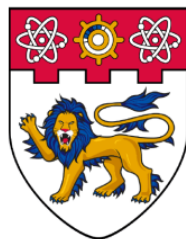


# Personalized Subgraph Federated Learning with Differentiable Auxiliary Projections

Wei Zhuo<sup>1</sup>, Zhaohuan Zhan<sup>2</sup>, Han Yu<sup>1</sup>

<sup>1</sup>Nanyang Technological University,

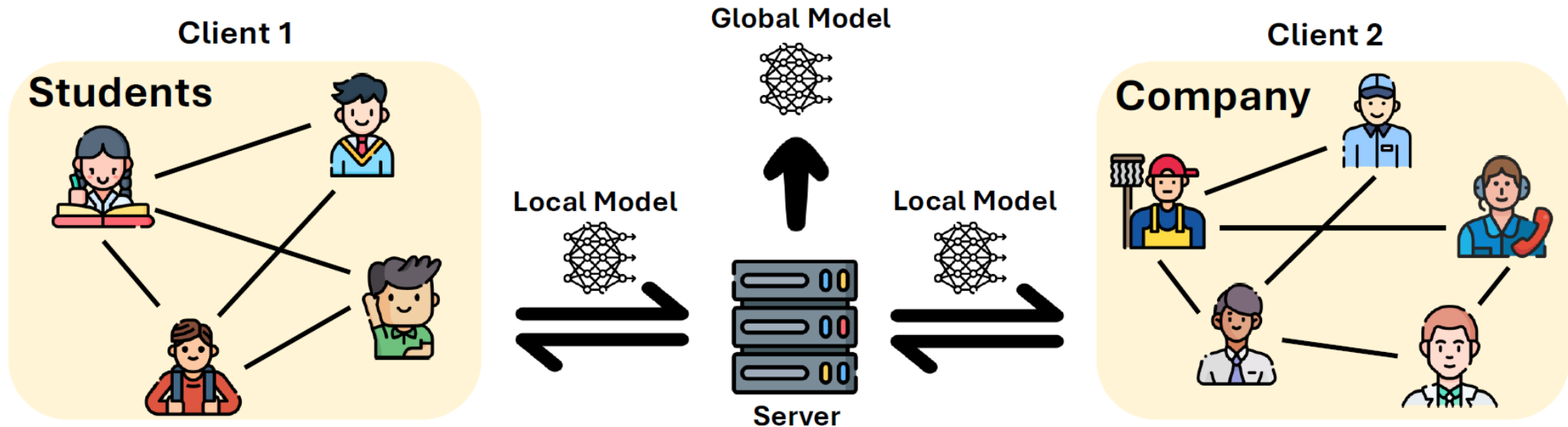
<sup>2</sup>Shenzhen MSU-BIT University



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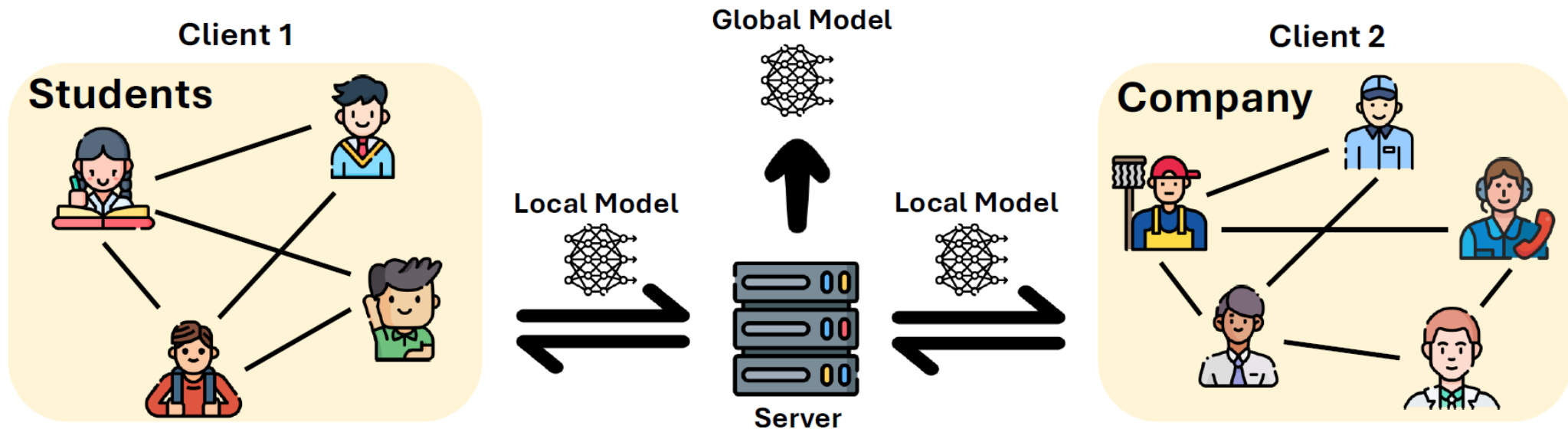
# Subgraph Federated Learning

