Let A Neural Network Be Your Invariant

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SAFETY

Nothing bad
ever happens.

LIVENESS

Something Good
Eventually Happens

SAFETY

Nothing bad ever happens.

Use an Inductive Invariant.

LIVENESS

Something Good Eventually Happens

Use a Ranking Function.

Shorter Runtime are always Nicer.

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Without it Developers Skip on Formal Verification

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Prior Neural Model checkers focus on Pure Liveness.

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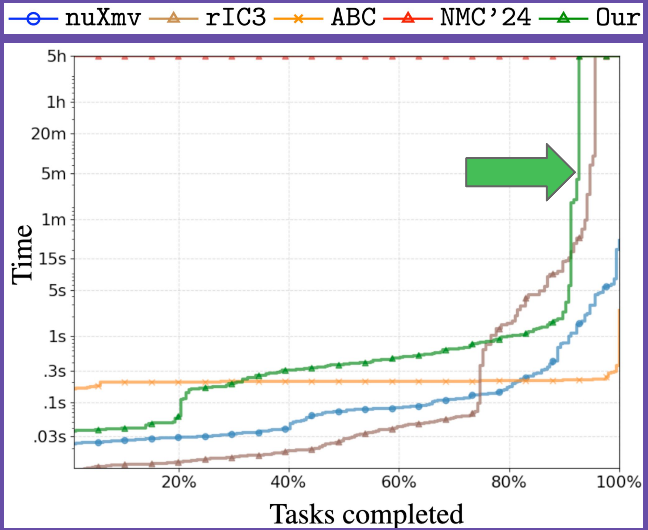
Prior Neural Model checkers focus on Pure Liveness.

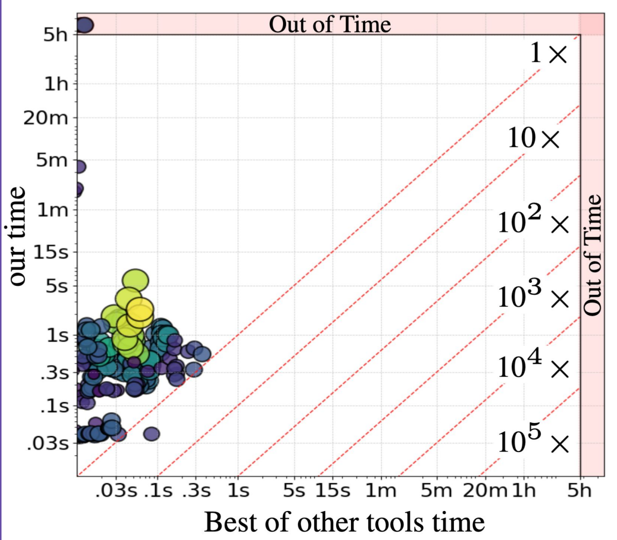
Picture a Fast Neural Model Checking Approach For both Safety and Liveness

Runtime Improvement

Pure Safety

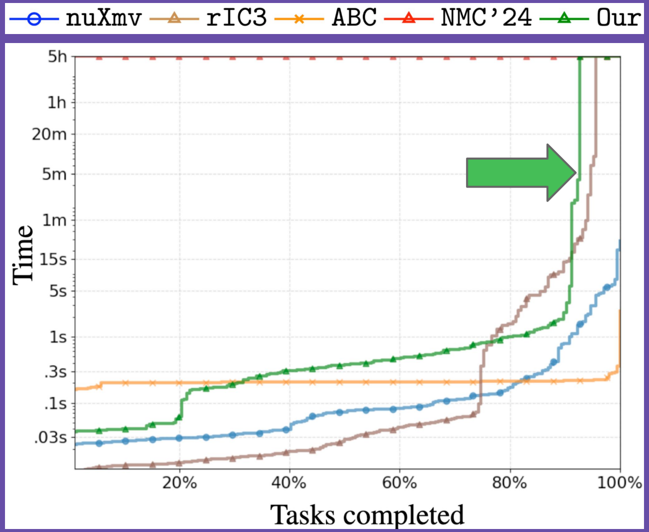


We are certainly not the fastest for pure safety—



—but 80% tasks complete in 1 sec for all tools.

Pure Safety



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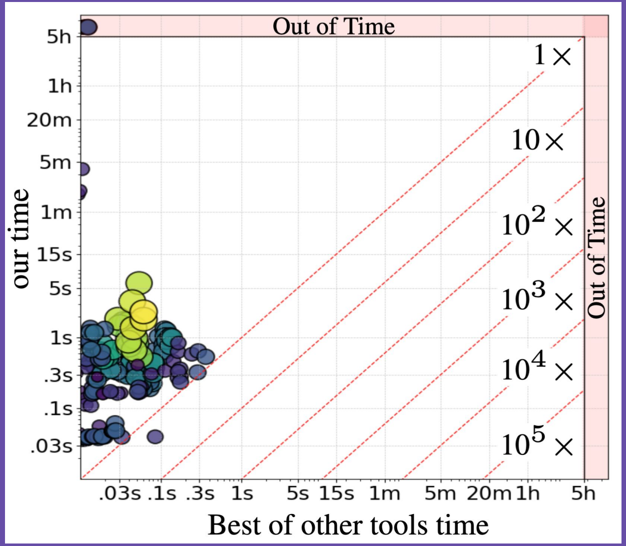

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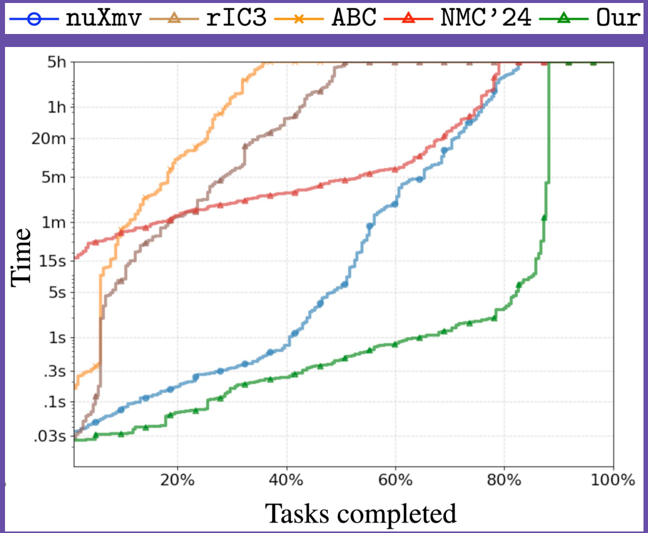
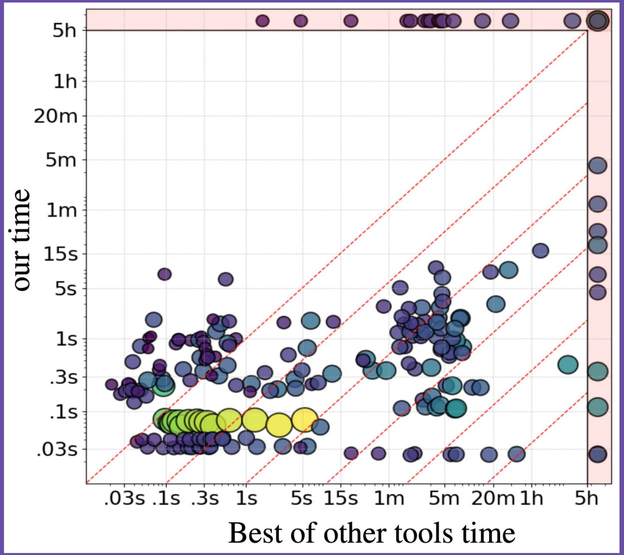
Pure Liveness

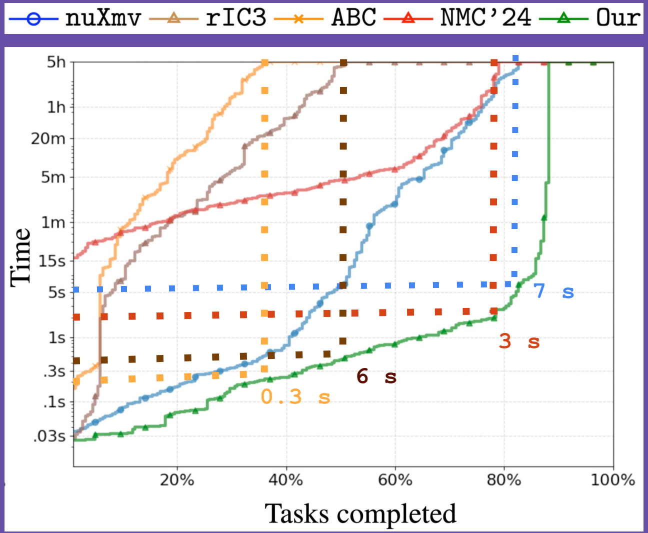


Tasks completed in 5 h per task by nuXmv, NMC'24, rIC3, and ABC are completed in under 7 s, 3 s, 0.6 s, 0.3 s by our method.



