



PHILAB



NEURAL INFORMATION
PROCESSING SYSTEMS

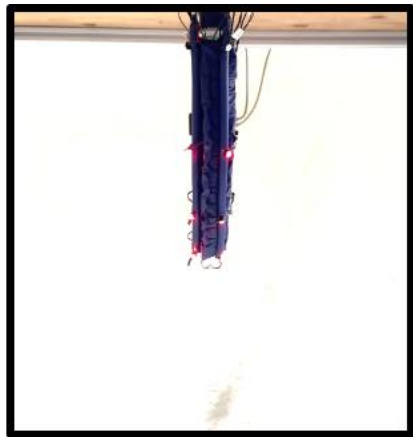
Input-to-State Stable Coupled Oscillator Networks for Closed-form Model-based Control in Latent Space

Maximilian Stölzle and Cosimo Della Santina

Problem setting: Learning Latent-space Dynamics



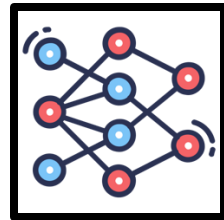
High-dimensional
observation



Encoder



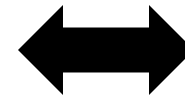
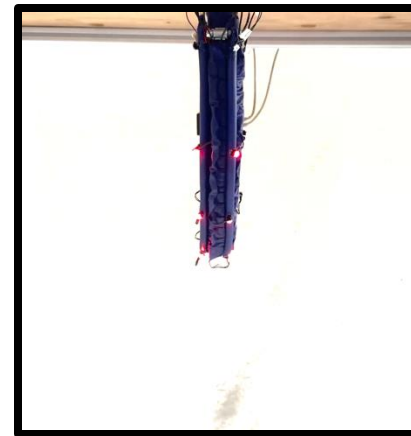
Latent dynamics
(e.g., MLP, RNN)



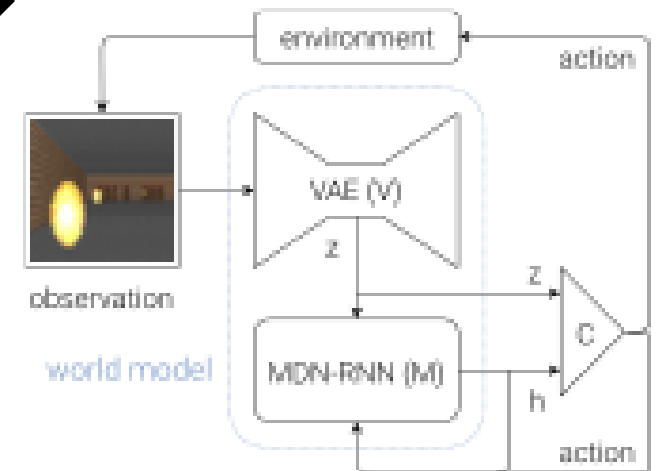
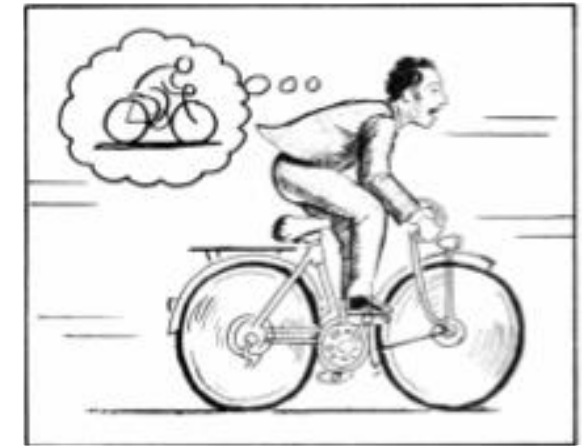
Decoder



Future
observations



World models



Haggerty, D. A., Banks, M. J., Kamenar, E.,
Cao, A. B., Curtis, P. C., Mezić, I., & Hawkes, E.
W. (2023). Control of soft robots with inertial
dynamics. *Science robotics*, 8(81).

Ha, D., & Schmidhuber, J. (2018). World
models. arXiv preprint arXiv:1803.10122.

