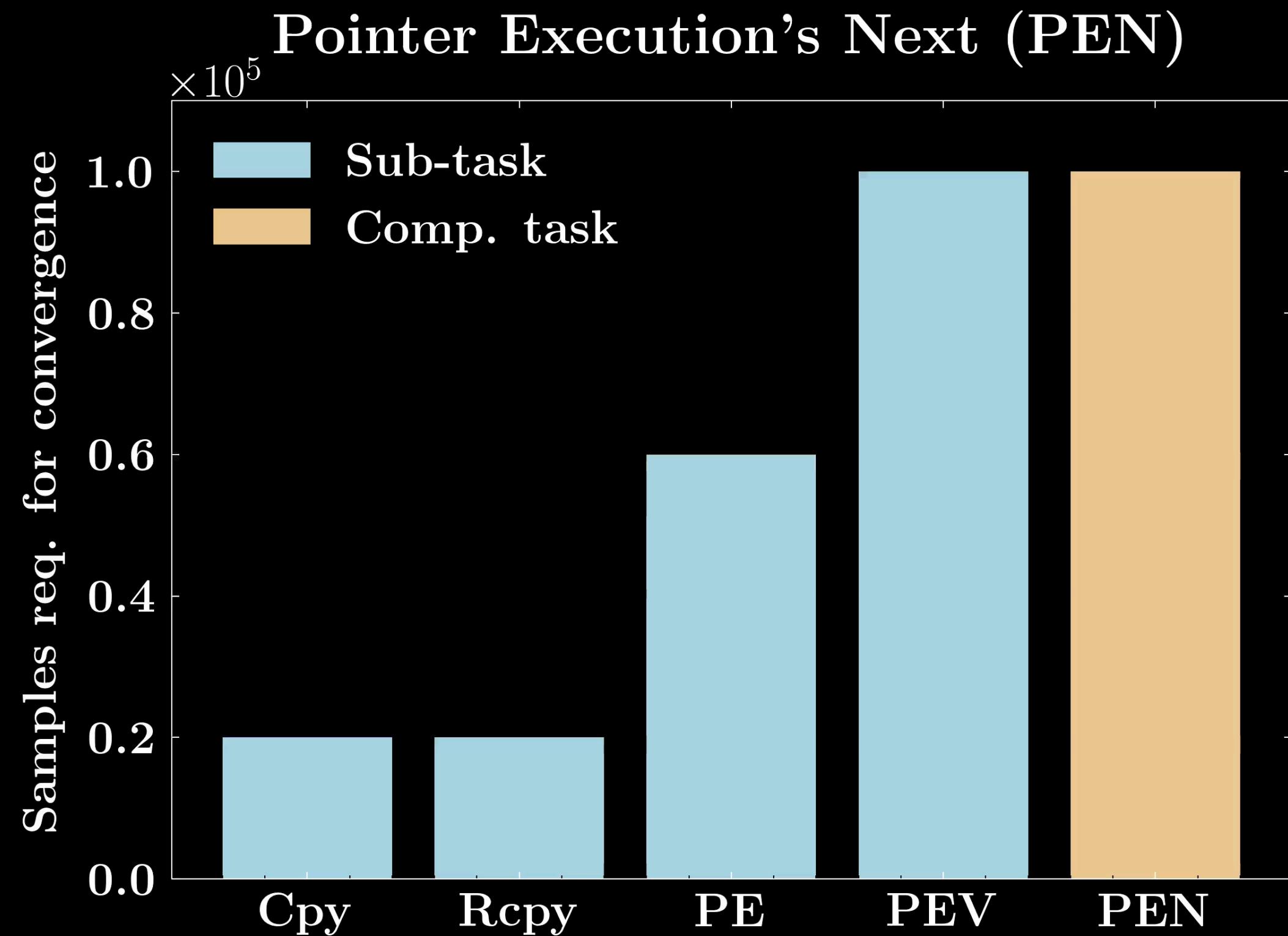
Transformer language models require an exponential number of samples to learn the composition of primitives (\mathcal{H}_4)