Against the per-task fastest, our method is faster on 66 % of tasks, 10× faster on 46 %, 1000× on 11 %, and 10000× on 4 %.

Pure Liveness



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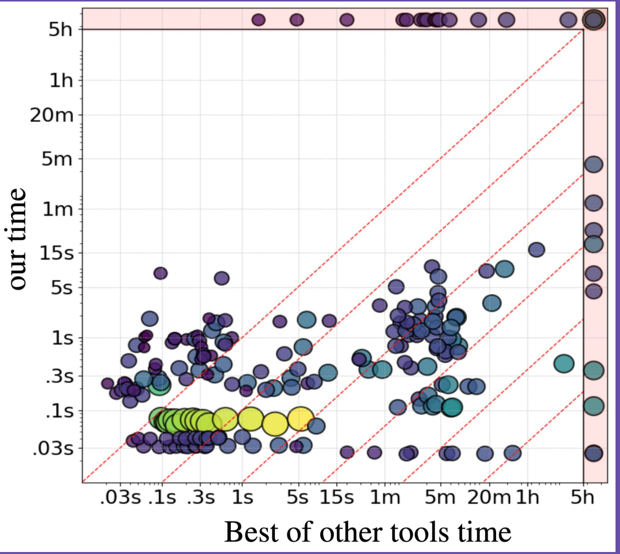

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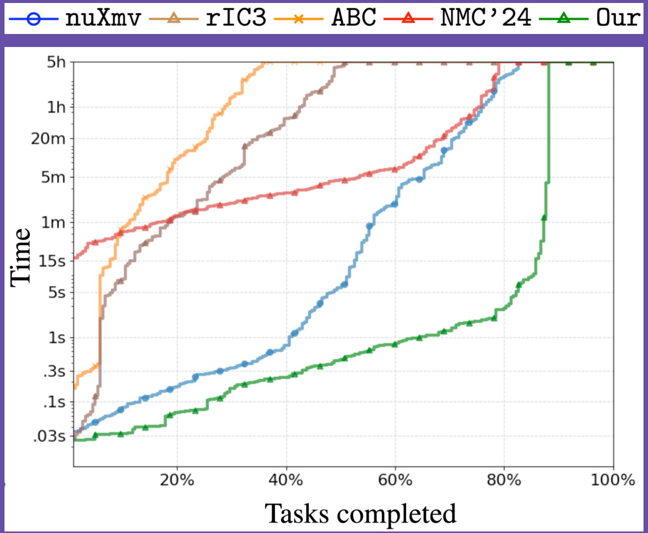

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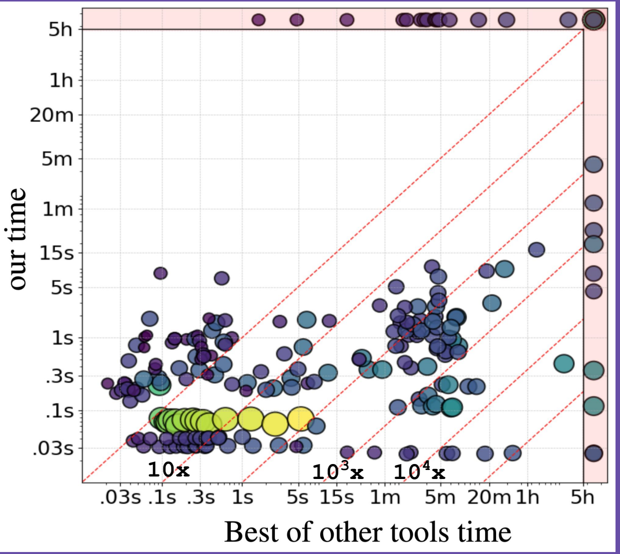

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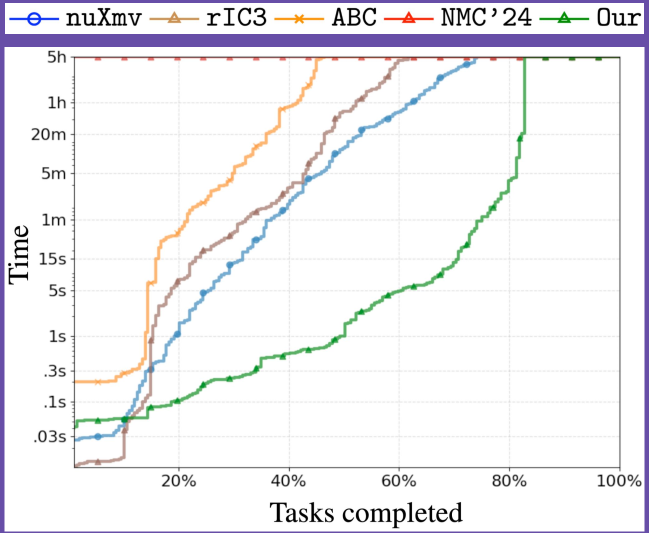
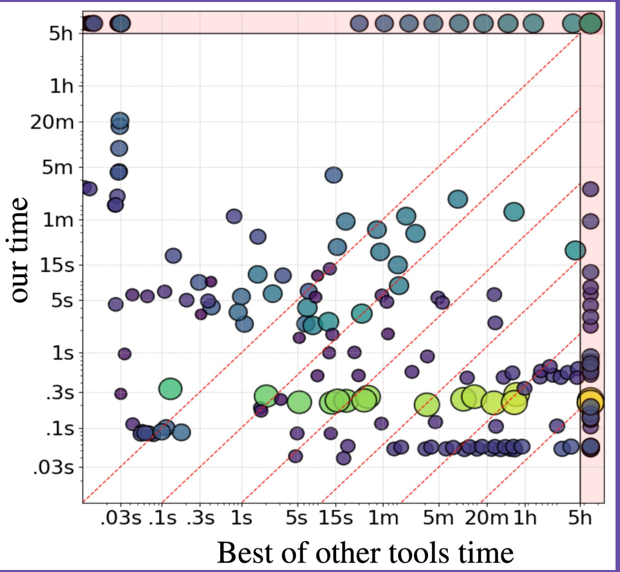
Safety + Liveness



Tasks that take nuXmv, ABC or rIC3 5 h we do in 56 s, 6 s, 0.8 s per task.



Faster than everyone else on 61 %, 100× faster on 43 %, 10000× faster on 27 %, and 100000× faster on 6 %.

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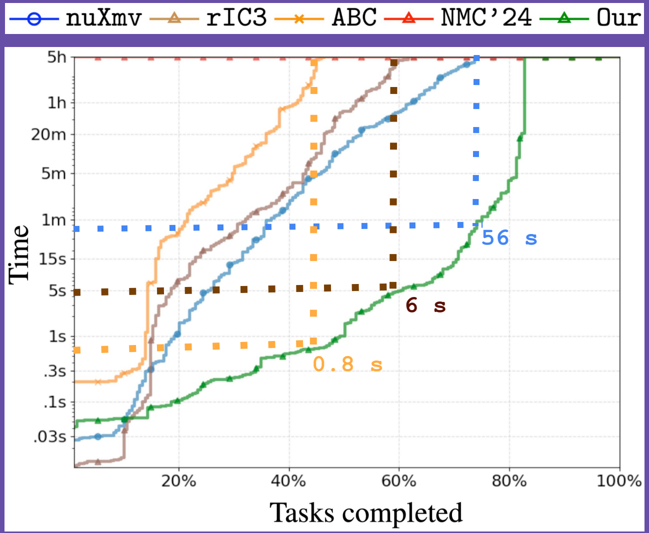

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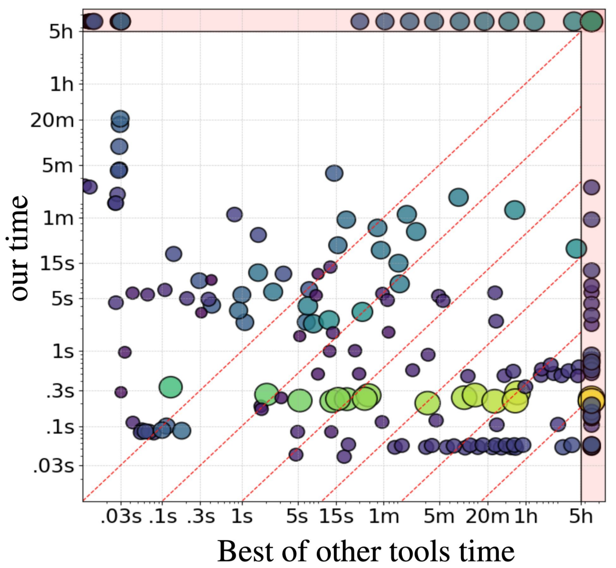