Limitations of Existing Latent-space Dynamic Models



X No (global) stability guarantees

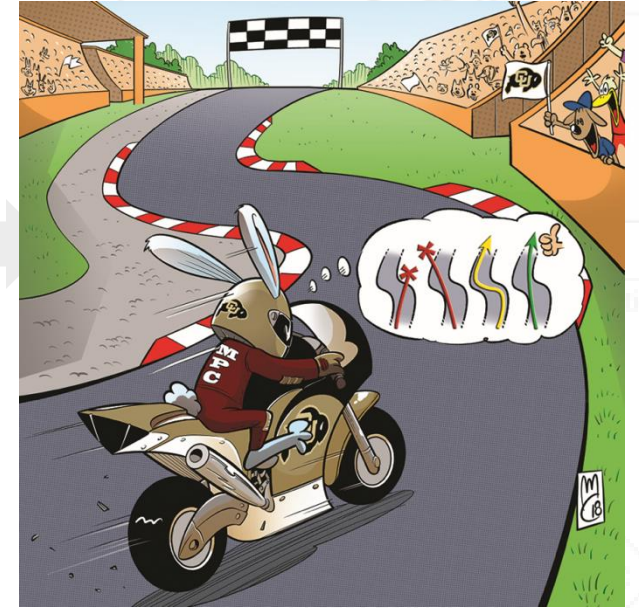
X No interpretation as a mechanical system

X No closed-form control policy
→ only RL, MPC, etc.

Encoder



Decoder



