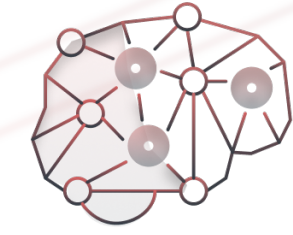




**POLITECNICO**  
MILANO 1863



NEURAL INFORMATION  
PROCESSING SYSTEMS



**RL<sup>3</sup>**

# Last-Iterate Global Convergence of Policy Gradients for Constrained Reinforcement Learning

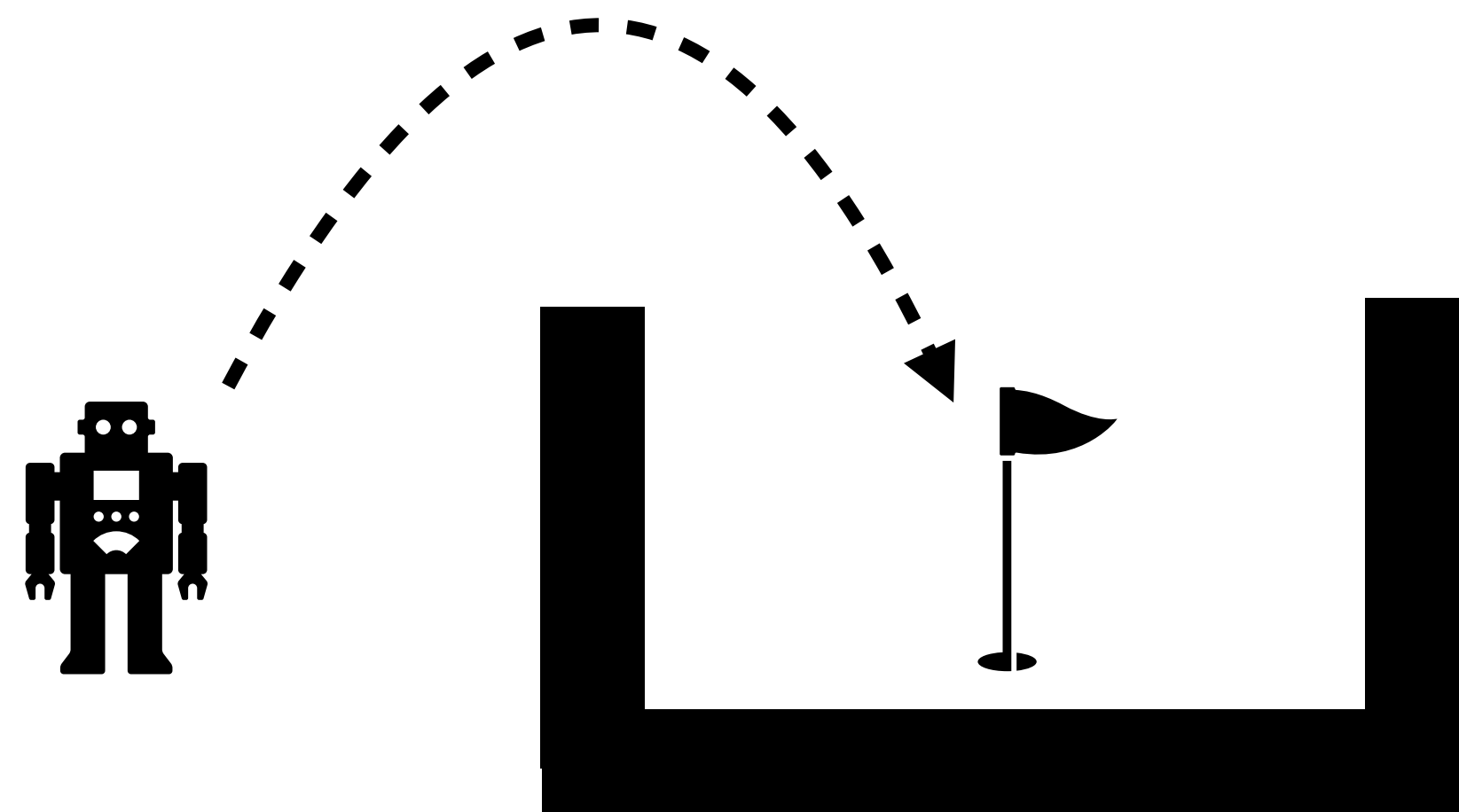
**A. Montenegro, M. Mussi, M. Papini, A. M. Metelli**

**38th Conference on Neural Information Processing Systems (NeurIPS 2024)**

# Constrained Reinforcement Learning (CRL)

## Introduction

- Real-world scenarios: reach a goal + meet structural/utility-based constraints
- Constrained RL: extension of RL with the possibility to account for constraints