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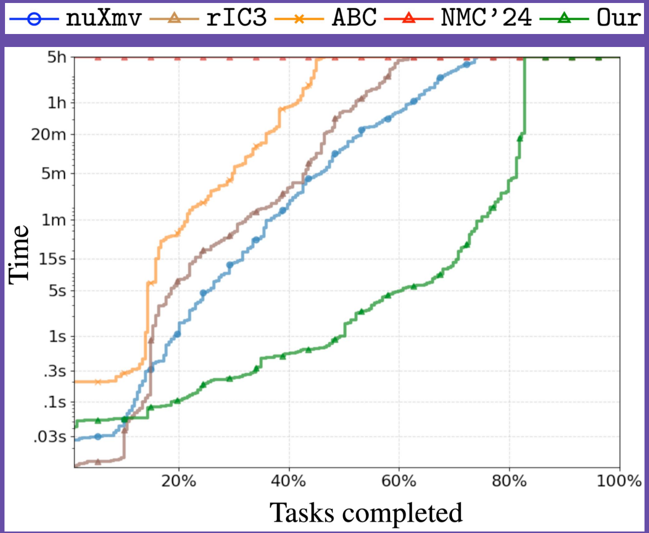

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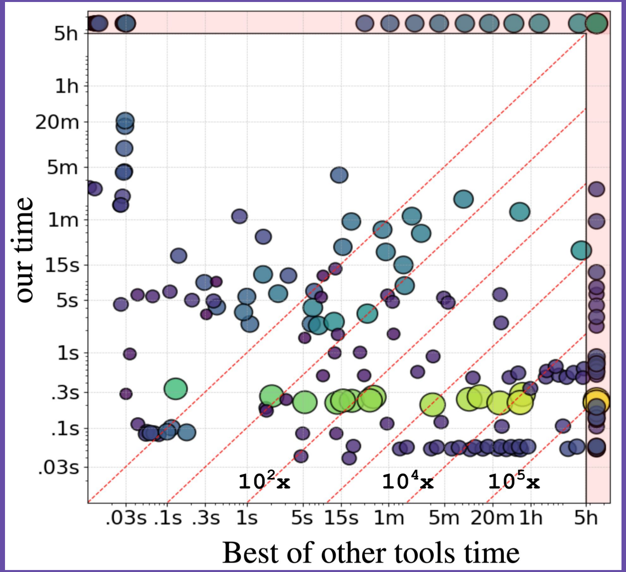

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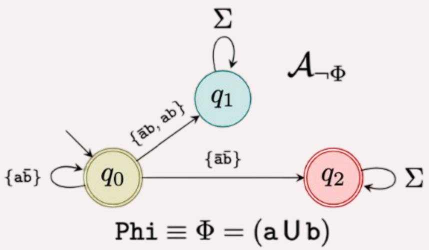
Our Approach

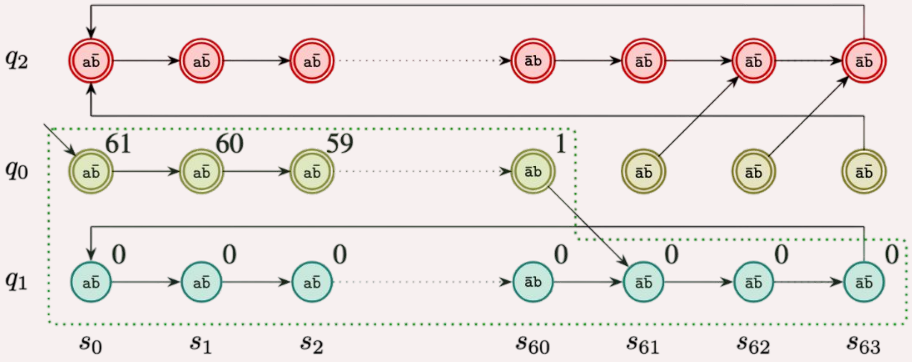
```
module MODEL (input clk, output reg a, b);
  reg [5:0] c = 0;
  assign a = (c < 60);
  assign b = (c == 60);
  always @(posedge clk) begin
       c <= c + 1;
  end
  Phi: assert property
     (@(posedge clk) a s_until b);
endmodule
```

(a)



(b)

Decide whether all trances of a **System Verilog** Design satisfies an **LTL specification**.



(c)

$$\boldsymbol{s} \in S_0 \implies (\mathrm{reg}\,\boldsymbol{s}, q_0) \in I, \tag{1}$$

$$(\boldsymbol{s}, q) \rightarrow_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (\boldsymbol{s}', q') \wedge (\mathrm{reg}\,\boldsymbol{s}, q) \in I \implies (\mathrm{reg}\,\boldsymbol{s}', q') \in I, \tag{2}$$

$$(\boldsymbol{s}, q) \rightarrow_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (\boldsymbol{s}', q') \wedge (\mathrm{reg}\,\boldsymbol{s}, q) \in I \implies V(\mathrm{reg}\,\boldsymbol{s}, q) \succeq V(\mathrm{reg}\,\boldsymbol{s}', q'), \tag{3}$$

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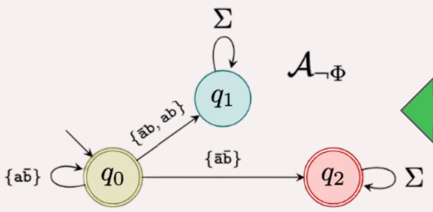
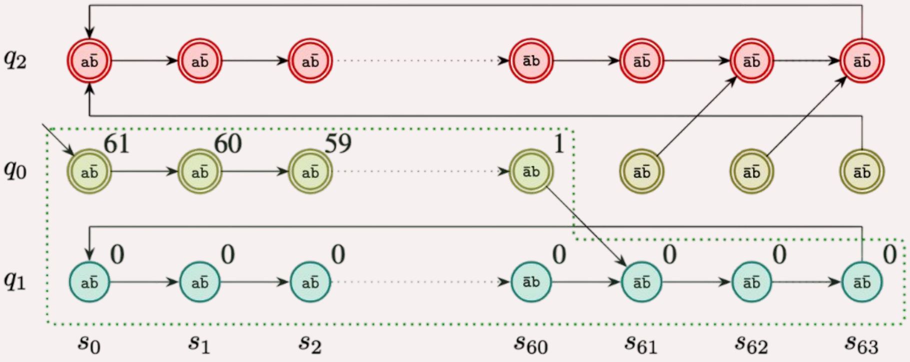


$\texttt{Phi} \equiv \Phi = (\mathsf{a}\,\mathsf{U}\,\mathsf{b})$

(b)



(c)

We encode the **negated specification** as an **ω**-regular automaton.

An **ω**-regular automaton's accepting traces **visit fair states infinitely often**.

$$\boldsymbol{s} \in S_0 \implies (\mathrm{reg}\,\boldsymbol{s}, q_0) \in I, \tag{1}$$

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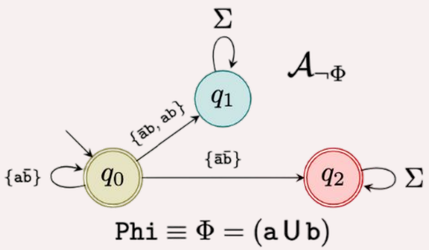
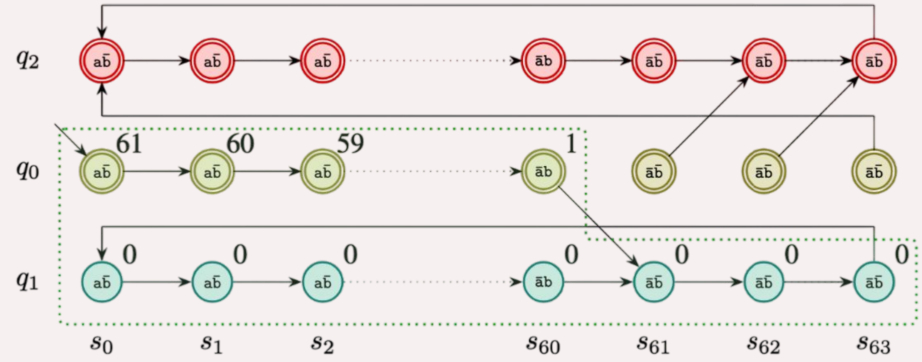


(b)



(c)

The **synchronous product** of model and negation automaton **accepts violation of the spec.**

$$\boldsymbol{s} \in S_0 \implies (\mathrm{reg}\ \boldsymbol{s}, q_0) \in I, \tag{1}$$

