# Interpret Your Decision: Logical Reasoning Regularization for Generalization in Visual Classification (NeurIPS24 Spotlight)

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# Generalization settings for visual classification

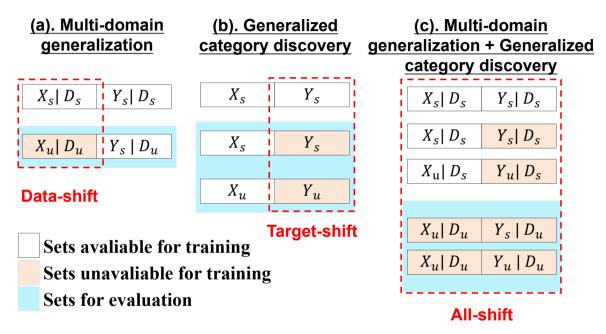


Figure 1: Diagrams of different generalization settings in visual classification tasks.

How can we improve generalization for all these settings? Can we even improve the interpretability with generalization?

# Interpretability and generalization in one: L-Reg

Facing the above questions, we introduce Logic regularization (L-Reg)

$$L_{L-Reg} = -\frac{1}{M} \sum_{i=1}^{M} \left[ \sum_{j=1}^{K} \sigma_{j,i}(\hat{Y}^{T}Z) \log \sigma_{j,i}(\hat{Y}^{T}Z) \right] + \sum_{j=1}^{K} \left[ \frac{1}{M} \sum_{i=1}^{M} \sigma_{j,i}(\hat{Y}^{T}Z) \log(\frac{1}{M} \sum_{i=1}^{M} \sigma_{j,i}(\hat{Y}^{T}Z)) \right], \quad (1)$$

where  $\sigma_{j,i}(\hat{Y}^TZ)$  denotes the value at the i,j position of  $softmax(\hat{Y}^TZ)$  and the soft-max function is applied at the last dimension.

L-Reg improves generalization with interpretability.

# What can L-Reg do? Improving interpretability

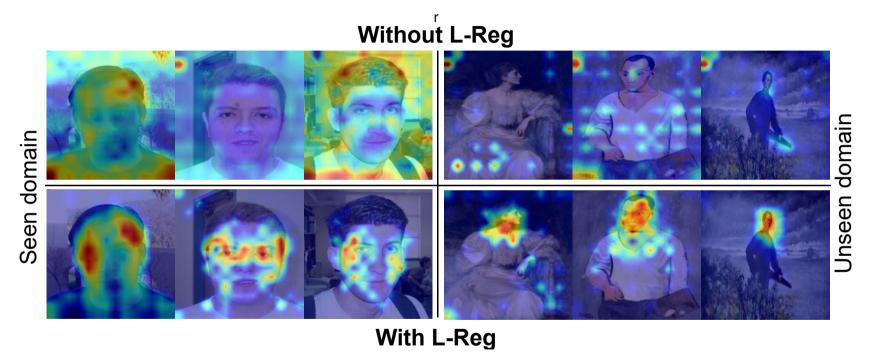


Figure 2: GradCAM [1] visualizations for the unknown class 'person' across seen and unseen domains of the GMDG baseline with *L*<sub>2</sub> regularization that is trained without and with L-Reg, respectively. Both experiments share the same hyper-parameters, except the latter is using the L-Reg.

# What can L-Reg do? Reducing classifier complexity

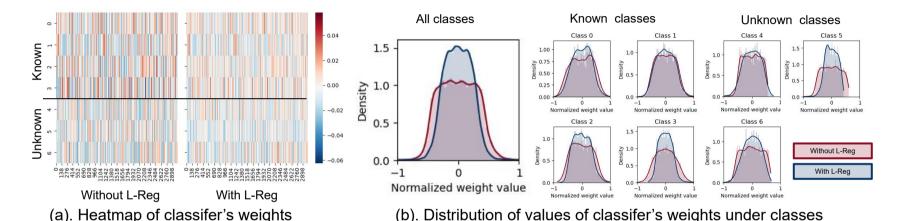


Figure 3: Visualizations of classifiers' weights form models trained using GMDG on PACS dataset without and with L-Reg under mDD+GCD setting, respectively.

Both experiments share the same hyper-parameters using Regnety-16g backbone, except the latter uses additional L-Reg.