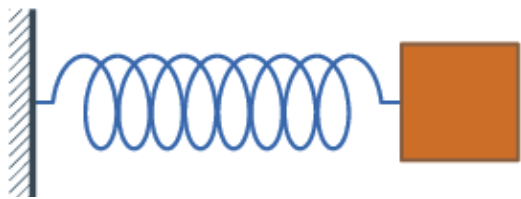
© Engineering Mechanics: by Knowledge flow

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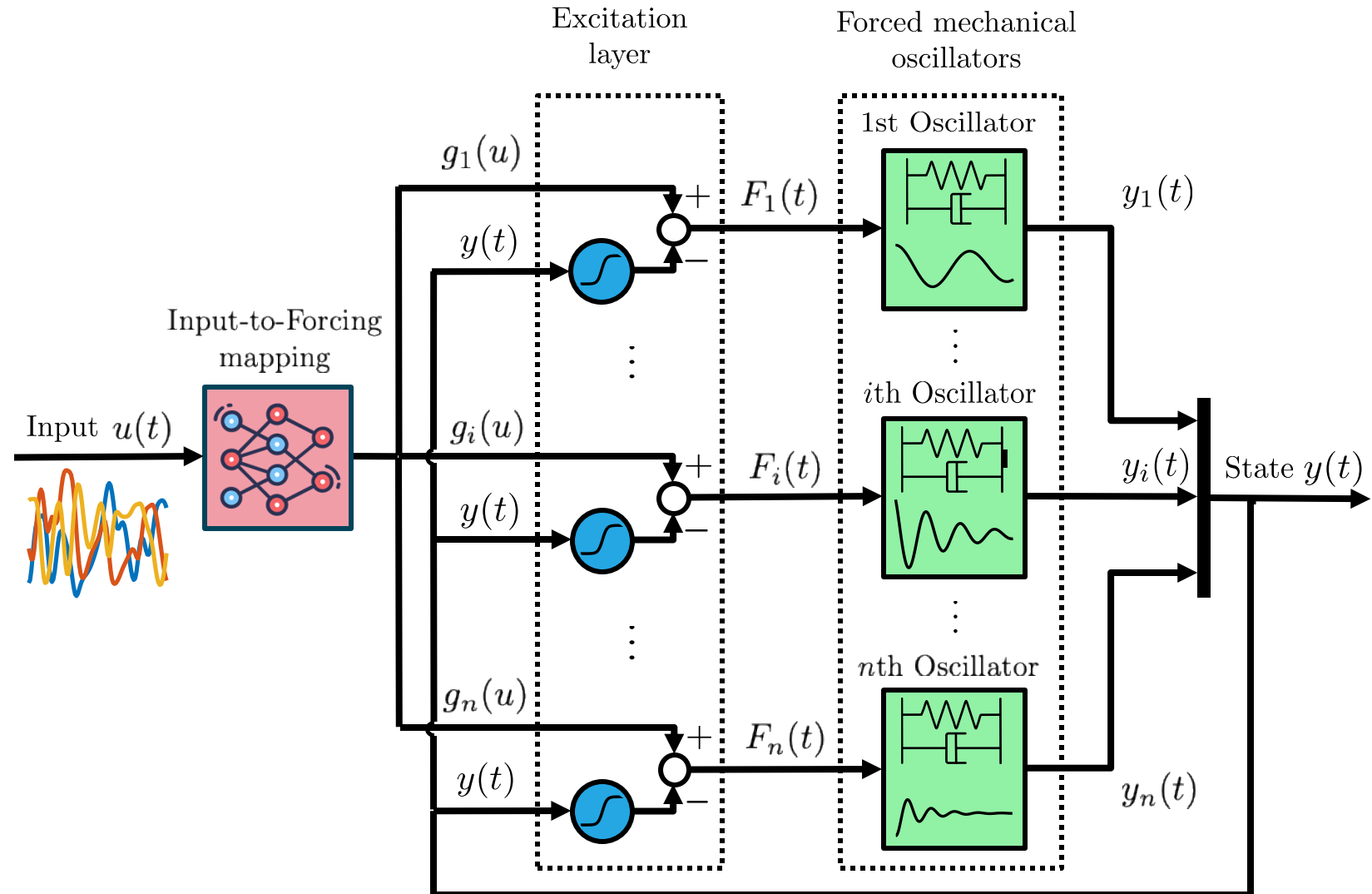
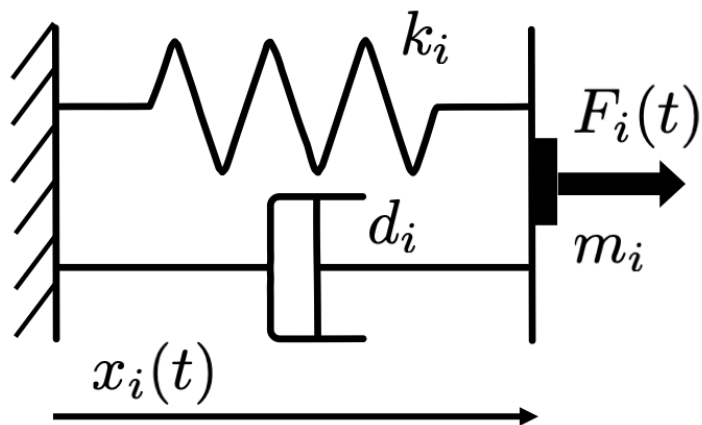
Coupling 1D Harmonic Oscillator Units via a Neuron-like Potential