\mathcal{H}_1 constant \times

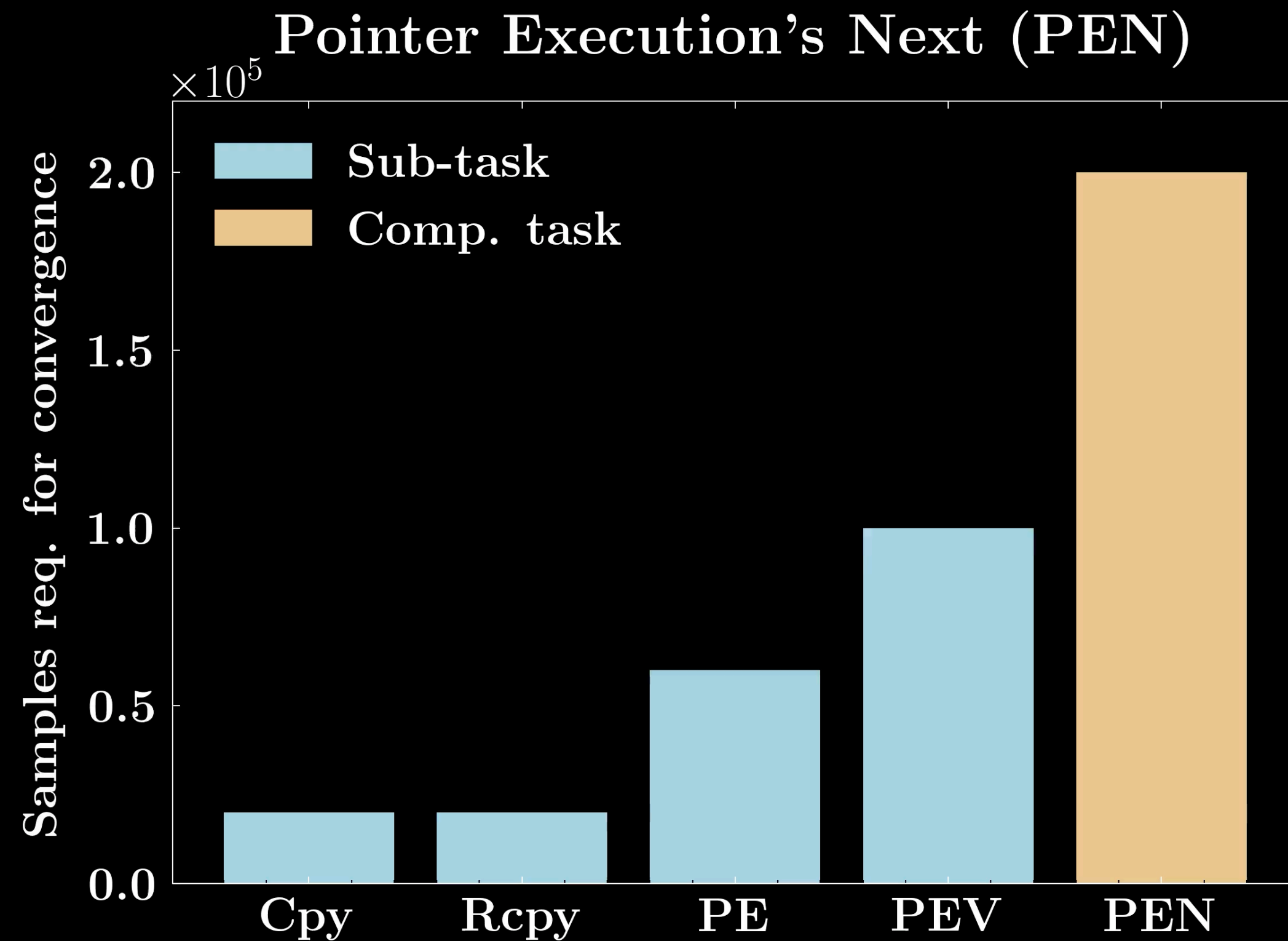
\mathcal{H}_2 $<$ most difficult subtask \times