# What can L-Reg do? Balancing feature complexity

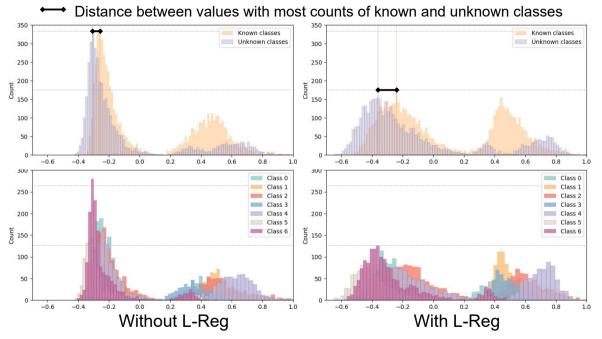


Figure 4: Visualizations of latent features form models trained using GMDG on PACS dataset without and with L-Reg under mDD+GCD setting using RegNetY-16G backbone, respectively.

## Logical analysis framework v.s. visual classification task

#### Definition

Following [2], a logic  $\mathcal{L}$  is a five-tuple defined in the form:

$$\mathcal{L} = \langle F_{\mathcal{L}}, M_{\mathcal{L}}, \models_{\mathcal{L}}, mng_{\mathcal{L}}, \vdash_{\mathcal{L}}, \rangle. \tag{2}$$

- $\blacksquare$   $F_{\mathcal{L}}$ : a set of all formulas of  $\mathcal{L}$ . Images and labels (X, Y) for computer vision cases.
- $M_{\mathcal{L}}$ : a class called the class of all models (or possible worlds) of  $\mathcal{L}$ . Different domains D of X.
- $\models_{\mathcal{L}}$ : a binary relation,  $\models_{\mathcal{L}} \subseteq M_{\mathcal{L}} \times F_{\mathcal{L}}$ , called the validity relation of  $\mathcal{L}$ . In the known set, the ground truth label of the image is given as truth, which is the validity relation.
- $mng_{\mathcal{L}}: F_{\mathcal{L}} \times M_{\mathcal{L}} \longrightarrow \text{Sets}$  where Sets is the class of all sets.  $mng_{\mathcal{L}}$  is a function with domain  $F_{\mathcal{L}} \times M_{\mathcal{L}}$ , called the meaning function of  $\mathcal{L}$ : Classifiers.
- $\vdash_{\mathcal{L}}$  represents the provability relation of  $\mathcal{L}$ , telling us which formulas are 'true' in which possible world and usually is definable from  $mng_{\mathcal{L}}$ . Estimation criteria.

We can correlate the image classification procedure in computer vision with the framework of logic studies perfectly:)

# 'Good general' logic

Following Definition 1, on the given *X*, *Y* sets, we specify:

$$\mathcal{L}_{(X_s, Y_s)} = \left\langle F_{(X_s, Y_s)}, D, \models_{(X_s, Y_s)}, h, \vdash_{(h(X), Y)} \right\rangle. \tag{3}$$

We aim to achieve a good general logic  $\mathcal{L}^*$  from  $\mathcal{L}_{(X_s,Y_s)}$  because:

■ A good general logic has strong generalizability.

By definition, we know that:

 $\blacksquare$   $F_{(g(X_s),Y_s)}$  and h in  $\mathcal{L}^*$  should form the *atomic formulas* to achieve the good general logic.

How to form the atomic formulas?

#### **Definition (Semantic support)**

We denote z = g(x), where  $z \in Z$ , as a set of compositions of these semantics:  $z := \{z^i\}_{i=1}^M$ , where M is the number of dimensions or semantics. Notably, not all semantics in z may be useful for deduction or inference. We define the subset  $\gamma$  of z, extracted from the sample  $x \sim \mathcal{X}$ , as the semantic support of x if  $\gamma$  is sufficient for deducing the relationship between x and a  $y \sim \mathcal{Y}$ .

Semantic supports gained in latent features combining with the classifier from the atomic formulas:  $h(g(x), y, d) \to True/False, s.t., \vdash_{(h \circ g(X), Y)} = \models_{(g(X_s), Y_s)}$ .

Based on the definition of good general logic, we present the constraints of learning semantic supports:

$$\min_{h,g} H(Y|g(\Gamma),D) - H(Y|g(\overline{\Gamma}),D), \tag{4}$$

which derives into Eq.1 as L-Reg.