Harmonic oscillator

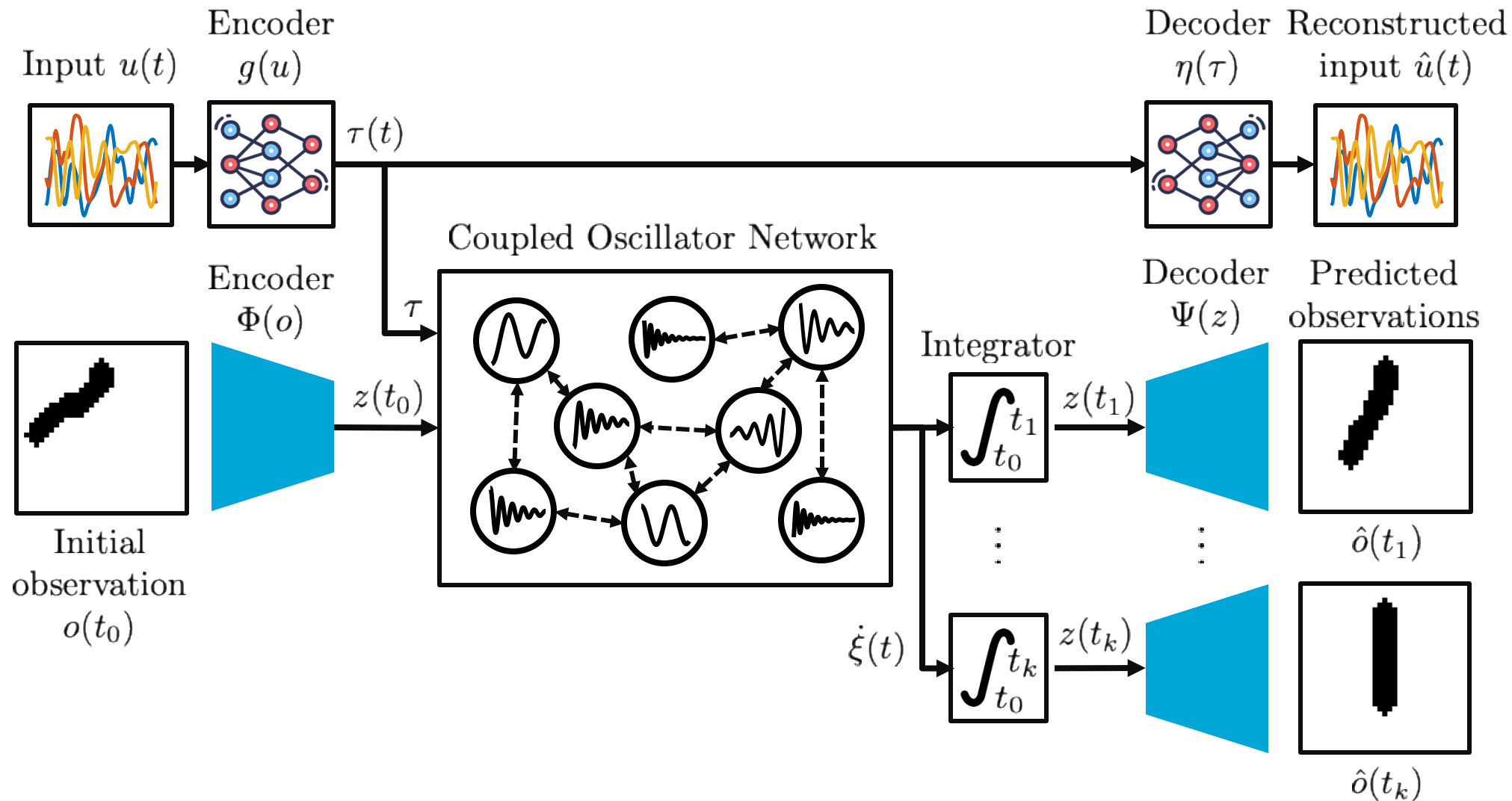


Parameters:



$$\ddot{x} + \kappa x + d\dot{x} + (K - \kappa)x + (D - d)\dot{x} + \tanh(Wx + b) = g(u)$$

Learning Latent-space Dynamics with Coupled Oscillator Networks



Results for Learning Latent Dynamics from Pixels



	Real-world Inspiration	Rendered Dataset Image
Mass-Spring + Friction (M-SP+F)		
Double Pendulum + Friction (D-P+F)		
Continuum Soft Robot (PCC-NS-3)		
Reaction Diffusion (R-D)		

