A/ Part/ition/ Cover/ Approach/ to/ Token/ization

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Contributions

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1. An intuitive proof that tokenization is NP-hard via a reduction from Vertex Cover

2. Partition cover formulation, independent of merge patterns

3. Proposed GreedTok, a novel base text tokenization algorithm with better compression.

4. Empirical approximation of objective.

5. Downstream language pretraining comparison.

Tokenization is NP-Hard

Finding: An optimal solution for *Tok* decision problem is the solution for vertex cover and vice versa for vertex cover problem instances.

Tokenization search problem (*Tok*)

Given

- Fixed alphabet $\boldsymbol{\Sigma}$
- Base token set $\mathbf{B} = \{(w) : w \in \boldsymbol{\Sigma}\}$ as all singleton characters
- Corpus $\mathscr{C} = (\mathbf{W}, \mathrm{COUNT})$
- Token budget $k \in \mathbb{N} > 0$
- Set of candidate tokens $\mathbf{T} \subseteq \boldsymbol{\Sigma}^+$

Goal: Find a subset $\mathbf{S} \subseteq \mathbf{T}$ such that $|\mathbf{S}| \leq k$ and minimise

$$\sum_{W \in \mathbf{W}} \mathrm{PARTITION}(\mathrm{W}, \mathbf{S} \cup \mathbf{B}) \cdot \mathrm{COUNT}(W).$$

Tokenization decision problem: given integer threshold $\mathscr{L} \in \mathbb{N} > 0$

Determine whether there exists a subset $\mathbf{S} \subseteq \mathbf{T}$ such that $|\mathbf{S}| \leq k$ and

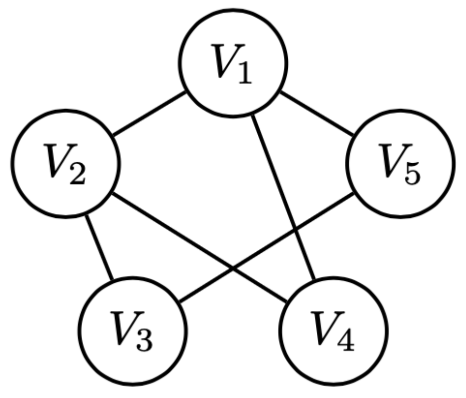
$$\sum_{W \in \mathbf{W}} \mathrm{PARTITION}(\mathrm{W}, \boxed{\mathbf{S} \cup \mathbf{B})} \cdot \mathrm{COUNT}(W) \leq \mathscr{L}.$$

Differentiate between singletons $\mathbf{B}$ and selected candidate tokens $\mathbf{S}$

Representing strings as graphs, given:

- Arbitrary vertex cover graph $\mathscr{G} = (\mathbf{V}, \mathbf{E})$.
- Set of words as $\mathbf{W} = \{W_{i,j} : V_i, V_j \in \mathbf{E}\}$
- Each word $W_{i,j} = (@, V_i, @, V_j, @)$, with a count of 1
- Set of candidate tokens as $\mathbf{T} = \{(@, V_i, @) : V_i \in \mathbf{V}\}$
- Set $\mathscr{L} = 3|\mathbf{W}| = 3|\mathbf{E}|$ and vertex cover's k to *Tok*'s k.

Notice that every edge ⇔ word and every vertex ⇔ token



$$\begin{aligned}
\boldsymbol{\Sigma} &= \{V_1, V_2, V_3, V_4, V_5, @\} \\
\mathbf{B} &= \{(V_1), (V_2), (V_3), (V_4), (V_5), (@)\} \\
\mathbf{W} &= \{(\underline{@, V_1, @}, V_2, @), (\underline{@, V_1, @}, V_4, @), (\underline{@, V_1, @}, V_5, @), \\
&\qquad (@, V_2, \underline{@, V_3, @}), (@, V_2, \underline{@, V_4, @}), (\underline{@, V_3, @}, V_5, @)\} \\
\textsc{count}(W) &= 1, \quad \forall W \in \mathbf{W} \\
\mathbf{T} &= \{\underline{(@, V_1, @)}, (@, V_2, @), \underline{(@, V_3, @)}, \underline{(@, V_4, @)}, (@, V_5, @)\} \\
k &= k \\
\ell &= 3|\mathbf{W}| = 3|\mathbf{E}| = 18
\end{aligned}$$