### Results: MDG results

Table 1: MDG results: Comparison between the proposed and previous non-ensemble and ensemble mDG methods. The best results for each group are highlighted in **bold**. Improvement and degradation in our approach from GMDG are highlighted in **red**.

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Test domain	PACS	VLCS	OfficeHome	Terralncognita	DomainNet	Avg.
MMD [3]	84.7±0.5	77.5±0.9	66.3±0.1	42.2±1.6	23.4±9.5	58.8
Mixstyle [4]	85.2±0.3	77.9±0.5	60.4±0.3	44.0±0.7	34.0±0.1	60.3
GroupDRO [5]	84.4±0.8	76.7±0.6	66.0±0.7	43.2±1.1	33.3±0.2	60.7
IRM [6]	83.5±0.8	78.5±0.5	64.3±2.2	47.6±0.8	33.9±2.8	61.6
ARM [7]	85.1±0.4	77.6±0.3	64.8±0.3	45.5±0.3	35.5±0.2	61.7
VREx [8]	84.9±0.6	78.3±0.2	66.4±0.6	46.4±0.6	33.6±2.9	61.9
CDANN [9]	82.6±0.9	77.5±0.1	65.8±1.3	45.8±1.6	38.3±0.3	62.0
DANN [10]	83.6±0.4	78.6±0.4	65.9±0.6	46.7±0.5	38.3±0.1	62.6
RSC [11]	85.2±0.9	77.1±0.5	65.5±0.9	46.6±1.0	38.9±0.5	62.7
MTL [12]	84.6±0.5	77.2±0.4	66.4±0.5	45.6±1.2	40.6±0.1	62.9
MLDG [13]	84.9±1.0	77.2±0.4	66.8±0.6	47.7±0.9	41.2±0.1	63.6
Fish [14]	85.5±0.3	77.8±0.3	68.6±0.4	45.1±1.3	42.7±0.2	63.9
ERM [15]	84.2±0.1	77.3±0.1	67.6±0.2	47.8±0.6	44.0±0.1	64.2
SagNet [16]	86.3±0.2	77.8±0.5	68.1±0.1	48.6±1.0	40.3±0.1	64.2
SelfReg [17]	85.6±0.4	77.8±0.9	67.9±0.7	47.0±0.3	42.8±0.0	64.2
CORAL [18]	86.2±0.3	78.8±0.6	68.7±0.3	47.6±1.0	41.5±0.1	64.5
mDSDI [19]	86.2±0.2	79.0±0.3	69.2±0.4	48.1±1.4	42.8±0.1	65.1
		Use R	egNetY-16GF [	20] as oracle mode	el.	
MIRO [21] (ECCV23)	97.4±0.2	79.9±0.6	80.4±0.2	58.9±1.3	53.8±0.1	74.1
GMDG [22] (CVPR24)	97.3±0.1	82.4±0.6	80.8±0.6	60.7±1.8	54.6±0.1	75.1
GMDG + L-Reg	<b>97.4</b> ±0.2 <sup>0.1</sup>	<b>82.4</b> ±0.0 <sup>0.1</sup> ↑	<b>80.9</b> ±0.5 <sup>0.1</sup> ↑	<b>62.9</b> ±0.9 <sup>2.2↑</sup>	<b>55.3</b> ±0.0 <sup>0.8</sup> ↑	75.8 <sup>0.7</sup> ↑

Table 2: GCD results: Average results across all datasets of PIM with L-Reg. Improvements and degradation are highlighted in red and blue, respectively.

Average	All	Known	Unknown
K-means [23]	44.7	46.0	43.9
RankStats+ [24] (TPAMI-21)	38.6	54.6	25.6
UNO+ [25] (ICCV-21)	51.2	74.5	36.7
ORCA [26] (ICLR-22)	46.3	51.3	41.2
ORCA - ViTB16	56.7	65.6	49.9
GCD [27] (CVPR-22)	60.4	71.8	52.9
RIM [28] (NeurlPS-10)	62.0	72.5	55.4
TIM [29] (NeurIPS-20)	62.7	72.6	56.4
PIM [30] (ICCV-23)	67.4	79.3	59.9
PIM + L-Reg	68.8 <sup>1.4</sup> ↑	79.0 <sup>0.3↓</sup>	62.7 <sup>2.8</sup> ↑

Table 3: **Results of Congestion prediction:** Congestion prediction is proposed for circuit design.

	pearson	spearman	kendall
Gpdl with UNet++	0.6085	0.5202	0.3855
CircuitFormer (SOTA)	0.6374	0.5282	0.3935
CircuitFormer + L-Reg (Ours)	0.6553	0.5289	0.3944

Table 4: MDG+GCD results: Averaged accuracy scores for all, known and unknown classes across all five datasets. Improvements and degradation are highlighted in red and blue respectively.