Transformer language models require an exponential number of samples to learn the composition of primitives (\mathcal{H}_4)



\mathcal{H}_1	constant	✗
\mathcal{H}_2	< most difficult subtask	✗
\mathcal{H}_3	< subtasks sum	✗

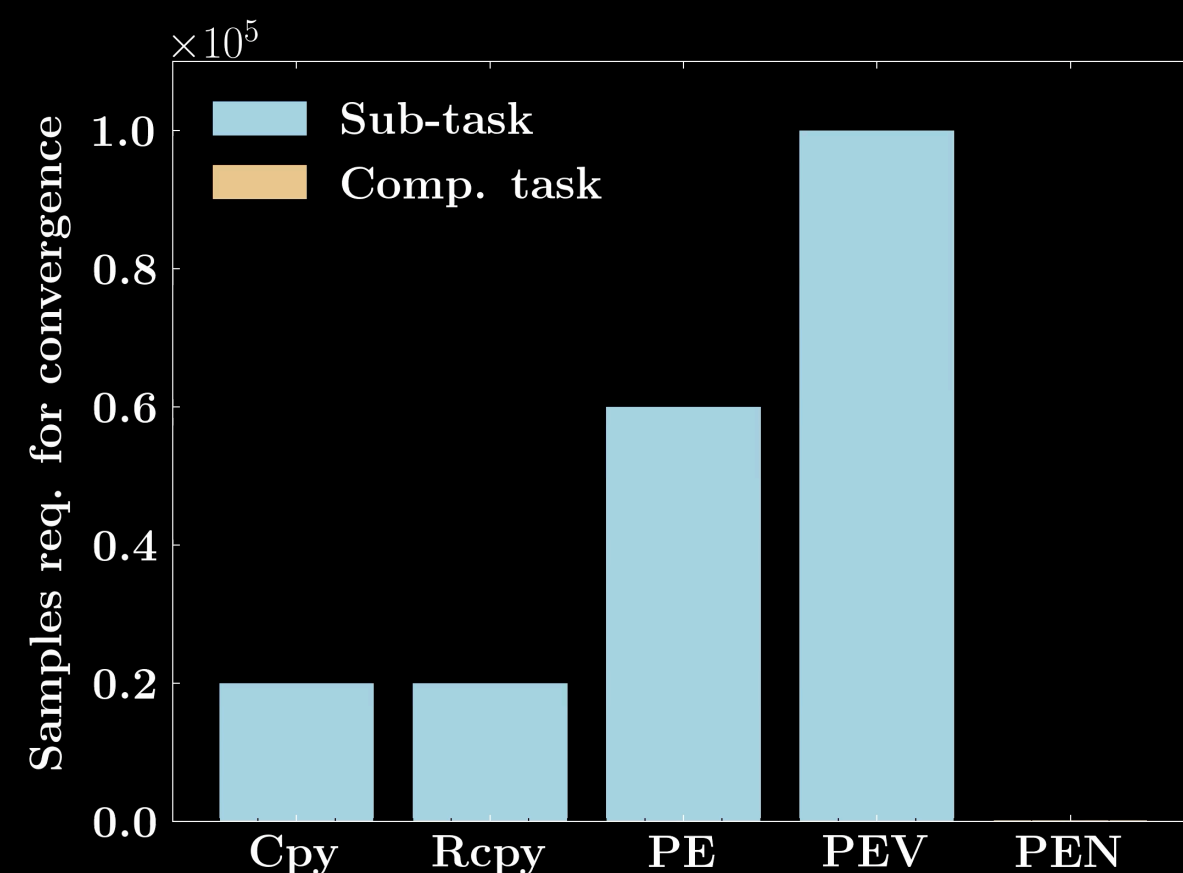
Transformer language models require an exponential number of samples to learn the composition of primitives (\mathcal{H}_4)



