

# Advection Augmented Convolutional Neural Networks

Niloufar Zakariaei<sup>1</sup>, Siddharth Rout<sup>1</sup>, Eldad Haber<sup>1</sup>, Moshe Eliasof<sup>2</sup>

<sup>1</sup>University of British Columbia, Vancouver, Canada

<sup>2</sup>University of Cambridge, Cambridge, United Kingdom

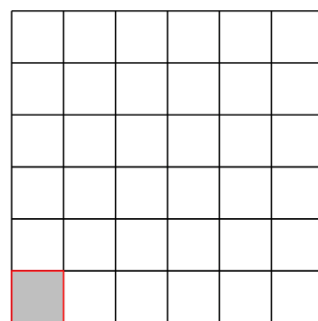
Project Page: <https://github.com/Siddharth-Rout/deepADRnet>

Paper: <https://arxiv.org/abs/2406.19253>

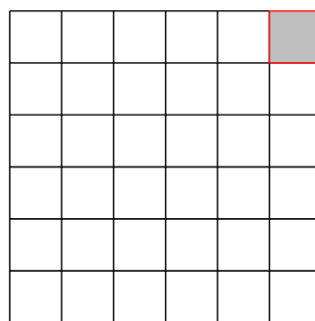


# Introduction and Motivation

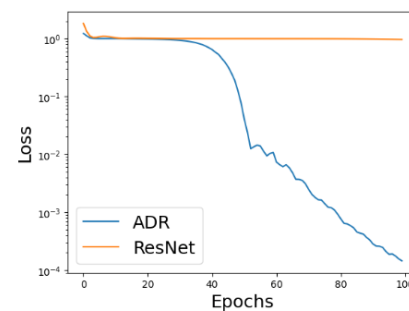
Convolutional Neural Networks (CNNs) struggle with tasks involving long-range information transport, which are common in applications like weather prediction and disease propagation.



(A) Source image



(B) Target image



(C) Convergence

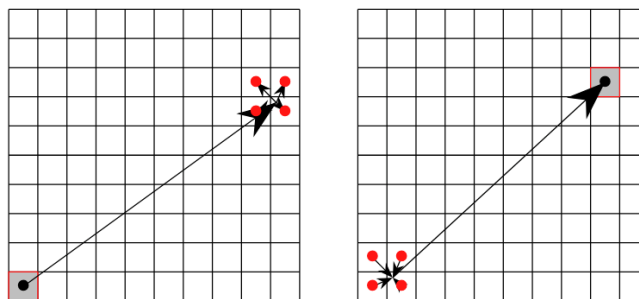
**Need for non-local operations in CNNs to handle such tasks efficiently**

# Key Ideas

## Augment CNNs with Advection, Diffusion, and Reaction (ADR) Process

- **Advection:** Non-local transport of information across the image.
- **Diffusion:** Smooths features locally between neighboring pixels.
- **Reaction:** Enables pointwise interaction among channels.

**Semi-Lagrangian Push Operator:** Allows efficient, non-local advection in a single step.



# ADR Network Structure

- **Overview:** Combines Advection, Diffusion, and Reaction within a CNN to capture complex transport processes in spatio-temporal data.