Method	Domain gap	All	Known	Unknown
ERM	Not	44.69	59.33	23.54
+L-Reg	minimized	45.50	61.43	21.63
Imp.		0.81	2.09	-1.91
PIM	Not	46.95	60.35	26.90
+L-Reg	minimized	47.27	60.83	26.34
Imp.		0.32	0.48	-0.57
MIRO	Not sufficiently	49.67	68.86	25.79
+L-Reg	minimized	52.11	71.26	26.49
Imp.		2.44	2.39	0.71
GMDG		47.94	68.75	20.68
+L-Reg	Minimized	51.94	69.87	27.68
Imp.		4.00	1.12	7.01

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### Advantages and limitations

# L-Reg forms atomic formulas and improves interpretability.

- For known classes:
  - h(has fingerboard, is guitar,  $d \in D$ )  $\rightarrow$  True
  - h(not has fingerboard, is guitar,  $d \in D$ )  $\rightarrow$  False
- For unknown classes:
  - $h(\text{has a face, is person}, d \in D) \rightarrow \text{True}$
  - h(not has a face, is person,  $d \in D$ )  $\rightarrow$  False

#### Limitations

- It may fail when the domain shift is enormous, e.g., sketch domain where human faces are missing and others.
- One crucial precondition highlighted in the theoretical analysis is that L-Reg operates effectively with a representation Z, where each dimension represents independent semantics.

Table 5: Averaged results of applying L-Reg to different layers across domains in PACS.

	All	Known	Unkown
GMDG	58.33	91.46	10.18
L-Reg: Deep layer	67.82	91.86	31.33
L-Reg: Earlier and the deep layers	58.97	80.73	35.05

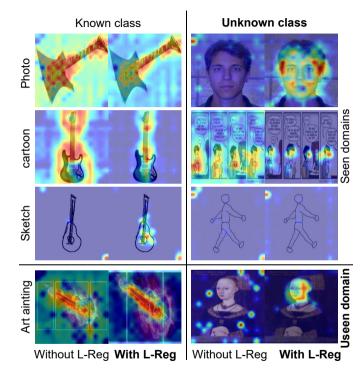


Figure 5: GradCAM visualizations of GMDG trained without and with L-Reg.
The seen, unseen domains and known, unknown classes are denoted.

### Future work

### We provide several possible solutions to the limitations of L-Reg.

- L-Reg should be applied to features from deep layers.
- Constraining the independence of dimensions in *Z* (e.g., using Ortho-Reg).

Table 6: **Results of GCD:** Averaged results across all datasets of PIM with different regularization applied to the latent features: Sparsity: achieved through Bernoulli Sample; Ortho-Reg: orthogonality regularization. +L-Reg outperforms other regularization terms when they are applied solely; +L-Reg+Ortho-Reg achieves the best performance and alleviates the performance degradation of unknown classes, validating our hypothesis in the paper that the improper Z may result in compromises and constraining the independence of each  $z^i \in z, z \in Z$  may be helpful.

	Avg				
	All	Known	Unknown		
PIM	67.4	79.3	59.9		
+Sparsity	66.6	77.3	60.0		
Improvements	-0.7	-2.0	0.1		
+Ortho-Reg	68.4	79.2	61.9		
Improvements	1.0	-0.1	2.0		
+L-Reg	68.8	79.0	62.7		
Improvements	1.4	-0.3	2.8		
+L-Reg+Ortho-Reg	69.3	79.6	63.4		
Improvements	2.0	0.3	3.5		

Table 7: **Results of GCD:** Detailed results across all datasets of PIM with different regularization applied to the latent features: Sparsity: achieved through Bernoulli Sample; Ortho-Reg: orthogonality regularization.