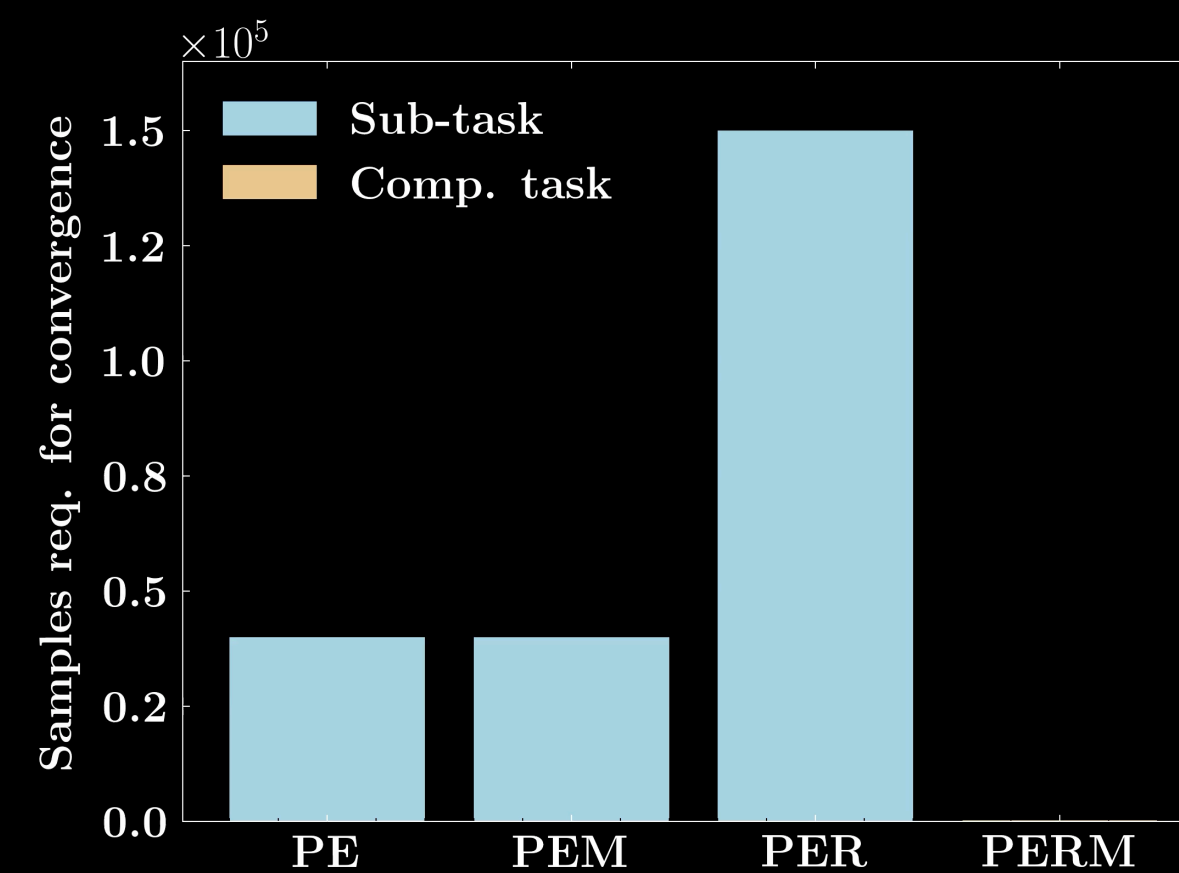
\mathcal{H}_1	constant	✗
\mathcal{H}_2	< most difficult subtask	✗
\mathcal{H}_3	< subtasks sum	✗
\mathcal{H}_4	> \mathcal{H}_3	✓