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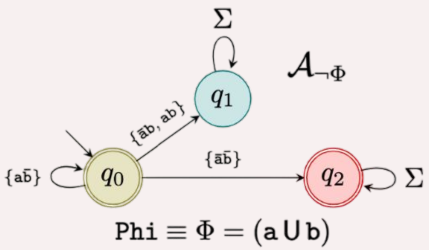
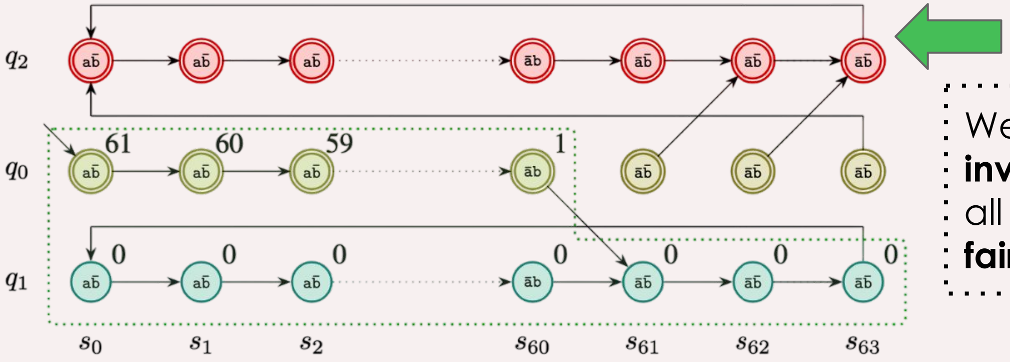


(b)



(c)

We first constructing an **invariant** that **excludes** all **cycles** containing **fair states**.

$$\begin{aligned}
\boldsymbol{s} \in S_0 \quad &\Longrightarrow (\mathrm{reg}\,\boldsymbol{s}, q_0) \in I, && (1)\\
(\boldsymbol{s}, q) \rightarrow_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (\boldsymbol{s}', q') \ \wedge\ (\mathrm{reg}\,\boldsymbol{s}, q) \in I \quad &\Longrightarrow (\mathrm{reg}\,\boldsymbol{s}', q') \in I, && (2)\\
(\boldsymbol{s}, q) \rightarrow_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (\boldsymbol{s}', q') \ \wedge\ (\mathrm{reg}\,\boldsymbol{s}, q) \in I \quad &\Longrightarrow V(\mathrm{reg}\,\boldsymbol{s}, q) \succeq V(\mathrm{reg}\,\boldsymbol{s}', q'), && (3)\\
(\boldsymbol{s}, q) \rightarrow_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (\boldsymbol{s}', q') \ \wedge\ (\mathrm{reg}\,\boldsymbol{s}, q) \in I \ \wedge\ q \in F \quad &\Longrightarrow V(\mathrm{reg}\,\boldsymbol{s}, q) \succ V(\mathrm{reg}\,\boldsymbol{s}', q'). && (4)
\end{aligned}$$

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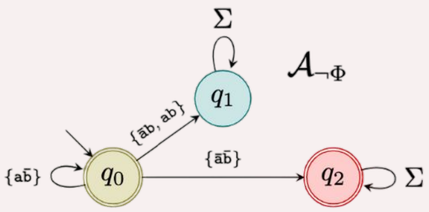
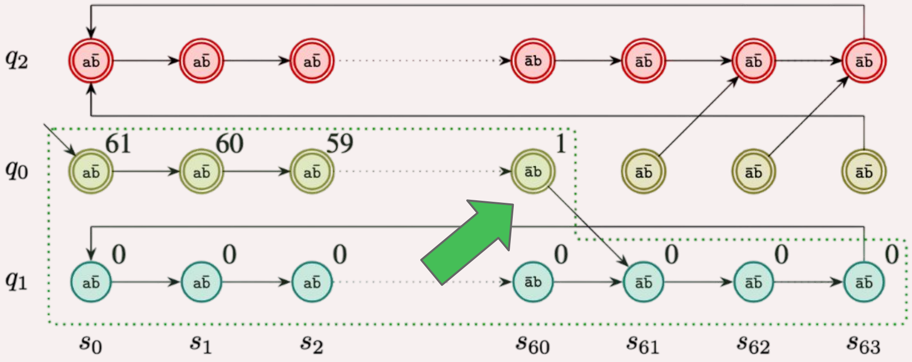


$\text{Phi} \equiv \Phi = (\mathsf{a}\,\mathsf{U}\,\mathsf{b})$

(b)

Inside this invariant we assign each state a **ranking bounded from below**, ensuring along any trace the rank **never increases** and **strictly decreases** whenever a **fair state** occurs.



(c)

Since the rank can only **decrease finitely many times**, every trace visits **fair states finitely many times** and thus cannot be accepting—which means the **property holds**.

$$s \in S_0 \implies (\mathrm{reg}\, s, q_0) \in I, \tag{1}$$

$$(s, q) \to_{\mathcal{M}\|\mathcal{A}_{\neg\Phi}} (s', q') \land (\mathrm{reg}\, s, q) \in I \implies (\mathrm{reg}\, s', q') \in I, \tag{2}$$

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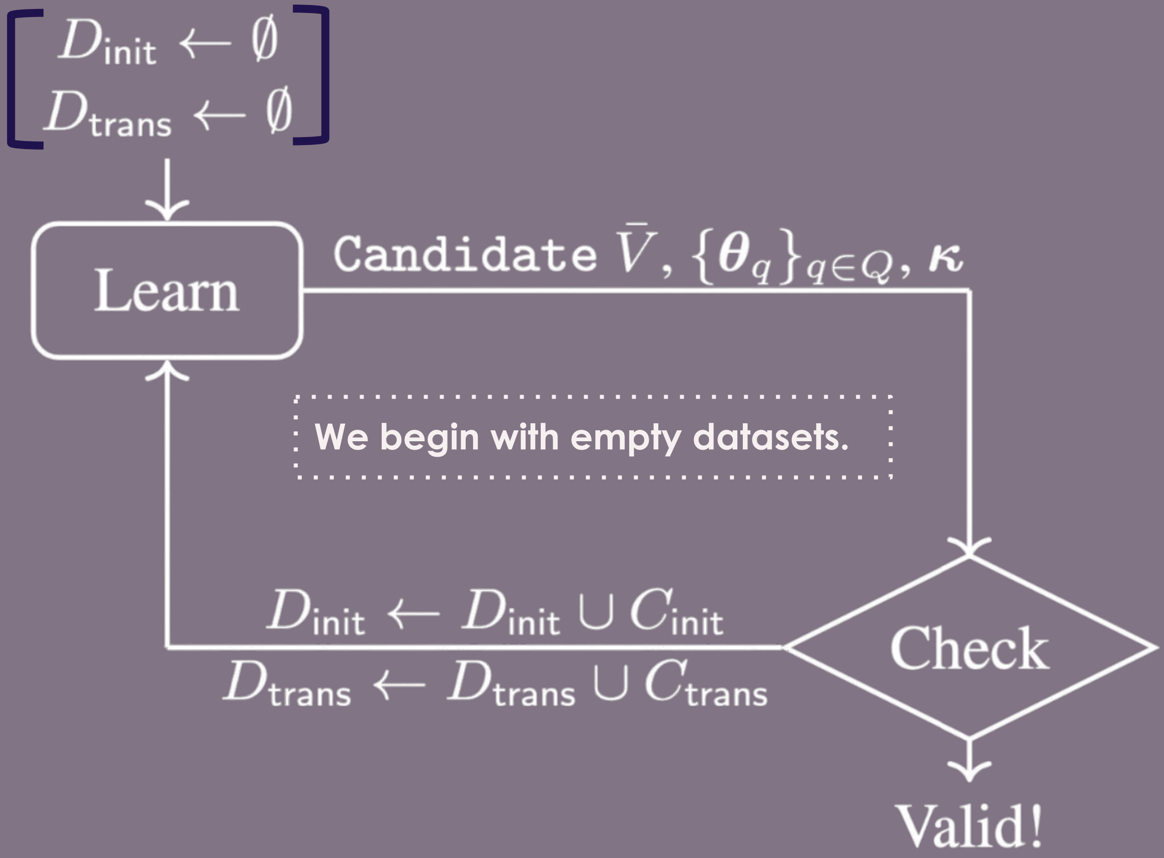
We learn a single function—represented by a neural network—that is both an invariant and a ranking function.