	CUB				Stanford	Cars	Herbarium19		
	All	Known	Unknown	All	Known	Unknown	All	Known	Unknown
PIM	62.7	75.7	56.2	43.1	66.9	31.6	42.3	56.1	34.8
PIM + Sparsity	60.1	72.7	53.8	40.4	61.7	30.1	42.0	53.7	35.8
Improvements	-2.6	-3.0	-2.4	-2.7	-5.2	-1.5	-0.3	-2.4	1.0
PIM + Ortho-Reg	64.9	76.7	58.9	44.3	65.6	34.1	42.9	57.2	35.1
Improvements	2.2	1.0	2.7	1.2	-1.3	2.5	0.6	1.1	0.3
PIM + L-Reg	65.3	76.0	60.0	44.8	66.0	34.6	43.7	55.8	37.2
Improvements	2.6	0.3	3.8	1.7	-0.9	3.0	1.4	-0.3	2.4
PIM + L-Reg + Ortho-Reg	66.8	77.3	61.6	45.8	67.3	35.5	43.3	57.5	35.6
Improvements	4.1	1.6	5.4	2.7	0.4	3.9	1.0	1.4	0.8
<u> </u>		CIFAR	10		CIFAR	100		ImageNe	t-100
	All	CIFAR Known	10 Unknown	All	CIFAR1 Known	I00 Unknown	All	ImageNe Known	t-100 Unknown
PIM	All 94.7			All 78.3					
PIM PIM + Sparsity		Known	Unknown		Known	Unknown	All	Known	Unknown
	94.7	Known 97.4	Unknown 93.3	78.3	Known 84.2	Unknown 66.5	All 83.1	Known 95.3	Unknown 77.0
PIM + Sparsity	94.7 94.2	97.4 97.4	93.3 92.6	78.3 79.7	Known 84.2 <b>84.6</b>	Unknown 66.5 69.7	All 83.1 83.4	95.3 93.7	77.0 78.2
PIM + Sparsity Improvements	94.7 94.2 -0.5	97.4 97.4 0.0	93.3 92.6 -0.7	78.3 79.7 1.4	84.2 84.6 0.4	Unknown 66.5 69.7 3.2	All 83.1 83.4 0.3	95.3 93.7 -1.6	77.0 78.2 1.2
PIM + Sparsity Improvements PIM + Ortho-Reg	94.7 94.2 -0.5 95.1	97.4 97.4 0.0 97.4	Unknown 93.3 92.6 -0.7 93.9	78.3 79.7 1.4 80.2	84.2 84.6 0.4 84.6	Unknown 66.5 69.7 3.2 71.4	83.1 83.4 0.3 83.0	95.3 93.7 -1.6 93.4	77.0 78.2 1.2 77.7
PIM + Sparsity Improvements PIM + Ortho-Reg Improvements	94.7 94.2 -0.5 95.1 0.4	97.4 97.4 0.0 97.4 0.0	93.3 92.6 -0.7 93.9 0.6	78.3 79.7 1.4 80.2 1.9	84.2 84.6 0.4 84.6 0.4	Unknown 66.5 69.7 3.2 71.4 4.9	All 83.1 83.4 0.3 83.0 -0.1	95.3 93.7 -1.6 93.4 -1.9	77.0 78.2 1.2 77.7 0.7
PIM + Sparsity Improvements PIM + Ortho-Reg Improvements PIM + L-Reg	94.7 94.2 -0.5 95.1 0.4 94.8	97.4 97.4 0.0 97.4 0.0 97.6	93.3 92.6 -0.7 93.9 0.6 93.4	78.3 79.7 1.4 80.2 1.9 80.8	Known 84.2 84.6 0.4 84.6 0.4 84.6	Unknown 66.5 69.7 3.2 71.4 4.9 73.2	All 83.1 83.4 0.3 83.0 -0.1 83.4	95.3 93.7 -1.6 93.4 -1.9 94.0	77.0 78.2 1.2 77.7 0.7 78.0

### Wait! You may also be interested in ...

#### Our previous studies in generalization:

- Multi-domain generalization from statistical perspective:

  Rethinking Multi-domain Generalization with A General Learning Objective (CVPR24). [22]
- An augmentation framework for enhancing generalization in text2image generation that based on group theory: Semantic-Aware Data Augmentation for Text-to-Image Synthesis (AAAI24). [31]

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Thank You!