An example tokenization problem instance. The tokens corresponding to the vertex cover $\mathbf{S} = \{V_1, V_3, V_4\}$ are underlined in $\mathbf{T}$. A possible tokenization of $\mathbf{W}$ using $\mathbf{S} \cup \mathbf{B}$ is also given with tokens in $\mathbf{S}$ being underlined, showing that each word in $\mathbf{W}$ only needs 3 tokens.

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GreedTok: Our greedy tokenizer

Algorithm design: from partition to covers

Cover Pairs and not singletons

Tokens

T1 : BA

T2 : NA

Example: B A N A N A

As pairs: T1 T2 T2 ✓

We set constraints such that the ends of the cover must be uncovered to demarcate valid covers/separations.

As singles: T1 T1 T2 T2 T2 T2 ?

Covering as singles introduces additional complications when verifying whether a cover is valid for updating scores.

Minimizing partitions is maximizing covers

$$\text{Length of word} = \text{\# covers} + \text{\# partitions} = 3 + 3 = 6$$

Relax MIP (Min. Partition) → Max. Cover with constraints → Greed tokens for potential covers

GreedTok: Our greedy tokenizer

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Design comparisons

Base Text Tokenizers

	BPE	Unigram	GreedTok
Starting Tokens	Singletons	All possible	All possible
Token Selection	Greed *k*	Eliminate all but *k*	Greed *k*
Objective	Max pairwise merges between selected tokens	Max subword probabilities	Max valid singleton-pair covers
Solving Complexity	$O(k \cdot \sum_{W \in \mathbf{W}} \lvert W \rvert)$	$O(\lvert \mathbf{T} \rvert \cdot \log k \cdot \sum_{W \in \mathbf{W}} \lvert W \rvert)$	$O(\lvert \mathbf{T} \rvert \cdot \boxed{\cancel{k}} \cdot \sum_{W \in \mathbf{W}} \lvert W \rvert)^{*}$
Encoding Complexity	$O(\lvert W \rvert^2)$	$O(k \lvert W \rvert)$	$O(\lvert W \rvert^2 \log W)$

Evaluating GreedTok's Compression

Finding: GreedTok has better compression than BPE and Unigram.

Dataset	$\lvert\mathbf{W}\rvert$	$\sum_{W}^{\mathbf{W}} c_W$	$\lvert\mathbf{T}\rvert$	max $\lvert\mathbf{S}\rvert$	Time
UN	105K	37M	884K	5K	6s
arχiv	881K	366M	7,626K	5K	63s
wiki	8,769K	2,949M	93.5M	10K	11m
PubMed	6,527K	4,149M	121M	10K	11m

Table 1 in paper

These corpora contains **100K to 8M unique space-delimited words**, with of **800K to 121M token candidates**. The runtime of GreedTok is competitive, with the longest duration being **11 minutes** on AMD EPYC 9654 @ 2.40GHz.

GreedTok (GTK) outperforms BPE and Unigram in larger corpora (arXiv, PubMed, wiki), with mean improvement of **2.88% over BPE** and **3.43% over UNIGRAM**.