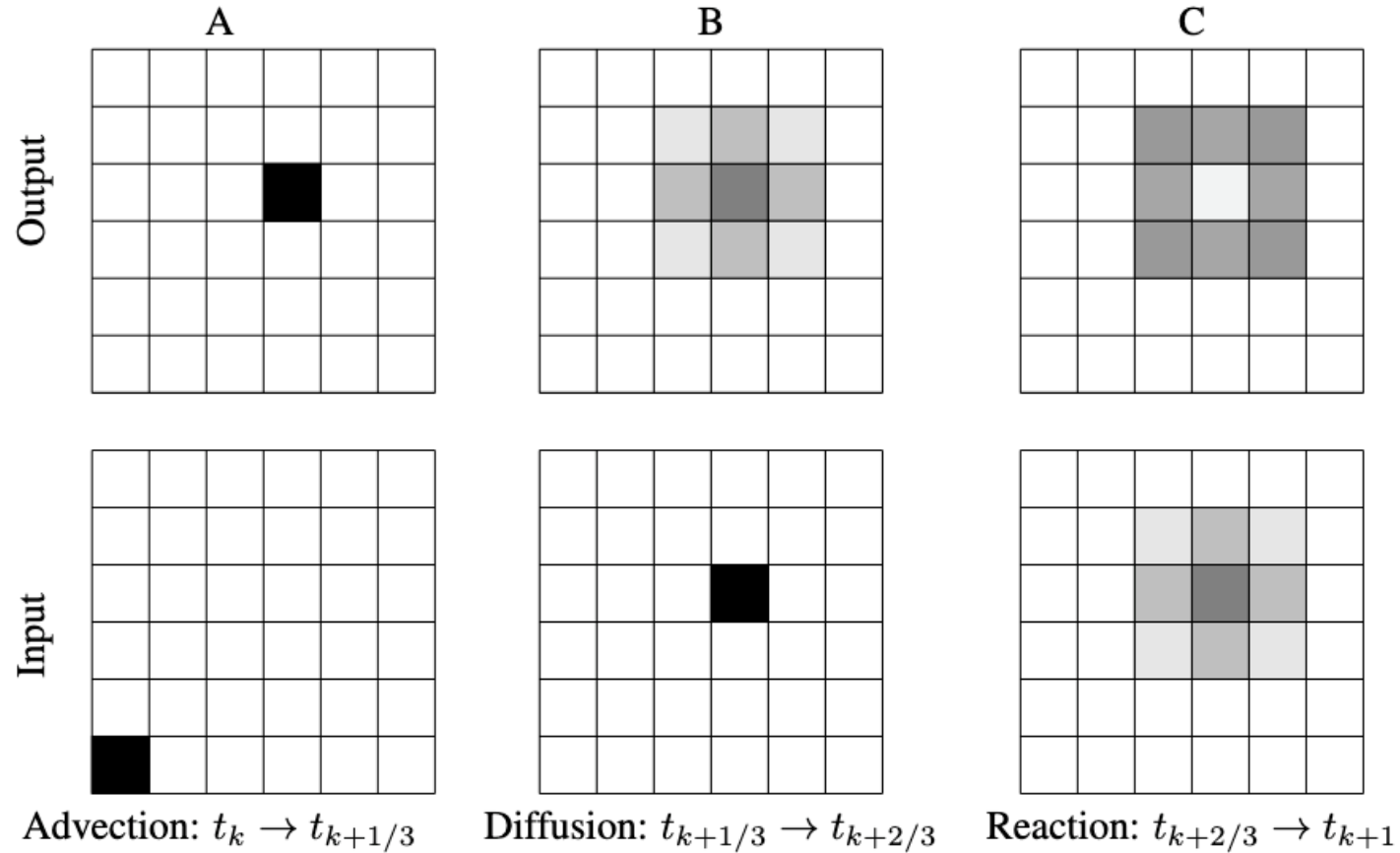
$$\frac{\partial \mathbf{I}(t, \mathbf{x})}{\partial t} = \kappa \Delta \mathbf{I}(t, \mathbf{x}) + \nabla \cdot (\mathbf{U} \mathbf{I}(t, \mathbf{x})) + R(\mathbf{I}(t, \mathbf{x}), \boldsymbol{\theta})$$

