NEURAL INFORMATION PROCESSING SYSTEMS

ReThinkLab SJTU

SHANGHAI JIAO TONG UNIVERSITY

Learning Plaintext-Ciphertext Cryptographic Problems via ANF-based SAT Instance Representation

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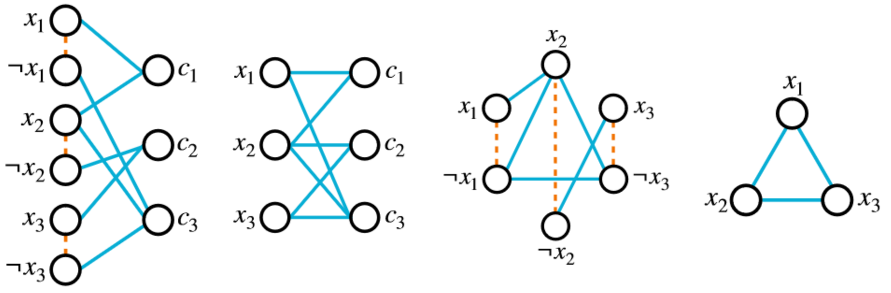
SAT for Cryptanalysis

Round function

$$T_1 = (L_i \lll a) \& (L_i \lll b),\ T_2 = L_i \lll c$$
$$T_3 = T_1 \oplus T_2 \oplus K_i$$
$$L_{i+1} = T_3 \oplus R_i$$
$$R_{i+1} = L_i$$

Cryptographic Problem
(like Plaintext-Ciphertext Problem)



(a) LCG (b) VCG (c) LIG (d) VIG

$$(x_1 \vee x_2) \wedge (\neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_3)$$

Satisfiability Problem
in CNF formula

ANF: ∧, ⊕

l-length Clause in ANF

CNF: ¬, ∧, ∨

Nearly 2^l-length Clause in CNF

$$x' \vee \neg x_2 \vee \neg x_3$$
$$\neg x' \vee x_2 \qquad \Longleftrightarrow \qquad x' + x_2 x_3 = 0$$
$$\neg x' \vee x_3$$

Problem size ballooning considerably

increasing computational complexity,
losing high-order operational information.

ANF formula for Crypto Operations

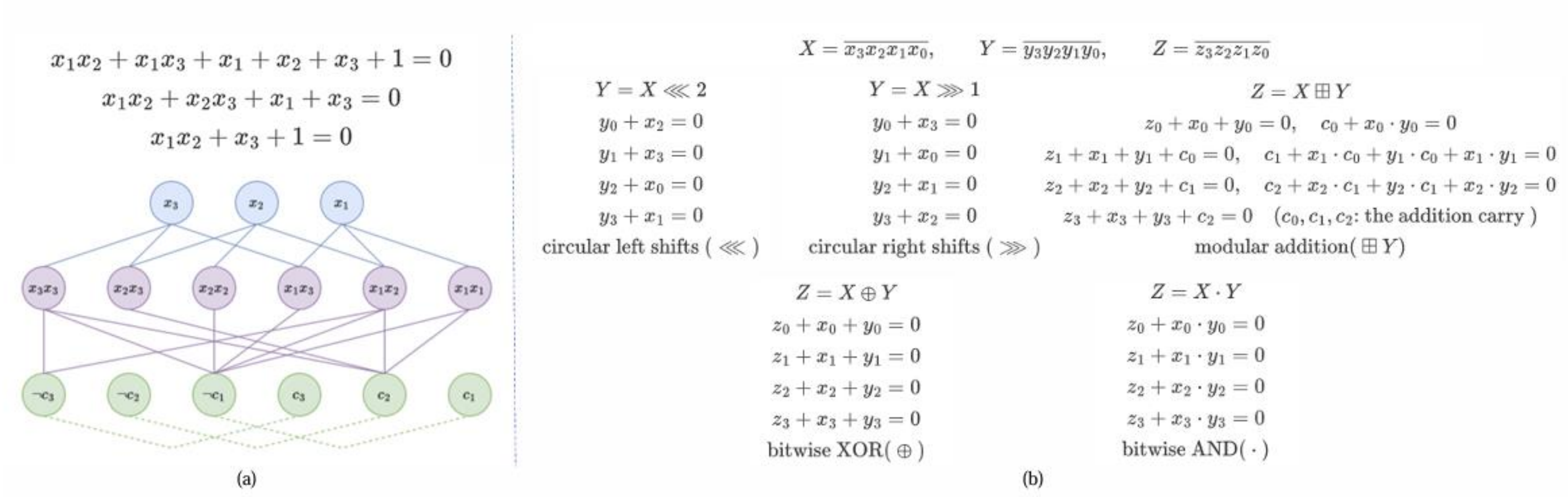


Figure 1: (a) Example ANF formula graph from MQ problem; (b) The transformations to express the circular left shifts ($\lll$), circular right shifts ($\ggg$), modular addition ($\boxplus$), bitwise XOR ($\oplus$), and bitwise AND ($\cdot$) operations in ANF.