Subgraph federated learning involves  $K$  clients, each holding a local subgraph from a larger global graph.



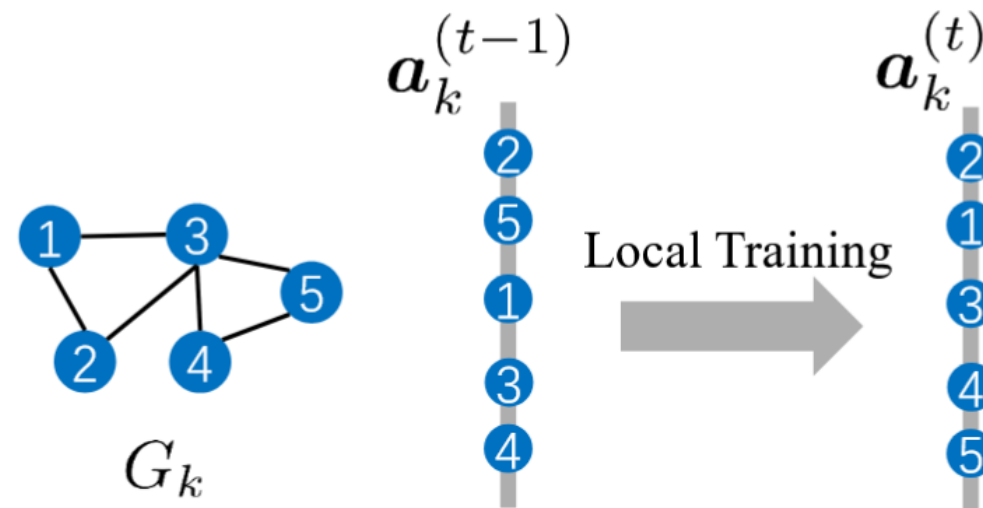
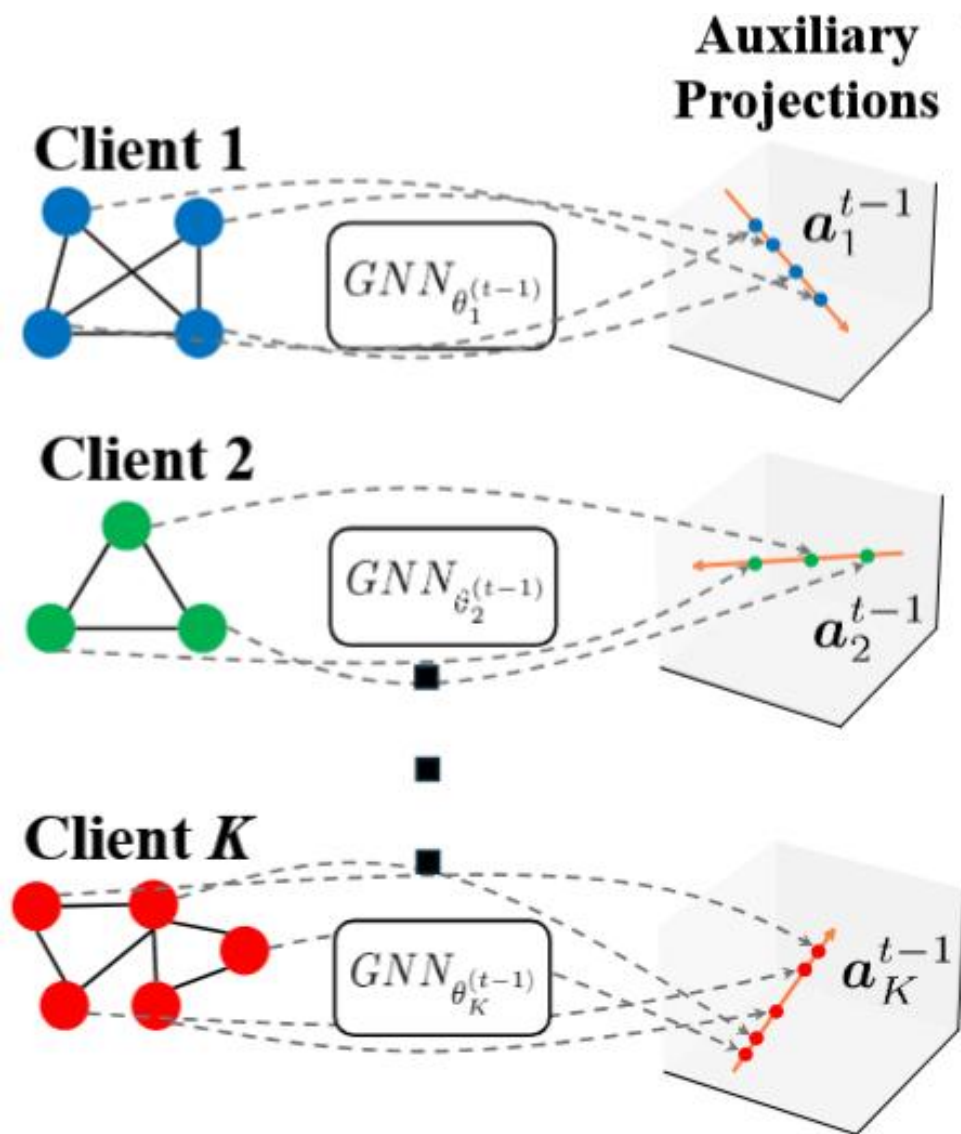
# Subgraph Federated Learning