**Diffusion**  
Smooths information locally

**Advection**  
Transports non-local  
information

**Reaction**  
Pointwise Interaction

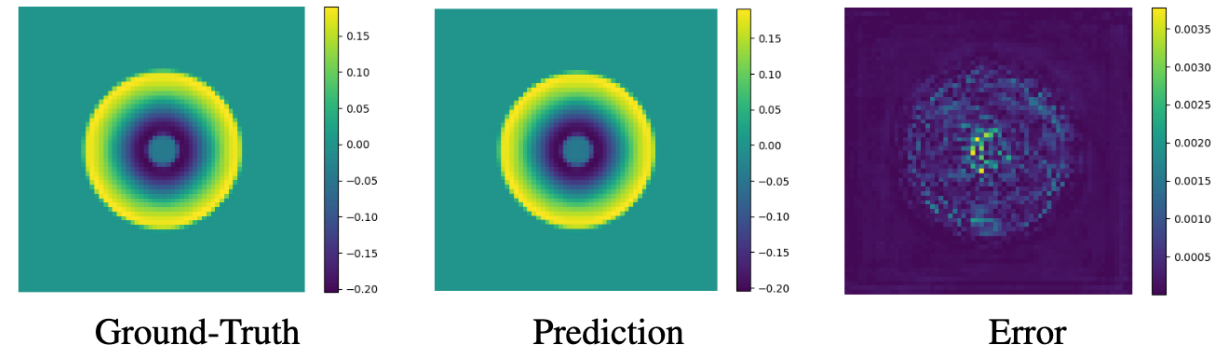
# ADR Network Structure (Cont.)



# Experimental Results

## Performance on Scientific Datasets

- CloudCast dataset
- PDEBench SWE Dataset



Prediction and error for the SWE problem using our ADRNet

Method	NRMSE ↓
UNet [52]	8.3e-2
PINN [52]	1.7e-2
MPP-AVIT-TI [36]	6.6e-3
ORCA-SWIN-B [47]	6.0e-3
FNO [52]	4.4e-3
MPP-AVIT-B [36]	<b>2.4e-3</b>
MPP-AVIT-L [36]	<b>2.2e-3</b>
ADARNet	<b>1.3e-4</b>

Results on PDEBench SWE Dataset

Method	SSIM (↑)	PSNR (↑)
AE-ConvLSTM [70]	<b>0.66</b>	<b>8.06</b>
MD-GAN [67]	<b>0.60</b>	<b>7.83</b>
TVL1 [56]	0.58	7.50
Persistent [70]	0.55	7.41
ADARNet	<b>0.83</b>	<b>38.17</b>

Results on CloudCast dataset

# Experimental Results (Cont.)

## Moving MNIST, KITTI

✓ Competitive on video prediction tasks

Method	MSE ↓	MAE ↓
MSPred [58]	34.4	-
MAU [6]	27.6	-
PhyDNet [19]	24.4	70.3
SimVP [53]	23.8	68.9
CrevNet [69]	22.3	-
TAU [54]	19.8	<b>60.3</b>
SwinLSTM [55]	<b>17.7</b>	-
IAM4VP [46]	<b>15.3</b>	<b>49.2</b>
ADNet	<b>16.1</b>	<b>50.3</b>

Results on Moving MNIST

Method	MS-SSIM ( $\times 10^{-2}$ ) ↑		LPIPS ( $\times 10^{-2}$ ) ↓	
	$t + 1$	$t + 3$	$t + 1$	$t + 3$
SADM [2]	83.06	72.44	14.41	24.58
MCNET [59]	75.35	63.52	24.04	37.71
CorrWise [15]	82.00	N/A	17.20	N/A
OPT [66]	82.71	69.50	12.34	20.29
DMVFN (w/o R) [23]	<b>88.06</b>	<b>76.53</b>	<b>10.70</b>	<b>19.28</b>
DMVFN [23]	<b>88.53</b>	<b>78.01</b>	<b>10.74</b>	<b>19.27</b>
ADNet	<b>85.86</b>	<b>83.62</b>	<b>7.54</b>	<b>9.26</b>

Results on KITTI



# Limitations and Future Work

## Limitations:

- Optimal for Scientific Data, Less effective for complex video prediction tasks requiring generative capabilities
- Niche Use Cases: Best suited for tasks requiring advection

## Future Work:

- Enhanced Generative Capabilities: Adapt for evolving video features.

