Efficient Streaming Algorithms for Graphlet Sampling

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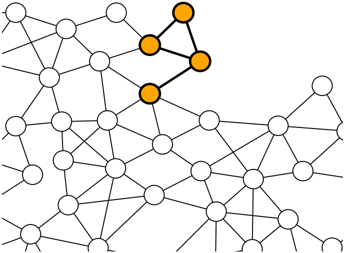
NEURAL INFORMATION PROCESSING SYSTEMS

Graphlet Sampling (GS)

INPUT: a simple undirected graph G and $k \geq 3$
OUTPUT: a *uniform* random connected k-vertex subgraph of G (a k-*graphlet*)



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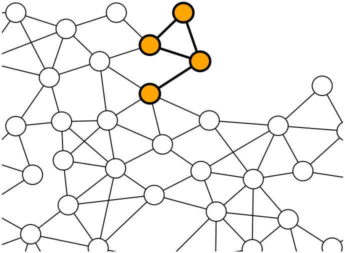
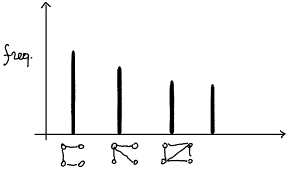


Applications:

- sampling k-graphlets $\rightarrow$ k-graphlet distribution $\rightarrow$ feature vector



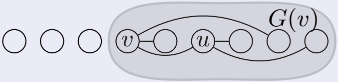
- graph classification, graph kernels, graph neural networks, clustered federated learning...

State-of-The-Art Algorithm

[Bressan, STOC 2021 and Algorithms 2023]

Two-phase uniform graphlet sampling

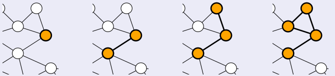
Preprocessing:



Degree-Dominating Order + Starting Probabilities

$O(n\,k^2 + m \log n)$ time

Sampling:



Random Grow + Rejection Sampling

Expected $k^{O(k)} \log n$ time per graphlet

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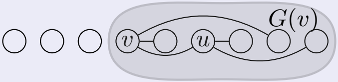

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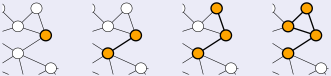

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Issue

Memory $O(m)$ – to store the whole graph

- E.g., Friendster: $n \approx 6.8 \times 10^7, m \approx 1.8 \times 10^9$.

Setting

Semi-Streaming Model

- Edges can only be accessed through *streaming passes*
  - 1 pass = scan the edge list sequentially in arbitrary order
- Memory $M = \tilde{O}(n)$

Goal: use a "small" number of passes

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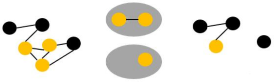
Challenges:

- Cannot store the graph in memory!
- Compute the order using $o(n)$ passes?

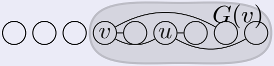
Our Results

Streaming Algorithm

Preprocessing:



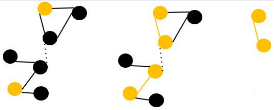
$\frac{1}{1+\epsilon}$-Degree-Dominating Order
$\tilde{O}(\log n)$ pass w.h.p.
$O(m \log n)$ time



Starting Probabilities

1 pass
$O(nk^2)$ time

Sampling:



$\Theta(Mk^{-O(k)})$ parallel samples w.h.p.
$2k-1$ pass
$O(M2^k \log n + mk \log n)$ time

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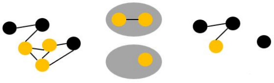

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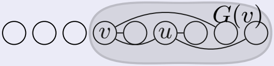

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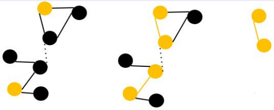

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Streaming Lower Bound

For $k \geq 3$, any p-pass streaming algorithm for GS requires $\Omega\left({}^{n}/_{p}\right)$ bits of memory.

- Nearly space-optimal!

Experiments

Dataset [Kunegis, 2013]	File Size (MB)	#Vertices	#Edges
Dense	1,858	20,000	159,993,472
NY Times	858	401,388	69,654,798
Twitter (WWW)	20,437	41,652,230	1,202,513,047
Twitter (MPI)	25,590	52,579,682	1,614,106,188
Friendster	32,300	68,349,466	1,811,849,343

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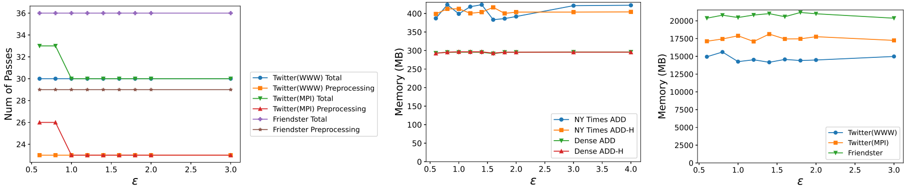

Number of passes and memory versus ϵ, fixing $k = 4$

Thank You!