ANF-based Graph Structures

A single iteration consists of the following two updates

Literal to clause

$$
\begin{aligned}
L_{l2c}^{(t)} &= L_{l2l}([L^{(t)}[I], L^{(t)}[J]]) \\
[C_{m,\text{pos}}^{(t)}, C_{m,\text{neg}}^{(t)}] &= M_{l2c}^{T} L_{\text{msg}}(L_{l2c}^{(t)}) \\
(C_{\text{pos}}^{(t+1)}, C_{h,\text{pos}}^{(t+1)}) &\leftarrow C_{u,\text{pos}}([C_{h,\text{pos}}^{(t)}, C_{\text{neg}}^{(t)}, C_{m,\text{pos}}^{(t)}]) \\
(C_{\text{pos}}^{(t+1)}, C_{h,\text{neg}}^{(t+1)}) &\leftarrow C_{u,\text{neg}}([C_{h,\text{neg}}^{(t)}, C_{\text{pos}}^{(t)}, C_{m,\text{neg}}^{(t)}])
\end{aligned}
$$

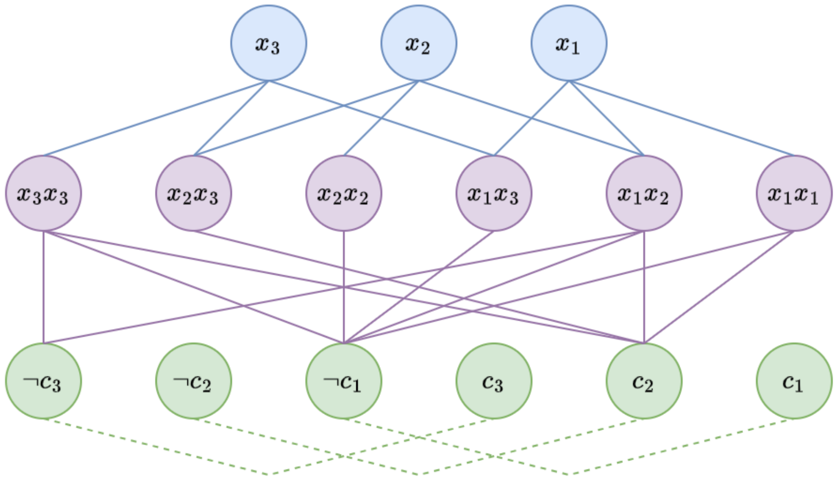
Clause to literal

$$
\begin{aligned}
L_{c2l}^{(t)} &= M_{l2c} C_{\text{msg}}([C_{\text{pos}}^{(t)}, C_{\text{neg}}^{(t)}]) \\
L_{m}^{(t)} &= M_{l2l} L_{l2m}(L_{c2l}^{(t)}) \\
(L^{(t+1)}, L_{h}^{(t+1)}) &\leftarrow L_{u}([L_{h}^{(t)}, L_{m}^{(t)}])
\end{aligned}
$$

$$x_1x_2 + x_1x_3 + x_1 + x_2 + x_3 + 1 = 0$$

$$x_1x_2 + x_2x_3 + x_1 + x_3 = 0$$

$$x_1x_2 + x_3 + 1 = 0$$



We only save the embedding of vanilla literals and pairs of complementary clauses.

Complexity for CNF and ANF formula

Table 1: Parameters of SAT problems in CNF and ANF

	Datasets	SR(5)	SR(25)	Scipher 3-8-16	Scipher 3-16-32	Scipher 6-8-16	Scipher 6-16-32	Speck 3-8-16	Speck 6-8-16
CNF	#Literals	6	424	25	49	49	97	57	129
	#Clauses	75	5492	195	225	735	1519	336	921
	#Nodes	87	6340	245	323	833	1713	450	1179
ANF	#Literals	5	25	24	48	48	96	56	128
	#Clauses	11	26	24	48	48	96	64	136
	#Nodes	27	77	72	144	144	288	184	400