Subgraph federated learning involves  $K$  clients, each holding a local subgraph from a larger global graph.



The key challenge is that these subgraphs exhibit significant **heterogeneity** in their structures, features, and label distributions, making traditional federated averaging ineffective.

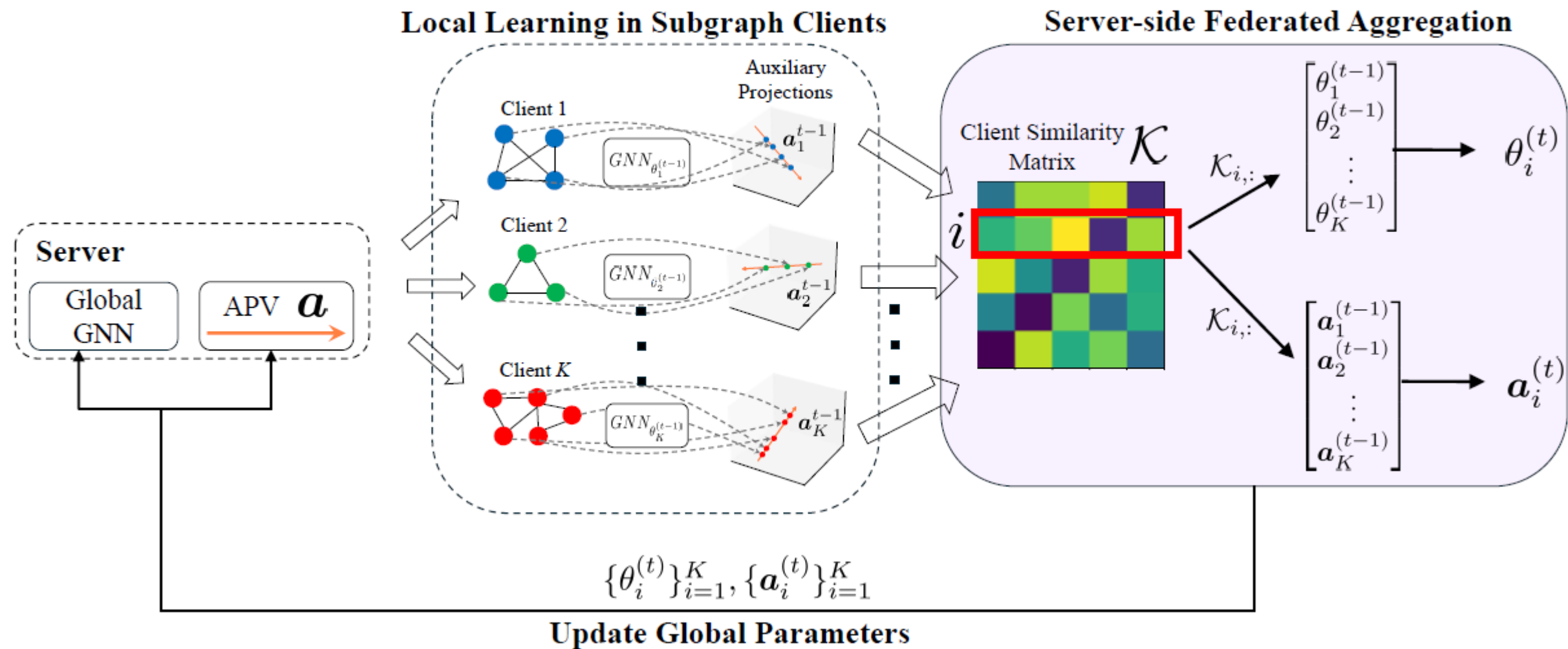
# Auxiliary Projection Vectors



**APV client-specific similarity proxies on the server**

APV serves as a privacy-preserving latent space that retains relative subgraph structure while concealing absolute node positions and raw features.

# Overall Framework



# Theoretical Analysis

**Theorem 3.1** (Fidelity of the APV  $\mathbf{a}$ ). *Let  $\mathbf{C} := \frac{1}{N} \sum_{i=1}^N h_i h_i^\top$  be the empirical covariance of node embeddings in the current client with size  $N$ . The gradient of the local loss  $\mathcal{L}$  w.r.t. the APV  $\mathbf{a}$  satisfies:*

$$\nabla_{\mathbf{a}} \mathcal{L} = -\frac{2}{\sigma^2} \mathbf{C} \mathbf{a} + \mathcal{R}(\sigma), \quad (9)$$

*where the remainder term obeys  $\|\mathcal{R}(\sigma)\| = \mathcal{O}(\sigma^0)$  as  $\sigma \rightarrow 0^+$ . Define  $\mathbb{S}^{d-1} = \{x \in \mathbb{R}^d : \|x\|_2 = 1\}$  as the unit Euclidean sphere embedded in  $\mathbb{R}^d$ , then the gradient descent on  $\mathcal{L}$  with unit-norm re-normalization reproduces Oja learning rule [22]:*

$$\mathbf{a} \leftarrow \Pi_{\mathbb{S}^{d-1}}(\mathbf{a} - \eta \mathbf{C} \mathbf{a}), \quad (10)$$

*whose unique stable fixed points are the eigenvectors of  $\mathbf{C}$ , and the global attractor is the principal eigenvector (largest eigenvalue).*