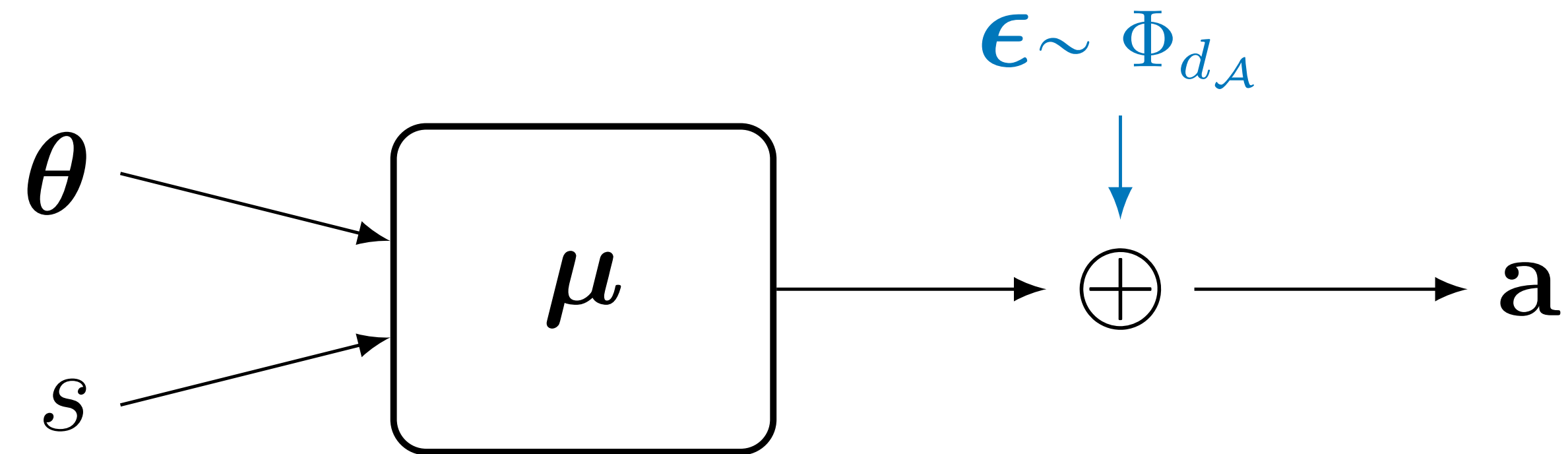
# Policy Gradients (PGs) for CRL

## Introduction

- Continuous State and Action Spaces
- Robustness to Actuators and Sensors Noise
- Robustness to Partial Observability
- Possibility to incorporate expert-knowledge in the Policy-design Phase

# Action-based (AB) Exploration

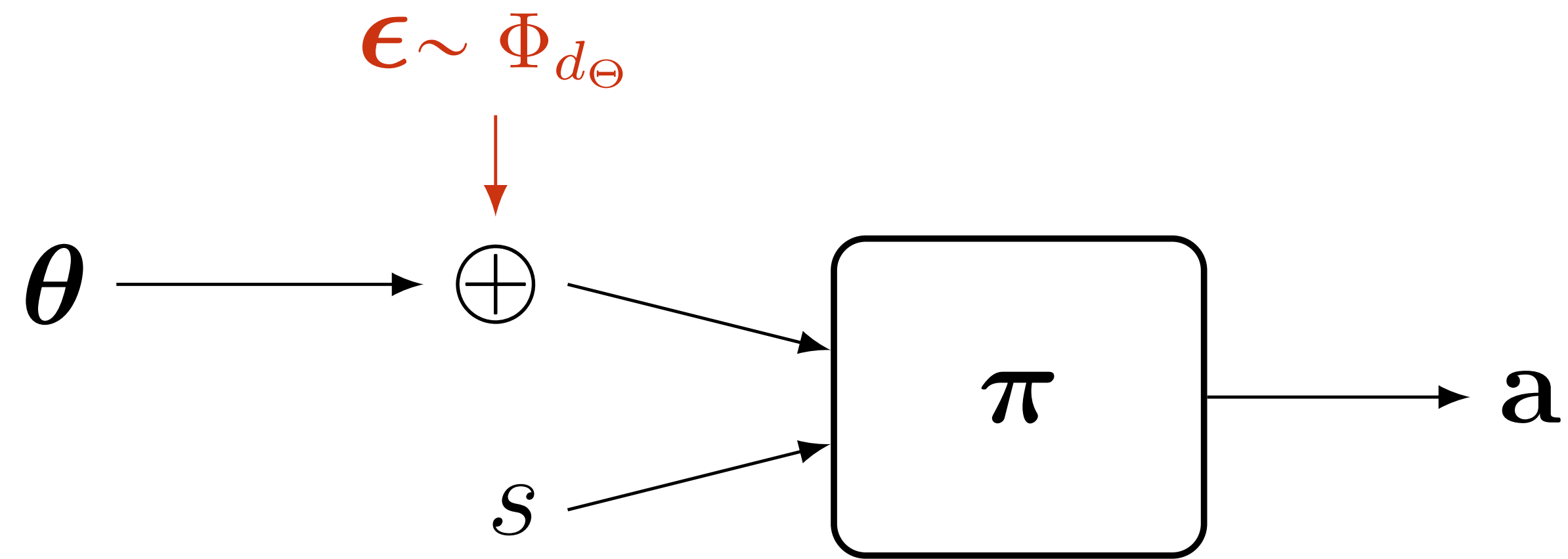
## PGs Exploration Approaches



$$J_A(\boldsymbol{\theta}) = \mathbb{E}_{\tau \sim p_A(\cdot | \boldsymbol{\theta})} [R(\tau)]$$