	k	1000	2000	3000	4000	5000		2000	4000	6000	8000	10000
GTK Tokens/Word	UN	1.607	1.374	1.268	1.205	1.163	PubMed	1.603	1.397	1.301	1.244	1.206
BPE Tokens/Word	UN	1.688	1.431	1.311	1.241	1.194	PubMed	1.650	1.431	1.328	1.266	1.225
GTK's Improvement (%)	UN	**4.86**	**3.99**	**3.33**	**2.92**	**2.54**	PubMed	**2.85**	**2.38**	**2.02**	**1.75**	**1.52**
UNIGRAM Tokens/Word	UN	1.655	1.385	1.261	1.193	1.148	PubMed	1.699	1.465	1.359	1.297	1.257
GTK's Improvement (%)	UN	**2.90**	**0.78**	-0.51	-0.97	-1.30	PubMed	**5.63**	**4.63**	**4.21**	**4.05**	**4.02**
GTK Tokens/Word	arχiv	1.742	1.475	1.349	1.275	1.226	wiki	1.692	1.489	1.389	1.326	1.283
BPE Tokens/Word	arχiv	1.837	1.551	1.407	1.320	1.263	wiki	1.731	1.519	1.413	1.347	1.301
GTK's Improvement (%)	arχiv	**5.12**	**4.94**	**4.15**	**3.41**	**2.89**	wiki	**2.26**	**1.98**	**1.71**	**1.53**	**1.37**
UNIGRAM Tokens/Word	arχiv	1.793	1.558	1.444	1.378	1.332	wiki	1.793	1.558	1.444	1.378	1.332
GTK's Improvement (%)	arχiv	**6.74**	**4.92**	**4.30**	**3.96**	**3.88**	wiki	**5.62**	**4.43**	**3.84**	**3.75**	**3.70**

Table 2 in paper

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Evaluating GreedTok's Approximability

We can empirically show that GreedTok is $d_{inst}(1 - 1/e)$!

Maximum Coverage Greedy is a $1 - 1/e$ algorithm, best for Maximum Coverage Problem.

However, *Tok* is *neither submodular nor supermodular*; uncertain if GreedTok's approximation ratio is $1 - 1/e$; see Appendix A

Weighted Maximum Coverage's Greedy (GreedWMC) is GreedTok's upper bound.

