Queueing Matching Bandits with Preference Feedback

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- Online labor service markets where tasks are recommended to freelance workers.

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- Given $S_{k,t}$ (set of agents assigned to arm k, 'assortment'), the service rate for arm k to serve a job of agent $n \in S_{k,t}$ follows Multi-nomial Logit (MNL) model as follows

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• It is allowed to have an outside option (n_0) , or not to choose any, for each arm k according to MNL with probability as $\mu(n_0 | \mathcal{S}_{k,t}, \theta_k) = \frac{1}{1 + \sum_{m \in \mathcal{S}_{k,t}} \exp(\mathsf{x}_m^\top \theta_k)}.$

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Definition 1 (System Stability)

The systems are denoted to be stable if $\lim_{T\to\infty} Q(T) < \infty$.

• As a preliminary study, we analyze an oracle strategy of MaxWeight under known service rates. It is shown to achieve $\mathcal{Q}(\mathcal{T}) = O\left(\frac{\min\{N,K\}}{\epsilon}\right)$ $\left(\frac{N,K}{\epsilon}\right)<\infty$, where ϵ is a traffic slackness.

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- We demonstrate the effectiveness of our algorithms using synthetic datasets.