Learn Check Loop



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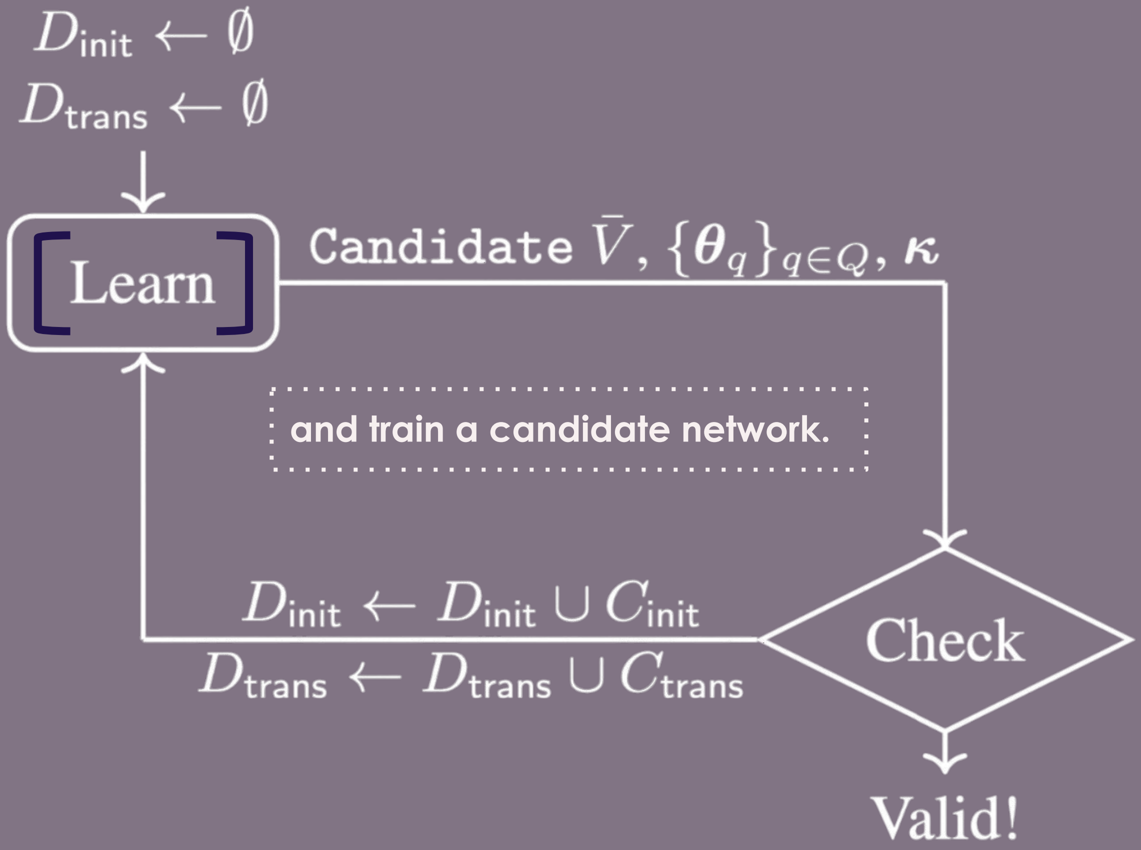

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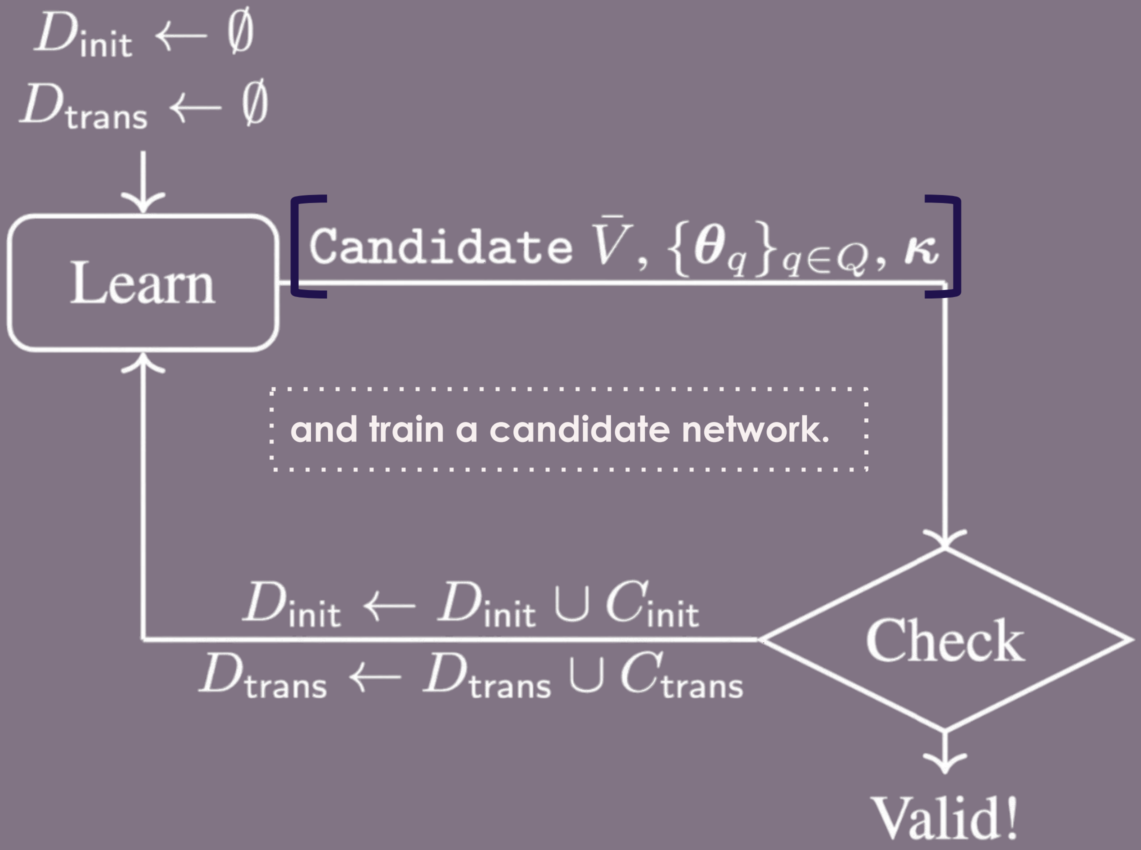

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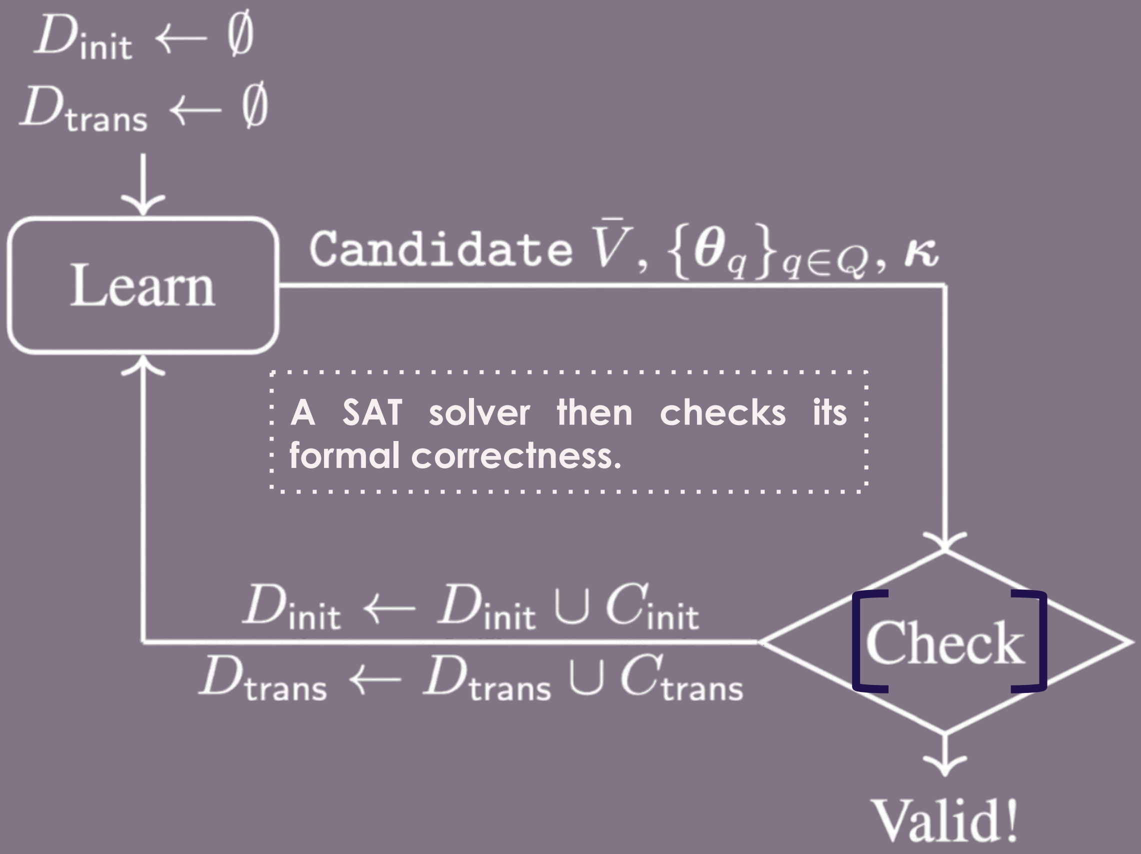

