

LM scaling laws & zero-sum learning



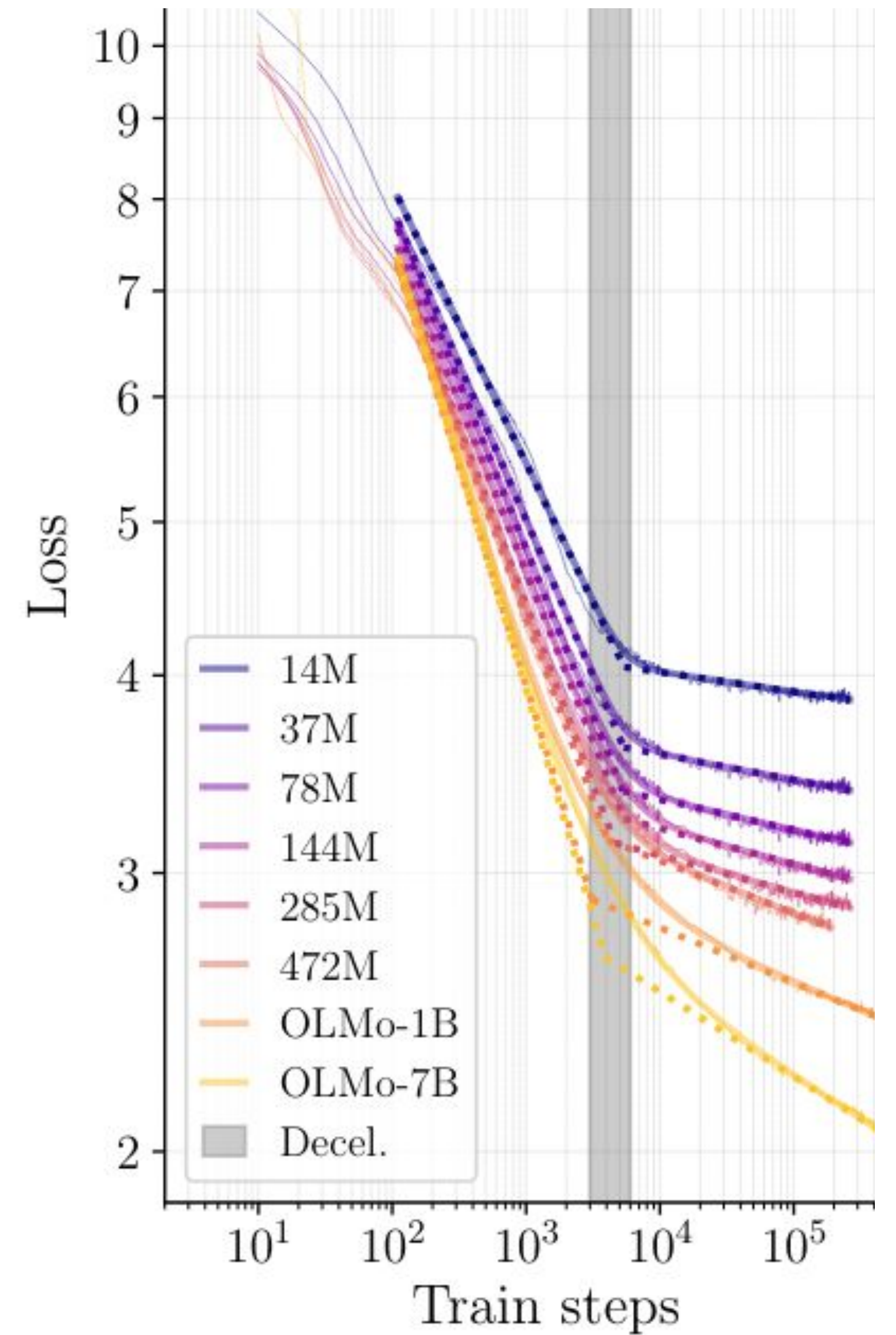
TL;DR

Can we explain LM scaling improvements in terms of training dynamics?
 Yes! Scaling improves LMs by mitigating loss deceleration (transition in training dynamics characterized by gradient opposition and zero-sum learning between tokens).

Explaining LM scaling laws: loss deceleration

Loss deceleration: rapid slow-down in rate of loss improvement observed early during LLM pretraining. Characterized by piece-wise linear behavior (log-log).

Quantifiable with 1-break BNSL. $L(t) - a = (bt^{-c_0}) \left(1 + (t/d_1)^{1/f_1}\right)^{-c_1 f_1}$
 t_d : d_1 , the step at which deceleration occurs.
 L_d : $bd_1^{-c_0}$, the loss at which deceleration occurs.
 r_d : $c_0 + c_1$, the log-log loss slope after deceleration.
 \hat{L}_T : $\log(L_T) \approx \log(\hat{L}_T) = \log(L_d) - r_d \log(T/t_d)$
 Can capture loss improvements due to scaling.

