Covariate Shift Corrected Conditional Randomization Test

Bowen Xu, Yiwen Huang, Chuan Hong, Shuangning Li, Molei Liu

Neural Information Processing System 2024

November 12, 2024

Motivation

(a) Source population.

(b) Target population.

Figure: Possible differences between the source and the target populations.

- **Conditional Independence Test:**
  - $H_0 : X \perp\!\!\!\perp Y \mid Z$
- **Covariate Shift Issue**
  - Difference between source and target population
  - Current Approach: Importance Sampling
- **Our Approach:**
  - Novel test within model-X framework
  - Incorporate density ratio for valid test procedure

Methodology And Key Ideas

- **Counterfeit Sampling**
  - For each datapoint (X_i, Y_i, Z_i), counterfeits $(X_i^{(t)}, Y_i, Z_i)$ are sampled from the distribution $X_i^{(t)} \sim p_T(X \mid Z)$.
- **Scoring and Labeling**
  - A label l_i is assigned to each data point based on its score among all the counterfeit scores through some scoring function
- **Uniformity Testing**
  - Calculate the weighted sum of scores $W_\ell = \sum_{i=1}^{n} w_i \cdot \mathcal{I}\{\ell_i = \ell\}$ with w_i as the density ratio, and the test statistic $U_{n,L} = \sum_{l=1}^{L} \left(W_\ell - \frac{n}{L}\right)^2$

Theorem (Valid Tests)

Under conventional assumptions, assume that the null hypothesis of $X \perp\!\!\!\perp Y \mid Z$ holds in the target population, then

$$\lim_{n \to \infty} \mathbb{P}[\text{Algorithm rejects}] = \alpha \tag{1}$$

Power Enhancement

- **Variance Reduction**
  - We define $\widetilde{W}_\ell$ with control variate a and parameter $\widehat{\gamma}_\ell$

$$\widetilde{W}_\ell = \sum_{j=1}^{n} w_j \left[\mathcal{I}\{\ell_i = \ell\} - \widehat{\gamma}_\ell \cdot a(X_i, Y_i, Z_i)\right] + n\widehat{\gamma}_\ell E[a(X, Y, Z)]$$

  - For arbitrary a, we obtain $\widehat{\gamma}_\ell$ by

$$\gamma_\ell = \frac{Cov[w_j \mathcal{I}[\ell_j = \ell], w_j a(X_j, Z_j, V_j)]}{Var[w_j a(X_j, Z_j, V_j)]} \tag{2}$$

Theorem (Variance Reduction)

Let W_l be the statistics computed in line 10 in Algorithm 2, and $\widetilde{W}_l$ be the statistics computed in Algorithm 2. Under conventional assumptions,

$$\limsup_{n\to\infty} \left(\frac{Var[\widetilde{W}_\ell]}{Var[W_\ell]} \right) \leq 1. \tag{3}$$

Simulation Results

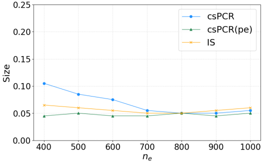
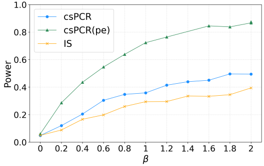
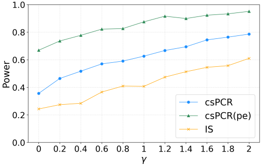


Figure: Comparison of Type-I error control across three methods.



(a) Indirect effect β



(b) Direct effect γ

Figure: Comparison of Power between different methods

Thank you!