# Experiments

Methods	Cora			CiteSeer			Pubmed		
	5 Clients	10 Clients	20 Clients	5 Clients	10 Clients	20 Clients	5 Clients	10 Clients	20 Clients
Local	81.30 $\pm$ 0.21	79.94 $\pm$ 0.24	80.30 $\pm$ 0.25	69.02 $\pm$ 0.05	67.82 $\pm$ 0.13	65.98 $\pm$ 0.17	84.04 $\pm$ 0.18	82.81 $\pm$ 0.39	82.65 $\pm$ 0.03
FedAvg	74.45 $\pm$ 5.64	69.19 $\pm$ 0.67	69.50 $\pm$ 3.58	71.06 $\pm$ 0.60	63.61 $\pm$ 3.59	64.68 $\pm$ 1.83	79.40 $\pm$ 0.11	82.71 $\pm$ 0.29	80.97 $\pm$ 0.26
FedProx	72.03 $\pm$ 4.56	60.18 $\pm$ 7.04	48.22 $\pm$ 6.81	71.73 $\pm$ 1.11	63.33 $\pm$ 3.25	64.85 $\pm$ 1.35	79.45 $\pm$ 0.25	82.55 $\pm$ 0.24	80.50 $\pm$ 0.25
FedPer	81.68 $\pm$ 0.40	79.35 $\pm$ 0.04	78.01 $\pm$ 0.32	70.41 $\pm$ 0.32	70.53 $\pm$ 0.28	66.64 $\pm$ 0.27	85.80 $\pm$ 0.21	84.20 $\pm$ 0.28	84.72 $\pm$ 0.31
GCFL	81.47 $\pm$ 0.65	78.66 $\pm$ 0.27	79.21 $\pm$ 0.70	70.34 $\pm$ 0.57	69.01 $\pm$ 0.12	66.33 $\pm$ 0.05	85.14 $\pm$ 0.33	84.18 $\pm$ 0.19	83.94 $\pm$ 0.36
FedGNN	81.51 $\pm$ 0.68	70.12 $\pm$ 0.99	70.10 $\pm$ 3.52	69.06 $\pm$ 0.92	55.52 $\pm$ 3.17	52.23 $\pm$ 6.00	79.52 $\pm$ 0.23	83.25 $\pm$ 0.45	81.61 $\pm$ 0.59
FedGTA	71.26 $\pm$ 2.93	68.33 $\pm$ 1.27	69.24 $\pm$ 0.91	69.39 $\pm$ 0.75	67.34 $\pm$ 1.08	65.29 $\pm$ 1.92	78.47 $\pm$ 0.25	82.79 $\pm$ 0.20	81.92 $\pm$ 0.60
FedSage+	72.97 $\pm$ 5.94	69.05 $\pm$ 1.59	57.97 $\pm$ 12.6	70.74 $\pm$ 0.69	65.63 $\pm$ 3.10	65.46 $\pm$ 0.74	79.57 $\pm$ 0.24	82.62 $\pm$ 0.31	80.82 $\pm$ 0.25
FED-PUB	83.72 $\pm$ 0.18	81.45 $\pm$ 0.12	81.10 $\pm$ 0.64	72.40 $\pm$ 0.26	71.83 $\pm$ 0.61	66.89 $\pm$ 0.14	86.81 $\pm$ 0.12	86.09 $\pm$ 0.17	84.66 $\pm$ 0.54
FedAux	<b>84.57<math>\pm</math>0.39</b>	<b>82.05<math>\pm</math>0.71</b>	<b>81.60<math>\pm</math>0.64</b>	<b>72.99<math>\pm</math>0.82</b>	<b>73.16<math>\pm</math>0.29</b>	<b>68.10<math>\pm</math>0.35</b>	<b>88.10<math>\pm</math>0.16</b>	<b>86.43<math>\pm</math>0.20</b>	<b>84.87<math>\pm</math>0.42</b>