Key solving algorithm for Plaintext-Ciphertext Problem

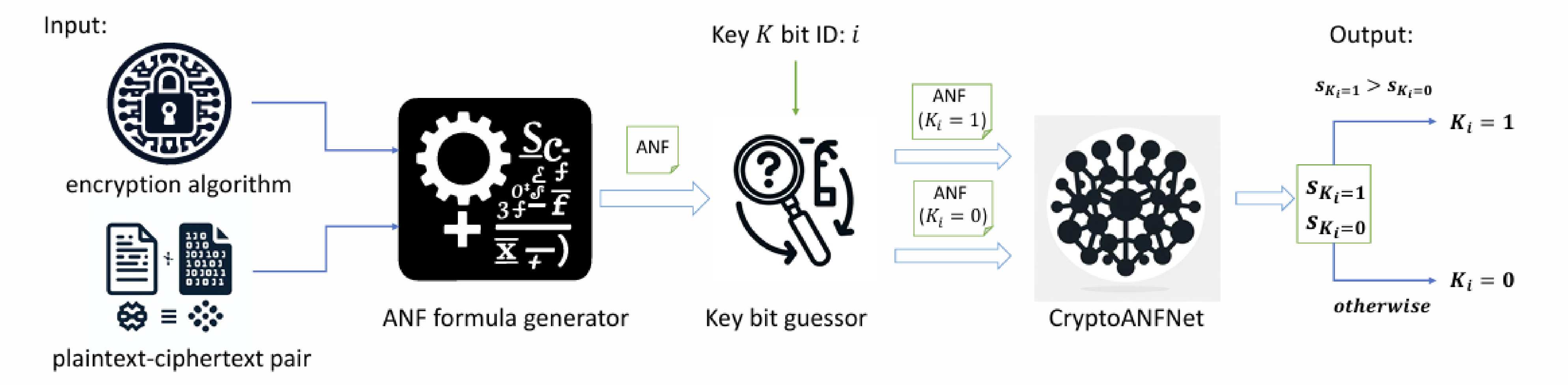


Figure 2: **The pipeline of the key-solving algorithm.** Given a plaintext-ciphertext pair and an encryption algorithm, we first transform them into an ANF-based instance of an MQ problem. Then, for a specific key bit K_i (the i-th bit of key), we guess its value as either 0 or 1 and generate two derived SAT instances. We then employ CryptoANFNet to predict the satisfiability of each instance. The final determination of K_i is based on which instance receives a higher satisfiability score.

Experimental Results for Satisfiability Prediction

Table 2: Performance of different learning-based solvers on synthetic datasets

Datasets	SR(5)	SR(25)	Scipher 3-8-16	Scipher 3-16-32	Scipher 6-8-16	Scipher 6-16-32	Speck 3-8-16	Speck 6-8-16
NeuroSAT	91.0%	57.0%	74.0%	72.7%	53.0%	51.0%	55.0%	52.5%
CryptoANFNet	**96.0%**	**72.0%**	**76.5%**	**75.6%**	**69.0%**	**66.5%**	**72.0%**	**68.5%**

Table 3: Performance for key-solving algorithm in solving MQ problems on synthetic datasets.

Datasets	Scipher 3-8-16	Scipher 3-16-32	Scipher 6-8-16	Scipher 6-16-32	Speck 3-8-16	Speck 6-8-16
NeuroSAT	74.0%	72.7%	53.0%	51.0%	55.0%	52.5%
CryptoANFNet	76.5%	75.6%	69.0%	66.5%	72.0%	68.5%
CryptoANFNet+ key-solving	**82.0%**	**78.4%**	**70.0%**	**69.0%**	**75.0%**	**71.0%**

Experimental Results for Efficiency Comparison

Table 4: Comparing the efficiency of different solvers for solving the MQ problem on synthetic datasets. (Average runtime: (SAT, UNSAT) ms/instance)

Datasets	SR(5)	SR(25)	Scipher 3-8-16	Scipher 3-16-32	Scipher 6-8-16	Scipher 6-16-32	Speck 3-8-16	Speck 6-8-16
NeuroSAT [5]	(3,3)	(20,20)	(7,7)	(10,10)	(7,7)	(14,14)	(13,13)	(18,18)
CryptoANFNet	(2,2)	(5,5)	(8,8)	(9,9)	(10,10)	(8,8)	(11,11)	(14,14)
WDSat [42]	(36,34)	(2470,5662)	(38,38)	(39,39)	(40,37)	(86,150)	(44,47)	(46,46)
CryptoMiniSat [44]	(4,4)	(13491,35912)	(4,4)	(7,9)	(8,9)	(410,1354)	(5,5)	(6,8)
Kissat [45]	(2,2)	(4922,14856)	(2,2)	(2,2)	(5,8)	(219,464)	(3,3)	(4,5)

Table 5: Comparing the efficiency of incomplete solvers for solving the MQ problem on synthetic datasets. (Average runtime: (SAT, UNSAT) ms/instance)

Datasets	SR(5)	SR(25)	Scipher 3-8-16	Scipher 3-16-32	Scipher 6-8-16	Scipher 6-16-32	Speck 3-8-16	Speck 6-8-16
WalkSAT [46]	(3,640)	(762,744)	(4,6)	(10,12)	(289,26)	(831,899)	(39,480)	(482,538)
RoundingSAT [47]	(3,3)	(36758,50122)	(3,5)	(7,10)	(28,20)	(664,1801)	(23,24)	(29,35)
FourierSAT [48]	(1275,8670)	(9620,9687)	(983,426)	(1779,459)	(8163,416)	(8830,8862)	(8733,8689)	(8799,8912)

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Thank you!

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