We observe the same trend across a wide range of compositional algorithmic datasets

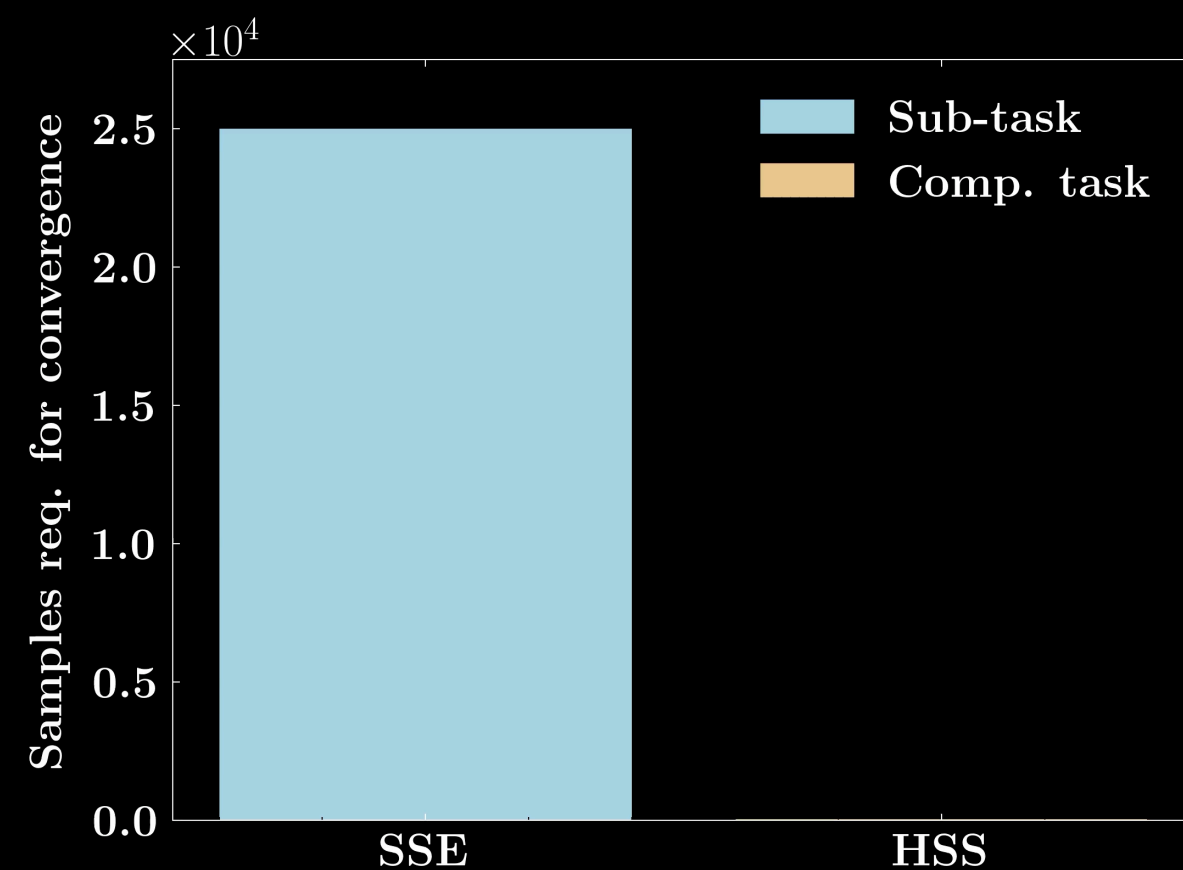
Pointer Execution's Next (PEN)