Mass-Spring

Prediction



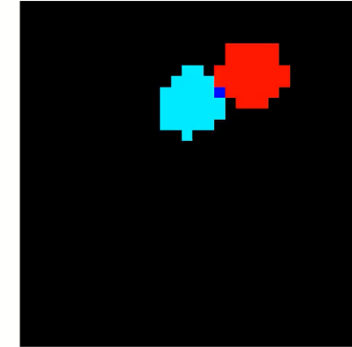
Ground-truth



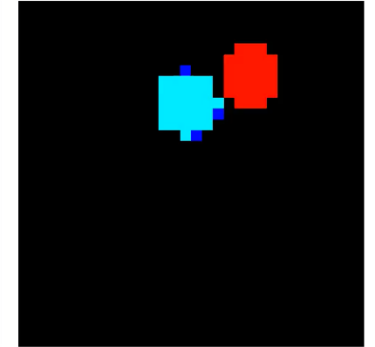
t = 0.05 s

Double Pendulum

Prediction



Ground-truth



t = 0.05 s

Continuum Soft Robot

Prediction



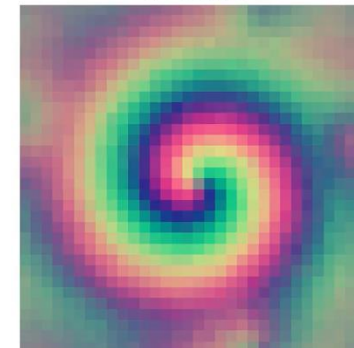
Ground-truth



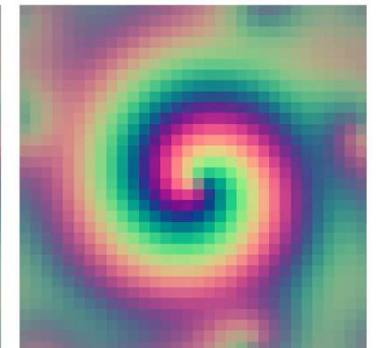
t = 0.02 s

Reaction Diffusion

Prediction



Ground-truth

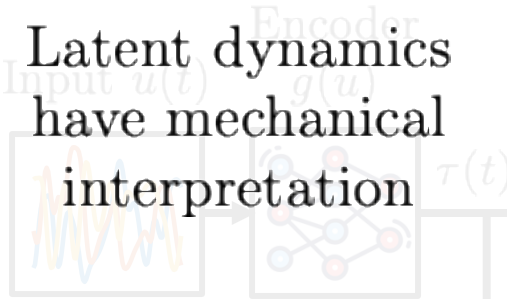


t = 0.05 s

Priors to the Rescue: Benefits of Coupled Oscillator Networks



Latent dynamics
have mechanical
interpretation

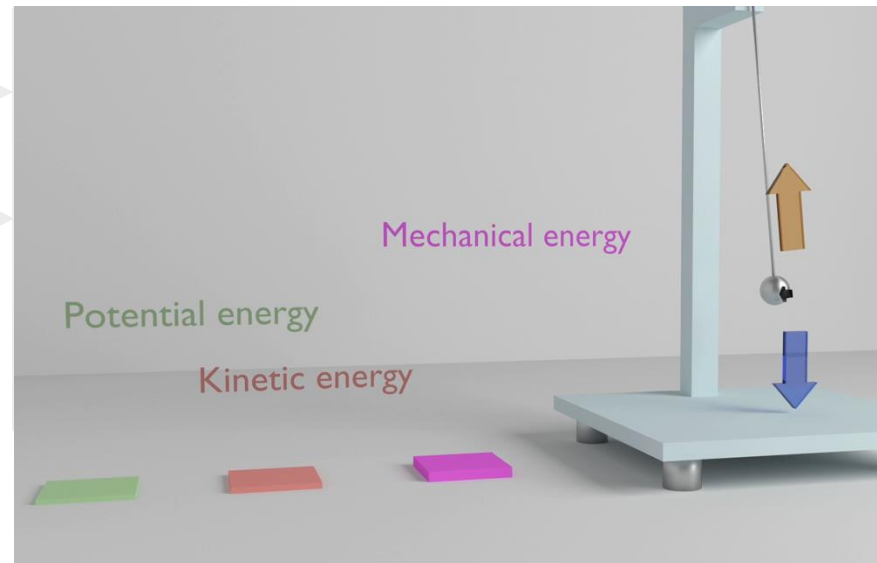
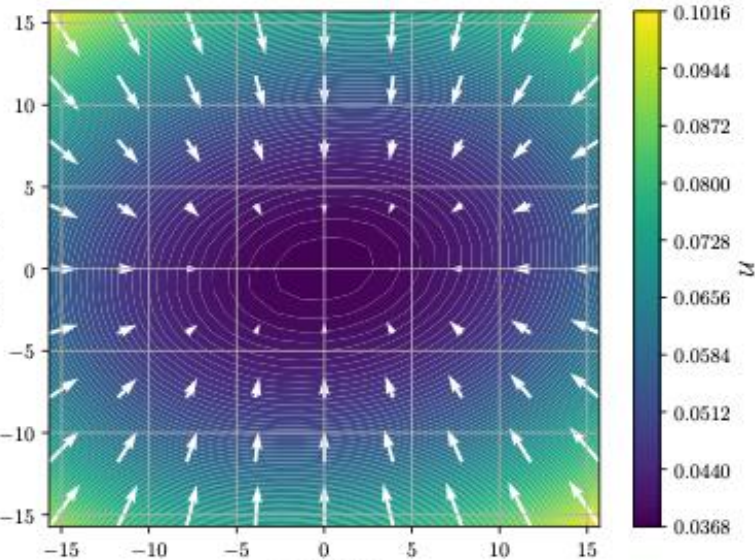


Shared stability
characteristics between
the original and latent
system (GAS / ISS)