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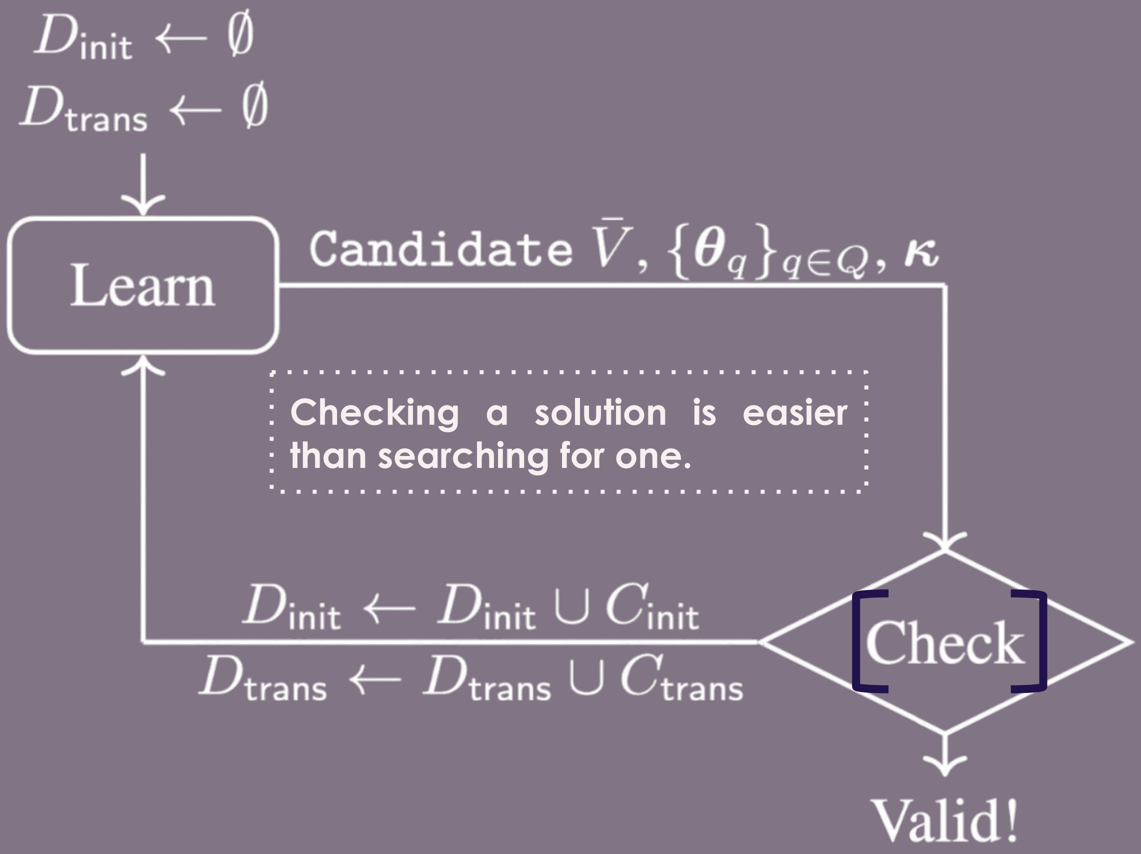

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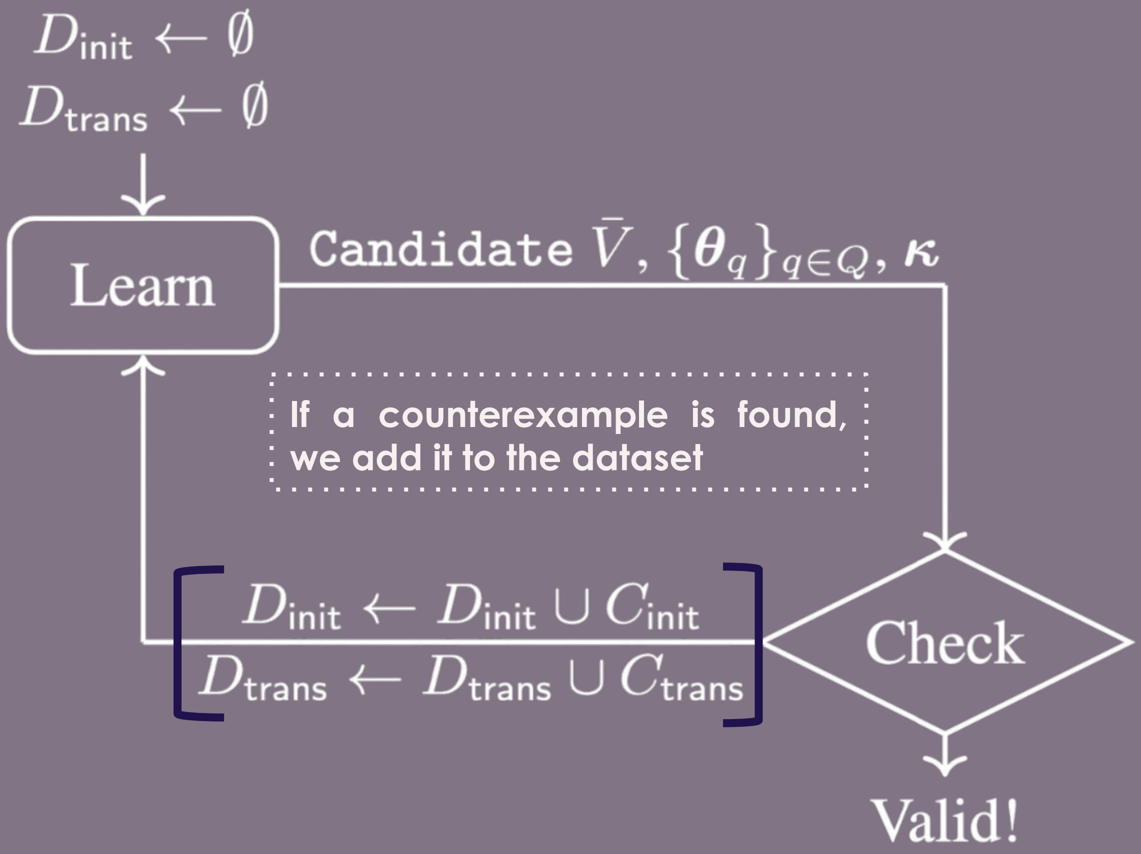

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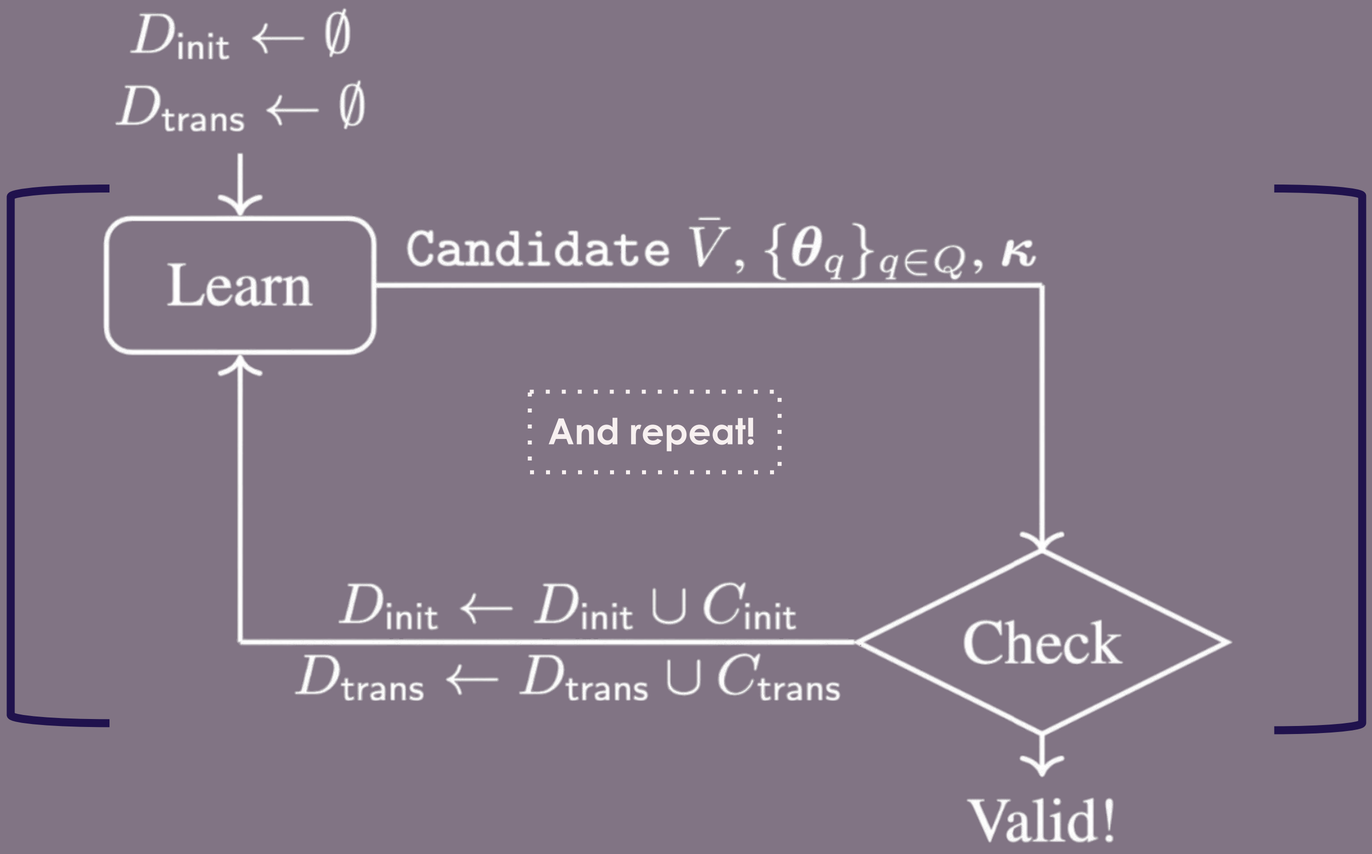