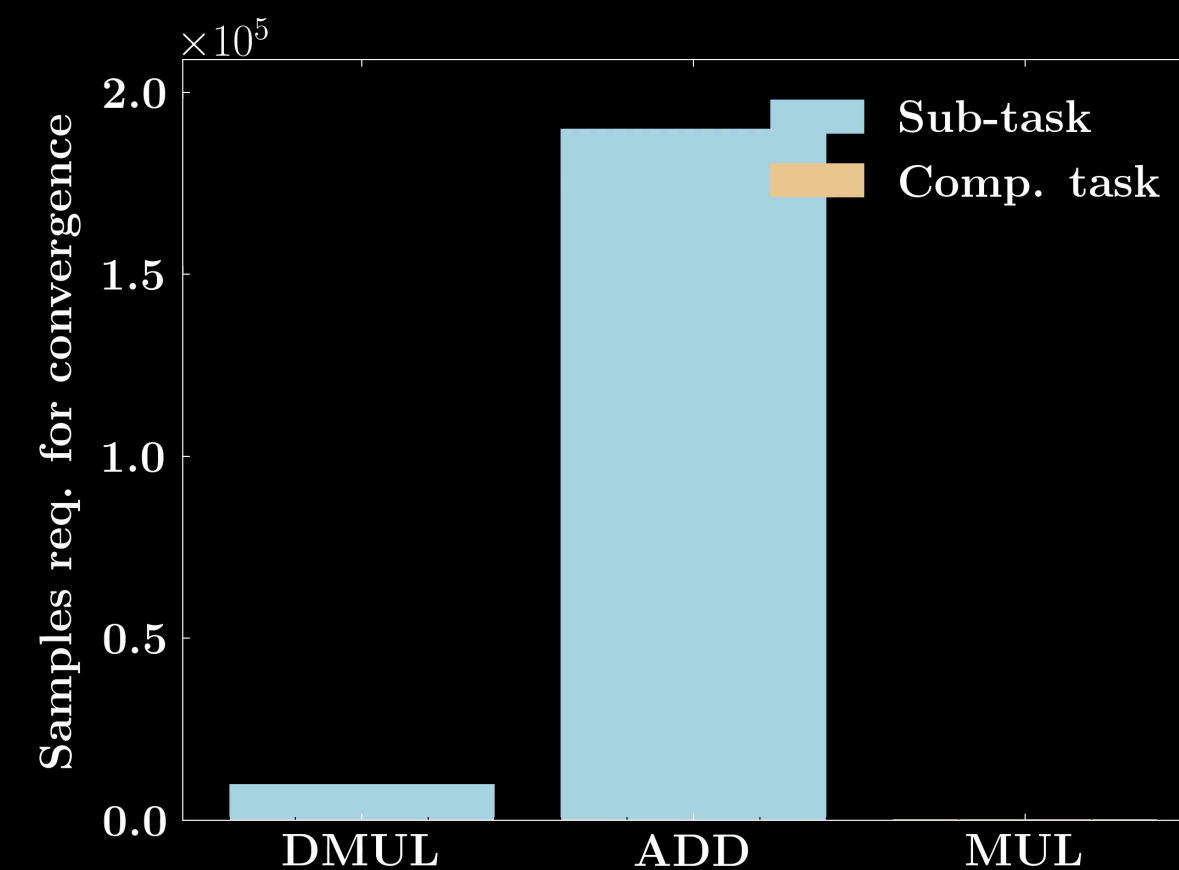
Pointer Execution Reverse Multicount (PERM)



Highest Subsequence Sum (HSS) [Dziri et al., 2022]