Coupled Oscillator Network



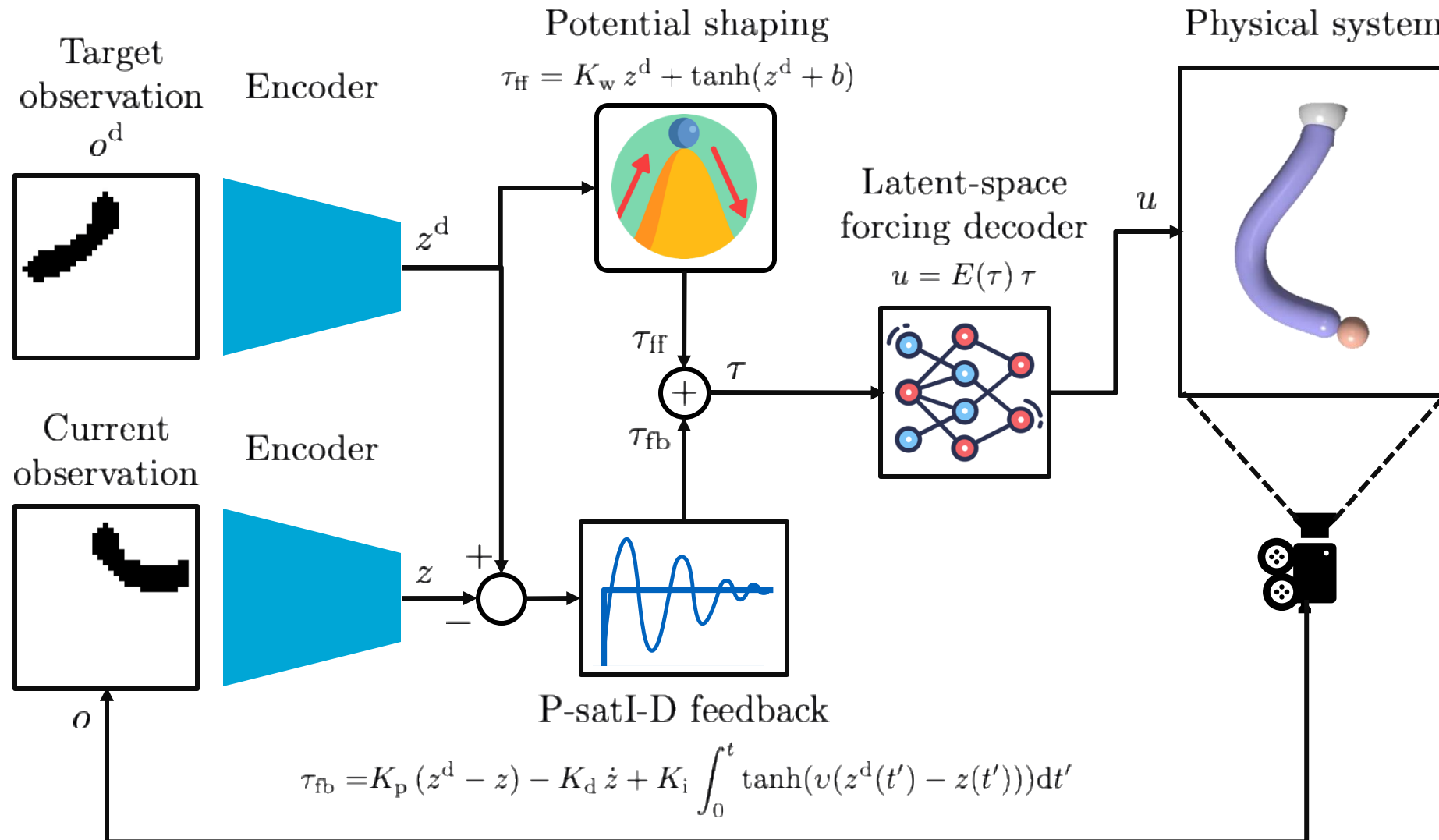
Allows for closed-form
model-based control
in latent-space through
potential shaping



Lee, R. H., Mulder, E. A., & Hopkins, J. B. (2022).
Mechanical neural networks: Architected materials
that learn behaviors. *Science Robotics*, 7(71).

<https://youtu.be/oWiuSp6qAPk>

Exploiting Coupled Oscillator Networks for Control



Results: Latent-space Control



Damped Mass-Spring

Actual behavior



Desired behavior



$t = 0.00$ s

Two-segment Continuum Soft Robot

Actual behavior



Desired behavior



$t = 0.00$ s

Conclusion



Summary:

- ✓ Proposed a new **oscillatory network** that is **Input-to-State (ISS) stable**.
- ✓ Leveraged the **Coupled Oscillator Networks** for learning latent dynamics from pixels.
- ✓ The **structure** of the latent dynamics can be exploited for **stability analysis & model-based control**.
- ✓ **Priors: shared characteristics** between **full- and reduced-order system**
 - Both systems are **mechanical** with **energy-like expressions**.
 - Both systems exhibit a **single, isolated equilibrium**.
 - Both systems **share stability guarantees**: Global Asymptotic Stability (GAS) & Input-to-State Stability (ISS).