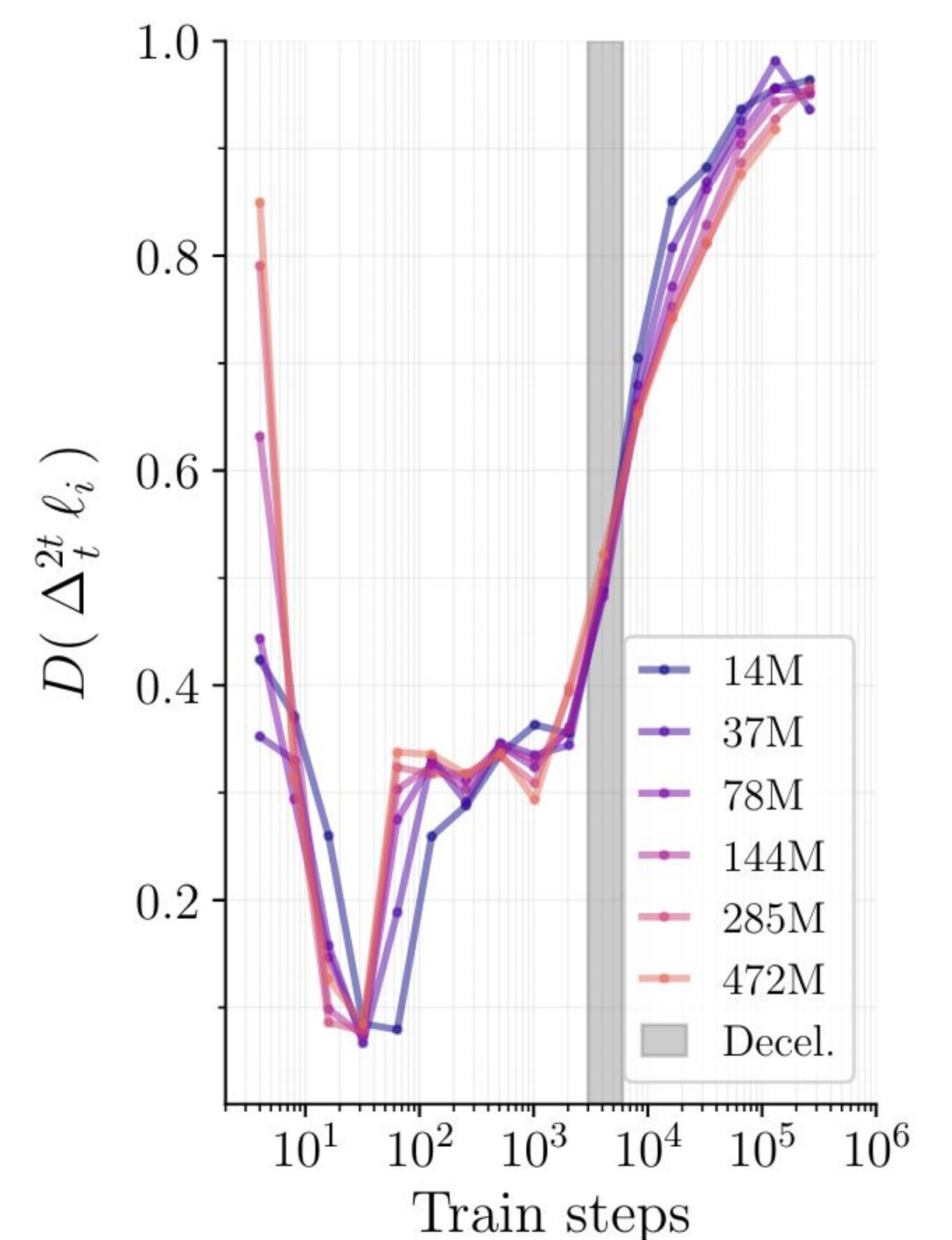


Explaining deceleration: zero-sum learning (ZSL)

Zero-sum learning: degenerate training dynamics where loss improvements in one set of examples are cancelled out by degradation in another.

ZSL can be quantified with **destructive interference**
 $D(\Delta \ell) = 1 - \text{abs}(\sum_i \Delta \ell_i) / \sum_i \text{abs}(\Delta \ell_i)$, $D(\Delta \ell) \in [0, 1]$
 where $D=1$ indicates complete interference and ZSL

- A.** Occurs simultaneously with deceleration and can be shown to fundamentally bottleneck loss improvements.
- B.** Scaling reduces ZSL after deceleration (improved slope)



Explaining ZSL: systematic gradient opposition

Systematic gradient opposition: model weight configuration with >99% destructive interference between per-example gradients.

$D(\nabla_{\theta} \ell) = 1 - \text{abs}(\sum_i \nabla_{\theta} \ell_i) / \sum_i \text{abs}(\nabla_{\theta} \ell_i)$

- A.** Occurs across parameters simultaneously with ZSL; shown to fundamentally cause ZSL.
- B.** Scaling reduces gradient opposition before deceleration (improved decel. loss)