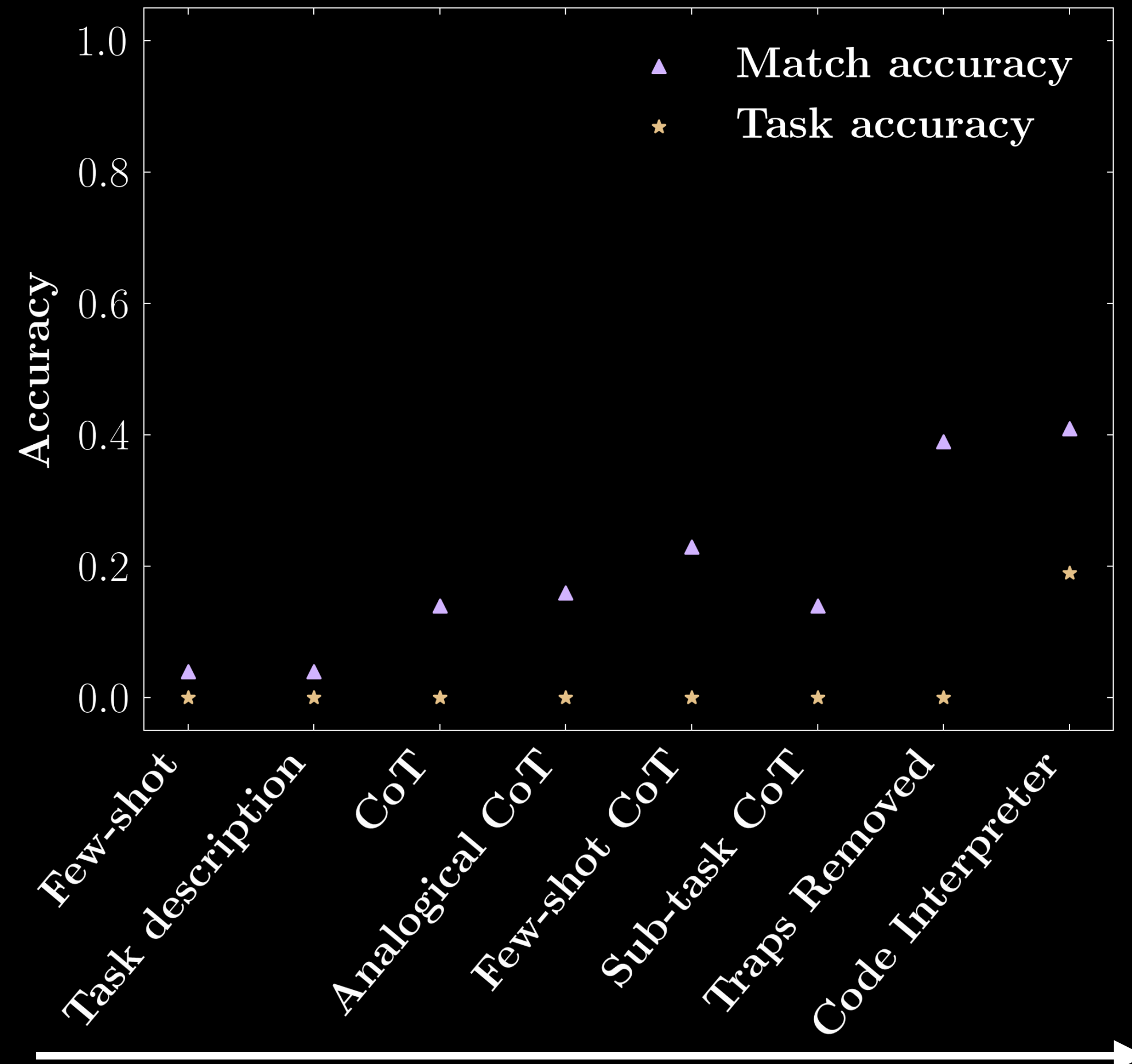


Multiplication (MUL) [Dziri et al., 2023]