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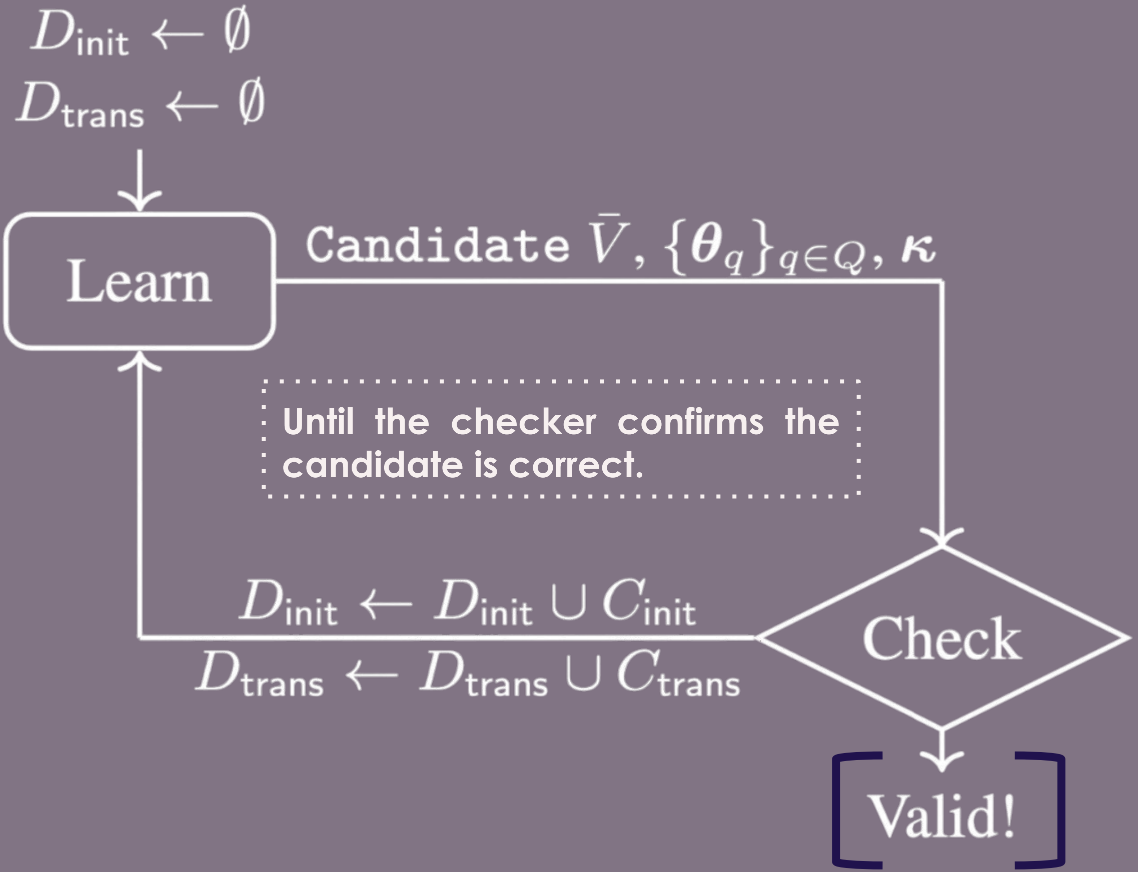
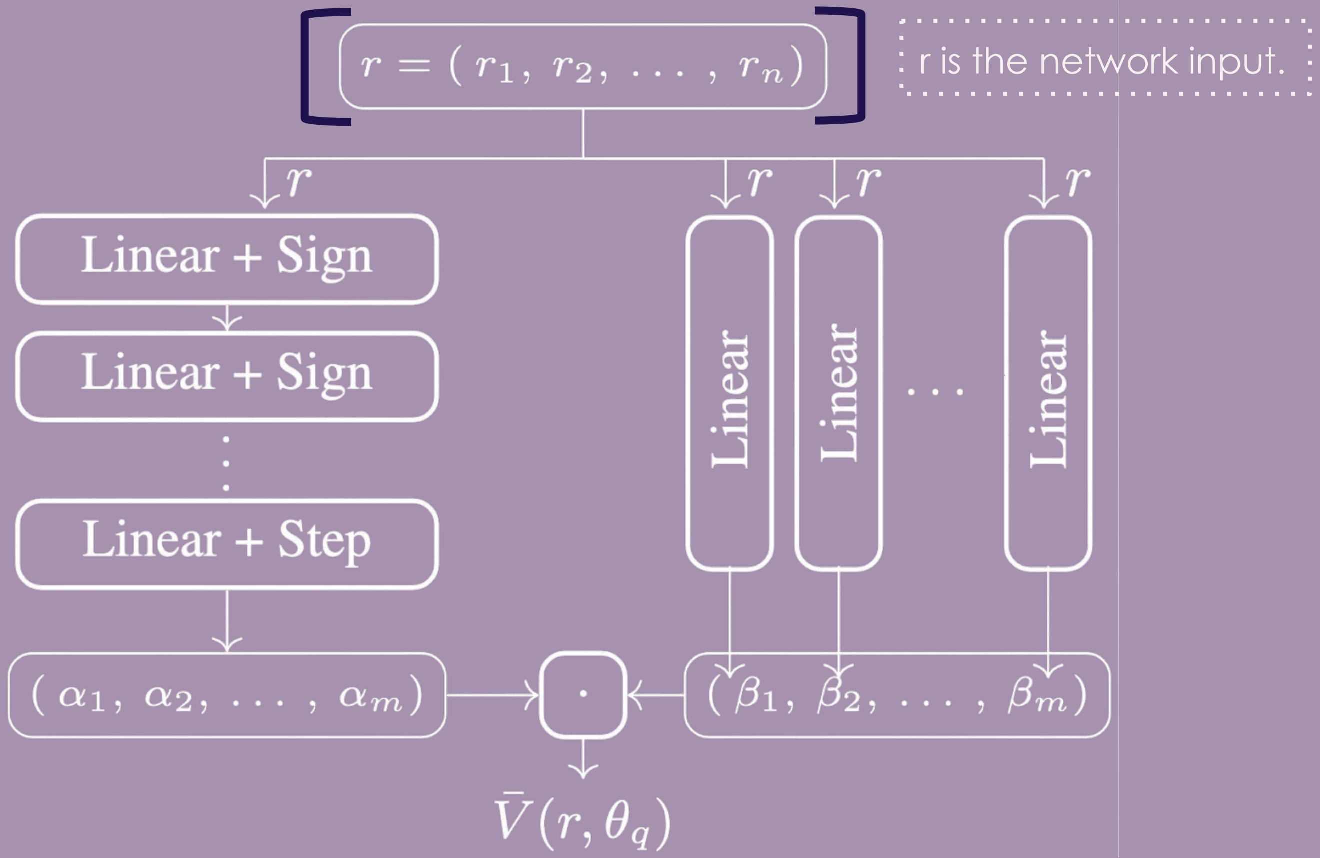