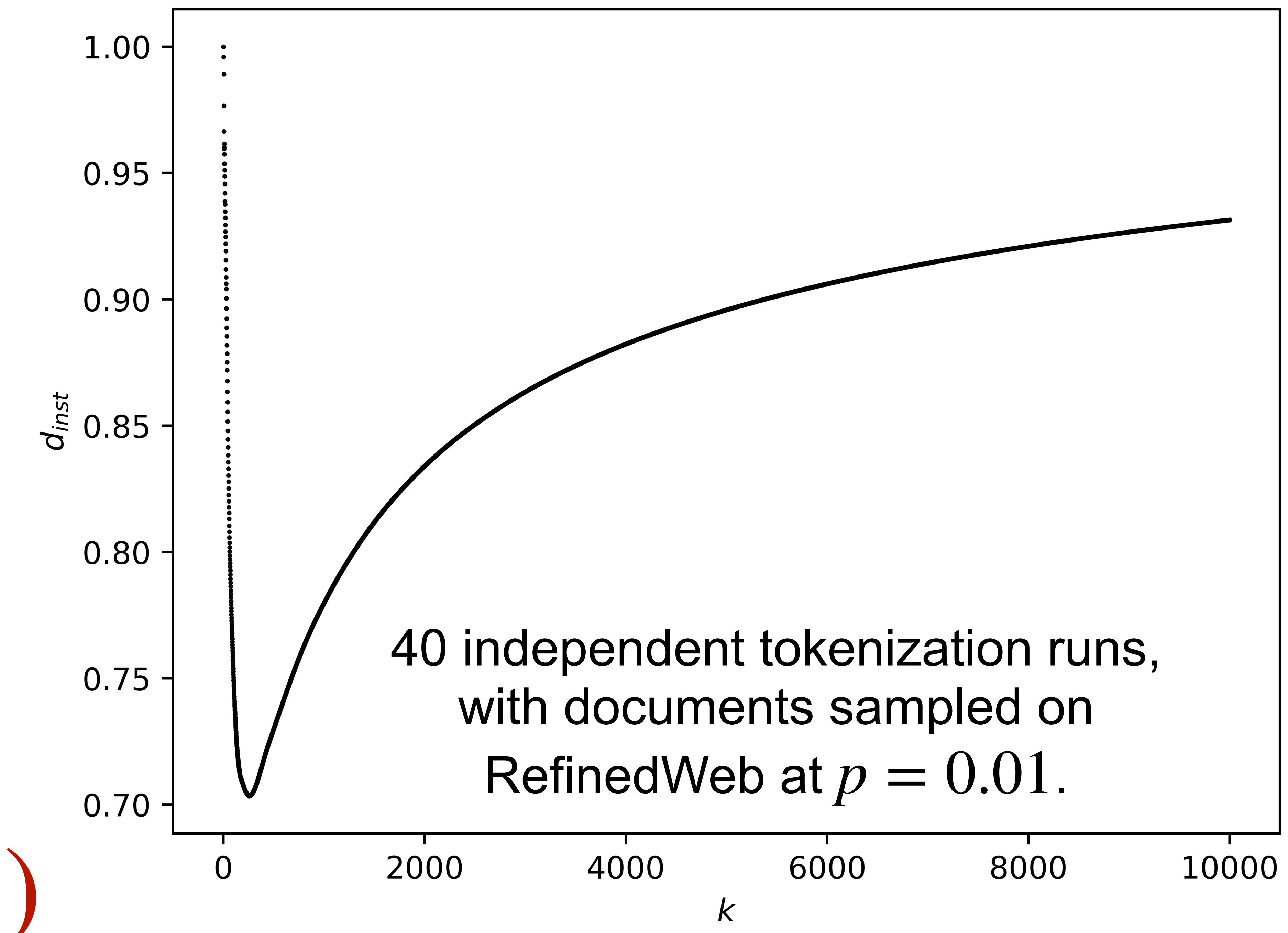
Score: # cover in *instance*

Optimal score for WMC $\geq$ Optimal for *Tok*

GreedWMC score

GreedTok score

$$d_{inst} = \frac{GreedTok}{GreedWMC}$$



For large *k*, GreedTok is $0.9(1 - 1/e)$

Evaluating GreedTok's Language Pretraining

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Experiment Design: GreedTok vs BPE, 1B-decoder-only Transformers, token sets 75% similar

The token count statistics for all three settings. GreedTok uses nearly 18% fewer tokens to represent the entire DCLM Dedup dataset. The total training tokens used is around 629B tokens. Dataset % differences due to tokenised compression and checkpointing.

Experiment Name	Tokenizer	Full dataset tokens	Training tokens	Dataset %
BPEM	BPE	$8.94 \cdot 10^{11}$	$6.29 \cdot 10^{11}$	70.35%
Equal Tokens (GTET)	GREEDTOK	$7.35 \cdot 10^{11}$	$6.29 \cdot 10^{11}$	85.58%
Equal Proportion (GTEP)	GREEDTOK	$7.35 \cdot 10^{11}$	$5.03 \cdot 10^{11}$	68.47%

Table 3 in paper

Our two GreedTok settings:

- **GTET**: trains on an equal amount of tokens compared to BPEM
- **GTEP**: trains on an comparable amount of text coverage compared to BPEM, note that this results in GTEP training on fewer tokens due to higher compression