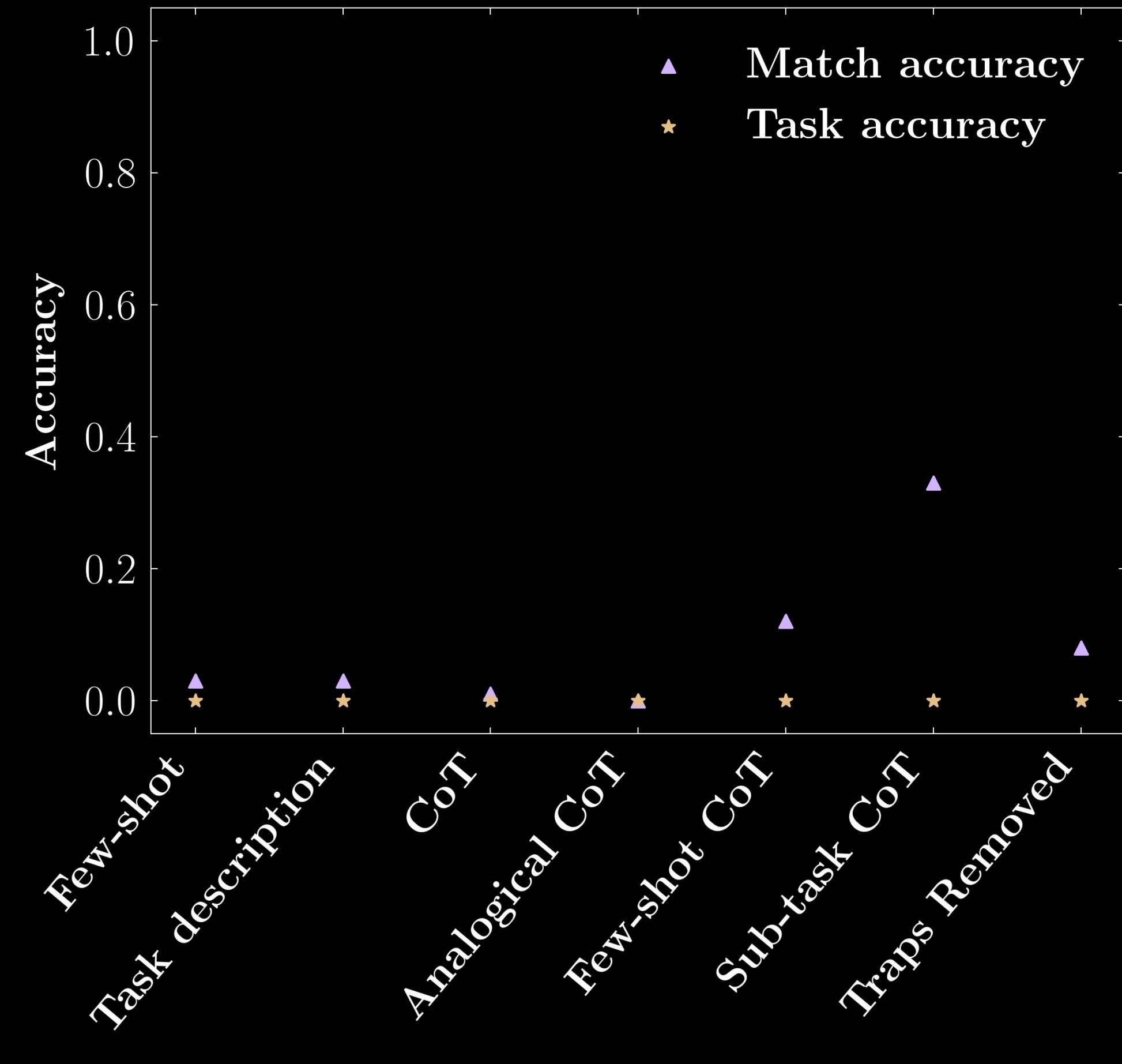
Pre-trained LLMs struggle on these tasks as well

GPT-4 accuracy on PEN



increasing prompt engineering complexity

Gemini-Pro accuracy on PEN



Message: Transformer LMs are inefficient learners of compositions of tasks, requiring more training samples than the sum of those required to learn each task individually.

Paper <https://arxiv.org/abs/2402.05785>

Code <https://github.com/IBM/limitations-lm-algorithmic-compositional-learning>



Paper



Code