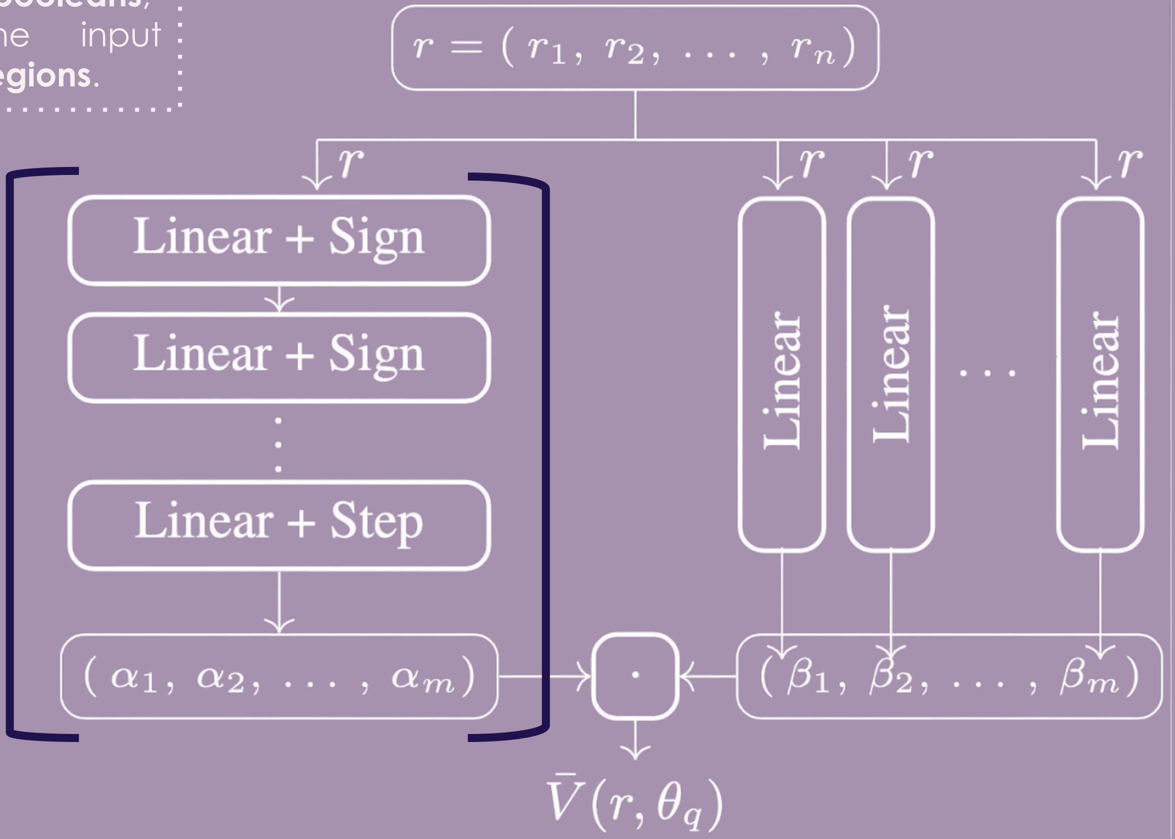
This allows for a task aware sampling approach keeping the dataset small allowing MILP learning.

Network Architecture



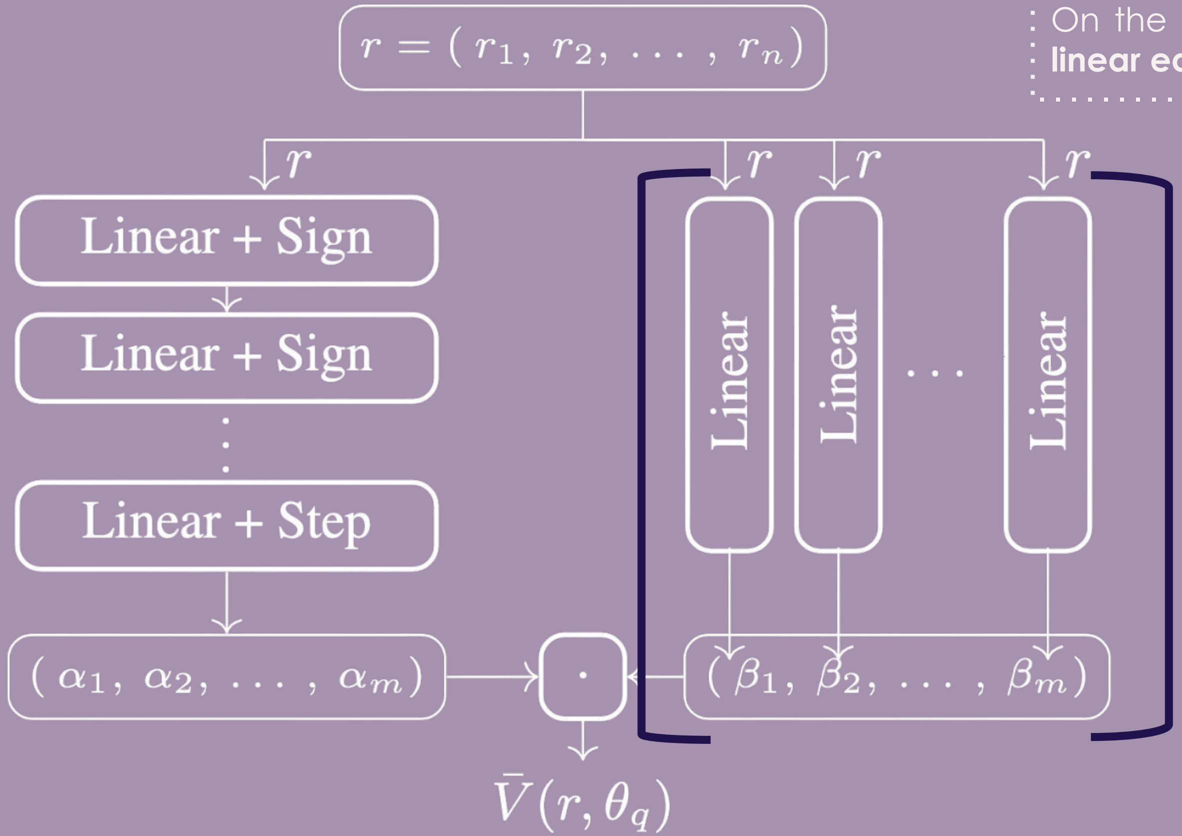
Network Architecture

On the **left**, a **fully connected** network with **sign and step** activations produces **m booleans**, partitioning the input space into **2^m regions**.

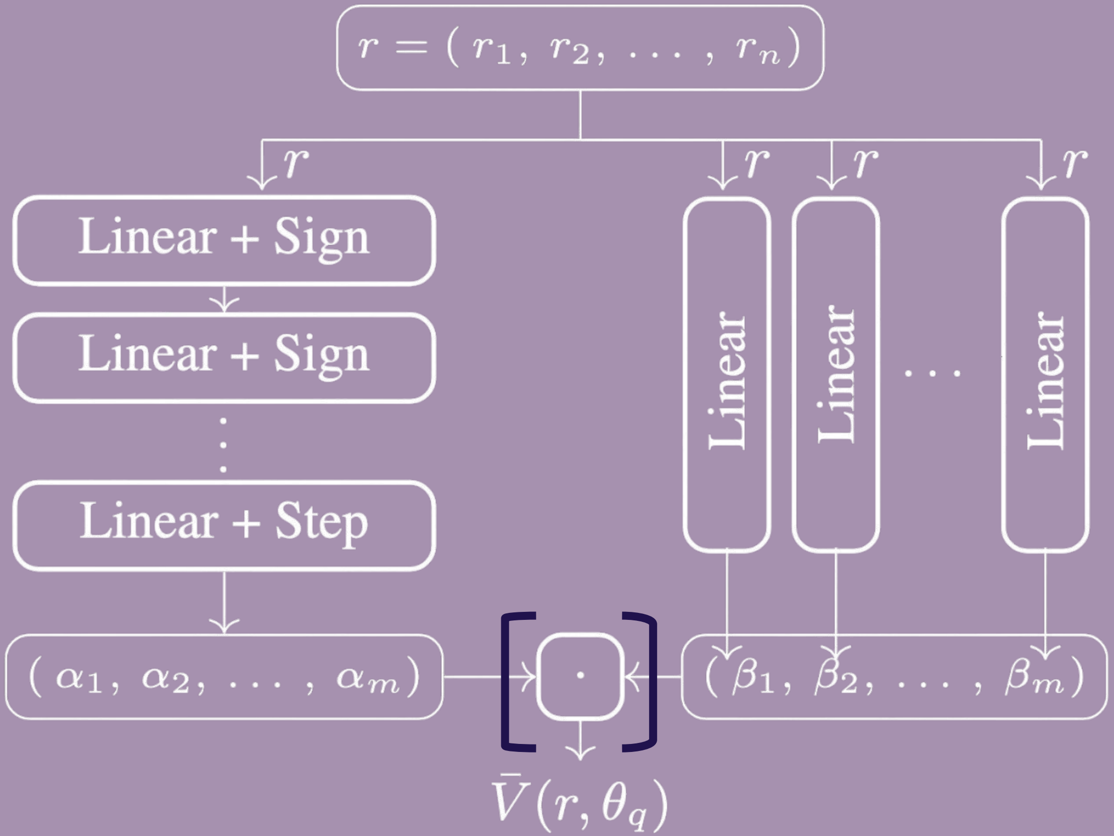


Network Architecture

On the **right**, we have **m linear equations** on input r.



Network Architecture



These linear **equations** on the **right** are **masked** by the **booleans on the left**, and their **dot product** assigns **one affine equation for each of the 2^m region**.

The architecture thus realises piecewise linear function, while allowing solver friendly MILP Encoding.

Thanks!