# Parameter-based (PB) Exploration

## PGs Exploration Approaches



$$J_P(\theta) = \mathbb{E}_{\theta \sim \nu_\rho} \left[ \mathbb{E}_{\tau \sim p_A(\cdot | \theta)} [R(\tau)] \right]$$

# Constrained Optimization Problem

## Setting

- Continuous State and Action spaces
- Multiple constraints on cost functions  $c_i$
- Both exploration paradigms are supported
- Inexact Gradients

# Constrained Optimization Problem

## Setting

$$\min_{\boldsymbol{v} \in \mathcal{V}} J_{\dagger,0}(\boldsymbol{v}) \quad \text{s.t.} \quad J_{\dagger,i}(\boldsymbol{v}) \leq b_i, \quad \forall i \in \llbracket U \rrbracket$$

# Constrained Optimization Problem

## Setting

$$\min_{\mathbf{v} \in \mathcal{V}} J_{\dagger,0}(\mathbf{v}) \quad \text{s.t.} \quad J_{\dagger,i}(\mathbf{v}) \leq b_i, \quad \forall i \in \llbracket U \rrbracket$$

AB or PB approaches on costs  $c_i$  with  $i \in \{0, 1, \dots, U\}$



# Constrained Optimization Problem

## Setting

$$\min_{\boldsymbol{v} \in \mathcal{V}} J_{\dagger,0}(\boldsymbol{v}) \quad \text{s.t.} \quad J_{\dagger,i}(\boldsymbol{v}) \leq b_i, \quad \forall i \in \llbracket U \rrbracket$$

$i$ -th threshold



# C-PG

## Exploration-Agnostic Algorithm

Algorithm

Projected Alternate Ascent Descent on the  $\omega$ -Regularized Lagrangian w.r.t. the Dual Variable

$$\downarrow \widehat{\nabla}_v \mathcal{L}_\omega(v, \lambda)$$

$$\uparrow \widehat{\nabla}_\lambda \mathcal{L}_\omega(v, \lambda)$$

# C-PG: Convergence

## Exploration-Agnostic Algorithm

Assumptions:

1.  $\psi$ -Gradient Domination on parameterization ( $\psi \in [1,2]$ )
2. Regularity of  $\mathcal{L}_\omega$
3. Existence of a saddle point

# C-PG: Convergence

## Exploration-Agnostic Algorithm

Theorem

$$\mathbb{E}[J_0(\mathbf{v}_k) - J_0(\mathbf{v}_0^*)] \leq \epsilon + \frac{\beta_1}{\alpha_1} + \frac{\omega}{2} \|\boldsymbol{\lambda}_0^*\|_2^2 \quad \text{and} \quad \mathbb{E}[(J_i(\mathbf{v}_k) - b_i)^+] \leq 4\epsilon + 4\frac{\beta_1}{\alpha_1} + \omega \|\boldsymbol{\lambda}_0^*\|_2, \quad \forall i \in \llbracket U \rrbracket$$

Holds for both exploration approaches

# C-PG: Convergence

## Exploration-Agnostic Algorithm

	$\psi = 1$	$\psi = 2$
Exact Gradients	$\mathcal{O}(\epsilon^{-2})$	$\mathcal{O}(\epsilon^{-1} \log(\epsilon^{-1}))$
Estimated Gradients	$\mathcal{O}(\epsilon^{-6} \log(\epsilon^{-1}))$	$\mathcal{O}(\epsilon^{-4} \log(\epsilon^{-1}))$

# Enforcing Constraints on Risks

## Risk and Exploration Agnostic Algorithms

- AB and PB explorations have a **semantic difference** when enforcing constraints
- In order to induce safer behaviors, we can enforce **constraints on risk** measures
- We introduce a **unified risk measure** that extends the framework previously described
- **Additional parameter** to learn required

# Conclusions

## Our Contribution

- **Framework** to handle CRL with PGs (both AB and PB) in continuous spaces and with multiple constraints
- Both approaches exhibit **last-iterate global convergence** to a feasible (hyper)policy guarantees
- We extend the framework to handle **risk-based constraints**
- We **numerically** validate our results

**Thank you!**