Methods	Amazon-Computer			Amazon-Photo			ogbn-arxiv		
	5 Clients	10 Clients	20 Clients	5 Clients	10 Clients	20 Clients	5 Clients	10 Clients	20 Clients
Local	89.22 $\pm$ 0.13	88.91 $\pm$ 0.17	89.52 $\pm$ 0.20	91.67 $\pm$ 0.09	91.80 $\pm$ 0.02	90.47 $\pm$ 0.15	66.76 $\pm$ 0.07	64.92 $\pm$ 0.09	65.06 $\pm$ 0.05
FedAvg	84.88 $\pm$ 1.96	79.54 $\pm$ 0.23	74.79 $\pm$ 0.24	89.89 $\pm$ 0.83	83.15 $\pm$ 3.71	81.35 $\pm$ 1.04	65.54 $\pm$ 0.07	64.44 $\pm$ 0.10	63.24 $\pm$ 0.13
FedProx	85.25 $\pm$ 1.27	83.81 $\pm$ 1.09	73.05 $\pm$ 1.30	90.38 $\pm$ 0.48	80.92 $\pm$ 4.64	82.32 $\pm$ 0.29	65.21 $\pm$ 0.20	64.37 $\pm$ 0.18	63.03 $\pm$ 0.04
FedPer	89.67 $\pm$ 0.34	89.73 $\pm$ 0.04	87.86 $\pm$ 0.43	91.44 $\pm$ 0.37	91.76 $\pm$ 0.23	90.59 $\pm$ 0.06	66.87 $\pm$ 0.05	64.99 $\pm$ 0.18	64.66 $\pm$ 0.11
GCFL	89.07 $\pm$ 0.91	90.03 $\pm$ 0.16	<b>89.08<math>\pm</math>0.25</b>	91.99 $\pm$ 0.29	92.06 $\pm$ 0.25	90.79 $\pm$ 0.17	66.80 $\pm$ 0.12	65.09 $\pm$ 0.08	65.08 $\pm$ 0.04
FedGNN	88.08 $\pm$ 0.15	88.18 $\pm$ 0.41	83.16 $\pm$ 0.13	90.25 $\pm$ 0.70	87.12 $\pm$ 2.01	81.00 $\pm$ 4.48	65.47 $\pm$ 0.22	64.21 $\pm$ 0.32	63.80 $\pm$ 0.05
FedGTA	85.06 $\pm$ 0.82	84.27 $\pm$ 0.71	79.46 $\pm$ 0.28	89.70 $\pm$ 0.67	76.53 $\pm$ 3.21	82.02 $\pm$ 0.78	65.42 $\pm$ 0.09	64.22 $\pm$ 0.08	63.75 $\pm$ 0.18
FedSage+	85.04 $\pm$ 0.61	80.50 $\pm$ 1.30	70.42 $\pm$ 0.85	90.77 $\pm$ 0.44	76.81 $\pm$ 8.24	80.58 $\pm$ 1.15	65.69 $\pm$ 0.09	64.52 $\pm$ 0.14	63.31 $\pm$ 0.20
FED-PUB	90.25 $\pm$ 0.07	89.73 $\pm$ 0.16	88.20 $\pm$ 0.18	93.20 $\pm$ 0.15	<b>92.46<math>\pm</math>0.19</b>	90.59 $\pm$ 0.35	67.62 $\pm$ 0.11	66.35 $\pm$ 0.16	63.90 $\pm$ 0.27
FedAux	<b>90.38<math>\pm</math>0.08</b>	<b>89.92<math>\pm</math>0.15</b>	88.35 $\pm$ 0.96	<b>93.37<math>\pm</math>0.26</b>	92.30 $\pm$ 0.29	<b>90.91<math>\pm</math>0.60</b>	<b>68.83<math>\pm</math>0.15</b>	<b>68.50<math>\pm</math>0.27</b>	<b>65.52<math>\pm</math>0.10</b>