Fast Proxy Experiment Design for Causal Effect Identification

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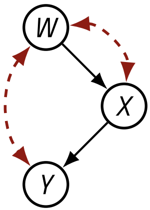

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The goal is to find min-cost interventions such that given the interventions, the causal effect of interest is identified.

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So the min-cost ID problem is **NP-hard!** $\implies$ impossible to solve in polynomial time.

Solving the MCID problem: old approach

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extremely slow in practice!

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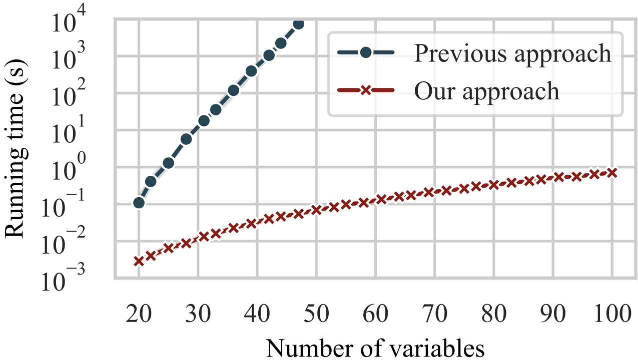

Proxy experiments for adjustment criterion

Adjustment criterion: a set Z s.t.:

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This special case is solvable in **polynomial time**!

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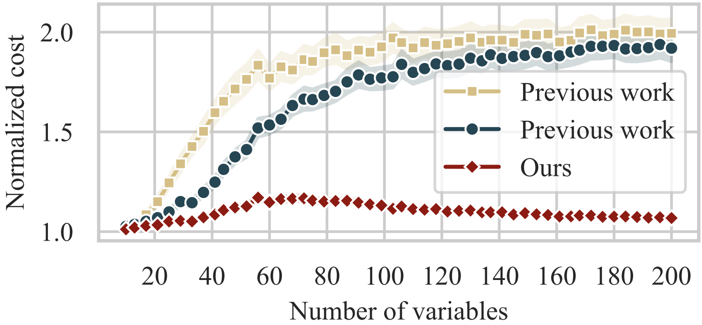

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Thank you for listening!