Evaluating GreedTok's Language Pretraining

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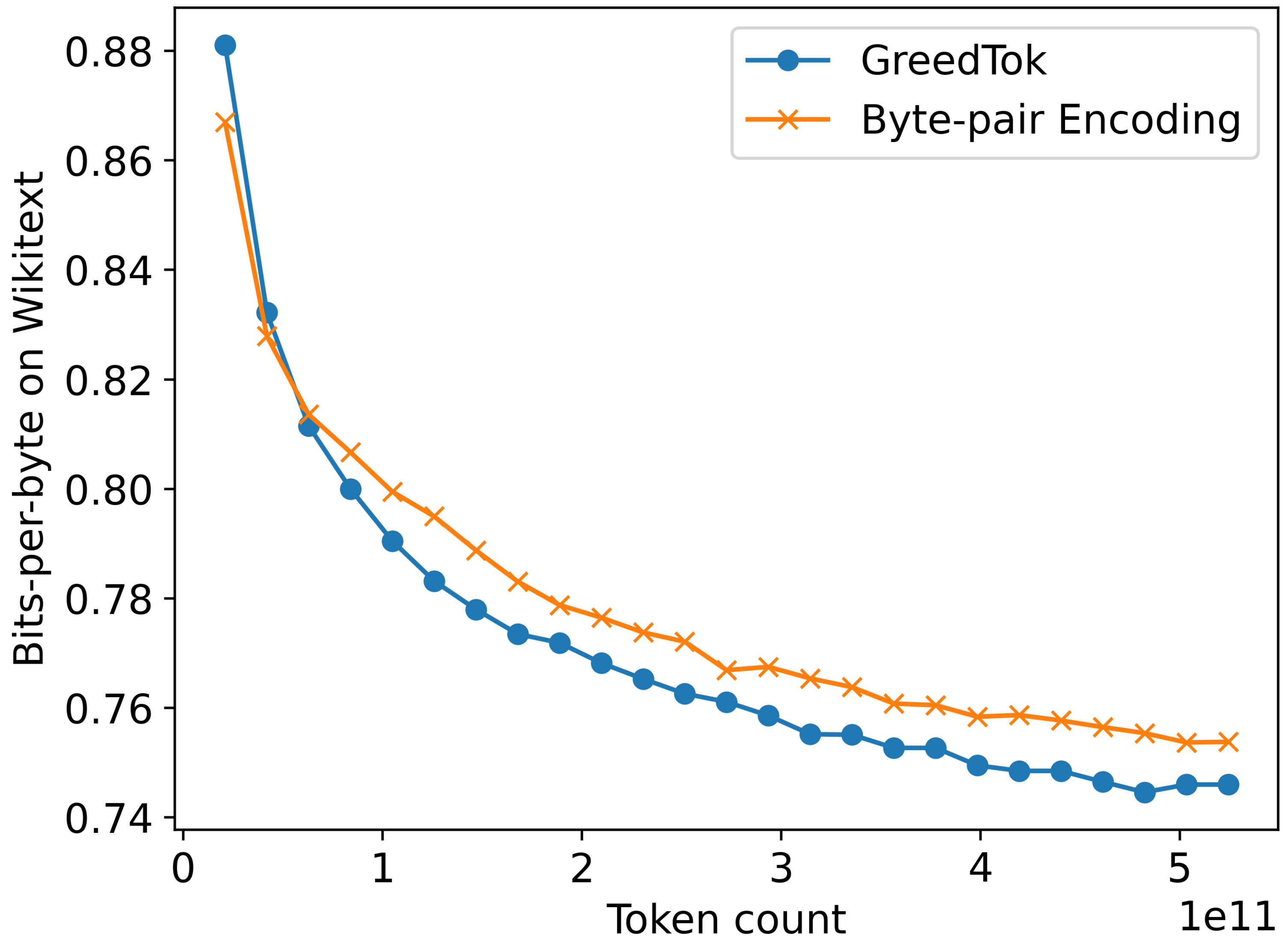
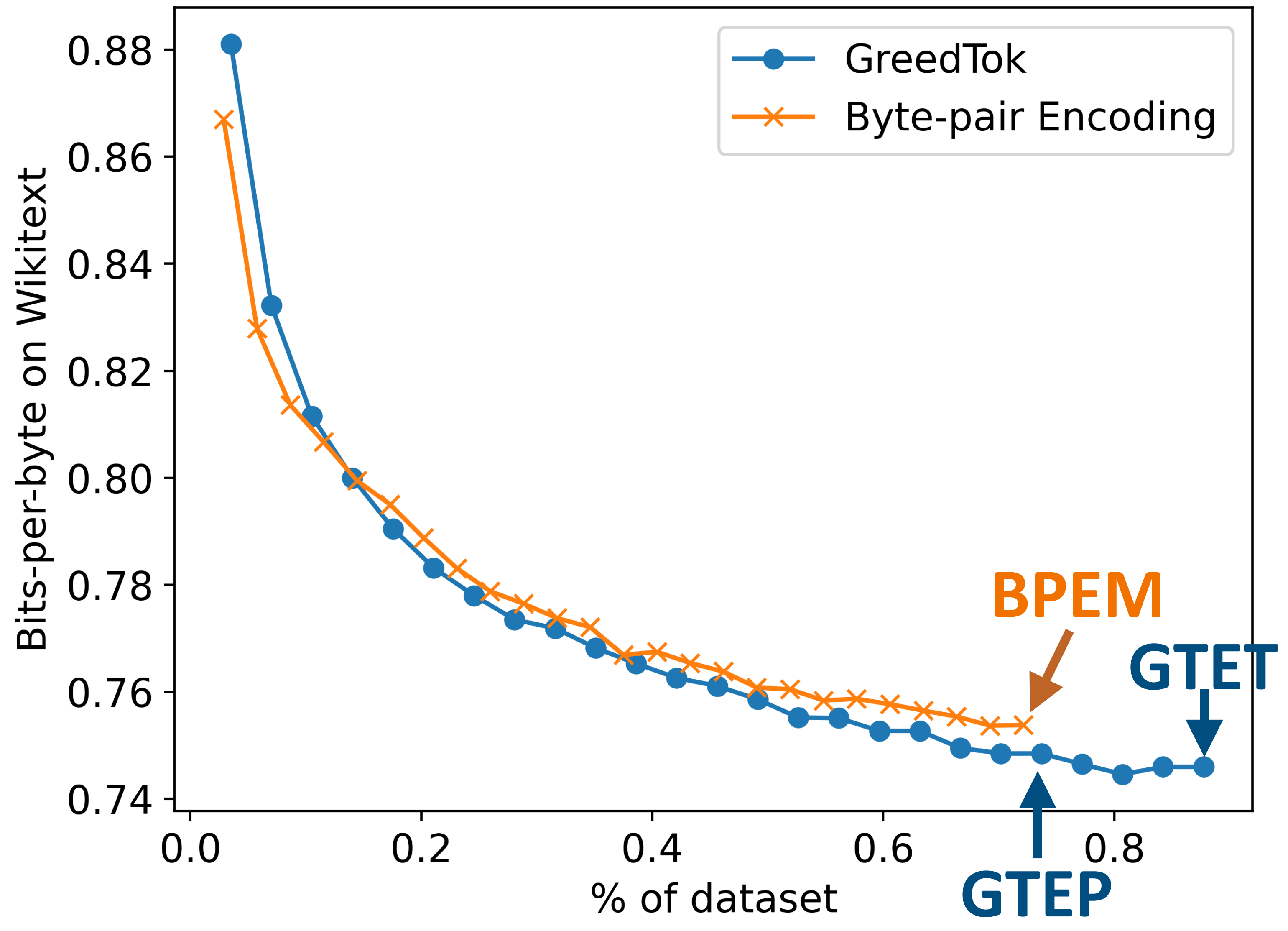
Results: GreedTok is ahead/comparable, on common benchmarks and bits/bytes.





We plot the bits/byte improvement across phase 1 training for model using GreedTok and BPE on different scales. The bits/byte metric is independent of tokenization and reflects true compression performance on the underlying data. Since both GTET and GTEP are equivalent in phase 1 for the first 100,000 steps, we examine bits/byte improvement on Wikitext with different scales on the x-axes.

	Accuracy (normalized)						*Accuracy*						*bits/byte*
	ARC-c	ARC-e	Hella-Swag	OBQA	PIQA	SciQ	BoolQ	COPA	LAMB-BADA	RACE	Wino-grande	*Avg.*	Wikitext
BPEM	36.2	67.9	65.6	40.0	75.7	89.8	65.8	81.0	61.1	36.4	62.8	62.0	0.7066
GTEP	37.6	68.8	64.9	39.6	75.6	90.0	67.6	79.0	63.9	36.8	**63.5**	62.5	0.7028
GTET	**38.3**	**70.0**	**65.7**	**40.6**	**75.8**	**90.5**	**67.7**	**82.0**	**64.6**	**37.7**	62.6	**63.2**	**0.6989**

Thanks!

Looking for industry positions!

Looking for academic positions!

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arXiv

Github