Data Fusion for Partial Identification of Causal Effects

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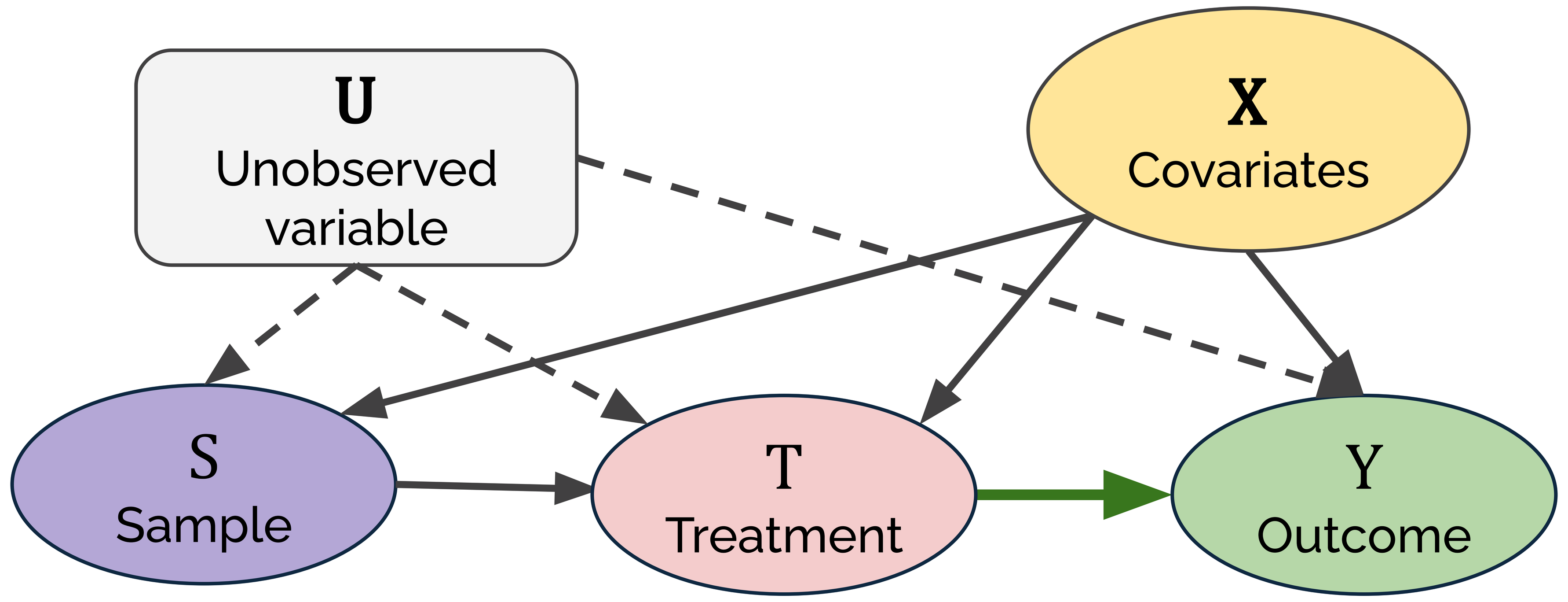
Full Paper

NEURAL INFORMATION PROCESSING SYSTEMS

Contributions

- *Interpretable sensitivity parameters*:
  - γ quantify the extent of external validity violations
  - ρ quantify extend of unmeasured confounding.
- *Doubly robust estimator* for estimating treatment effect bounds as a function of (γ, ρ), without relying on strong distributional or parametric assumptions.
- Operationalize an efficient *breakdown frontier analysis*, which characterizes regions in the (γ, ρ) space where the treatment effect remains conclusively positive (or negative)
  - Allowing for assessment of the robustness of causal conclusions under simultaneous assumption violations.

Setup & Assumptions



We aim to estimate the ATE (i.e. the arrow from T → Y).

For identification, we require either*:

A.1. No Unobserved Confounding in Observational Data

$$(Y(1),Y(0)) \perp T \mid \mathbf{X}=\mathbf{x}, S=0$$

A.2. Study exchangeability

$$(Y(1),Y(0)) \perp S \mid \mathbf{X}=\mathbf{x}$$

*In addition to SUTVA, consistency, positivity, and internal validity of the RCT.

Partial ID Framework

Estimable quantities:

$g_s(\mathbf{x})$	Sample probability: $P(S=s \mid \mathbf{X}=\mathbf{x})$
$e_t(\mathbf{x},s)$	Propensity score: $P(T=t \mid \mathbf{X}=\mathbf{x}, S=s)$
$\mu(\mathbf{x},s,t)$	Expected outcome: $E[Y(t) \mid \mathbf{X}=\mathbf{x}, S=s]$

Target: $$E[Y(t)|\mathbf{X}=\mathbf{x}] = g_1(\mathbf{x})\mu(\mathbf{x},1,t) + g_0(\mathbf{x})\mu(\mathbf{x},0,t)$$

Identified — Not identified without A1 or A2

ρ: Level of violation to conditional ignorability.

γ: Level of violation to study exchangeability.

$$\rho = \left|1 - \frac{E[Y(t)|\mathbf{X},S=0,T=1-t]}{E[Y(t)|\mathbf{X},S=0,T=t]}\right| \quad \gamma = \left|1 - \frac{E[Y(t)|\mathbf{X},S=0]}{E[Y(t)|\mathbf{X},S=1]}\right|$$

Without Assumption A or B, We use ρ and γ to bound $\mu(x,0,t)$

$$g_1(\mathbf{X})\mu(\mathbf{X},1,t)+g_0(\mathbf{X})\max\{w(\mathbf{X}, t, -\rho), v(\mathbf{X}, t, -\gamma)\} \leq E[Y(t)|\mathbf{X}] \leq g_1(\mathbf{X})\mu(\mathbf{X},1,t)+g_0(\mathbf{X})\min\{w(\mathbf{X}, t, \rho), v(\mathbf{X}, t, \gamma)\}$$

where

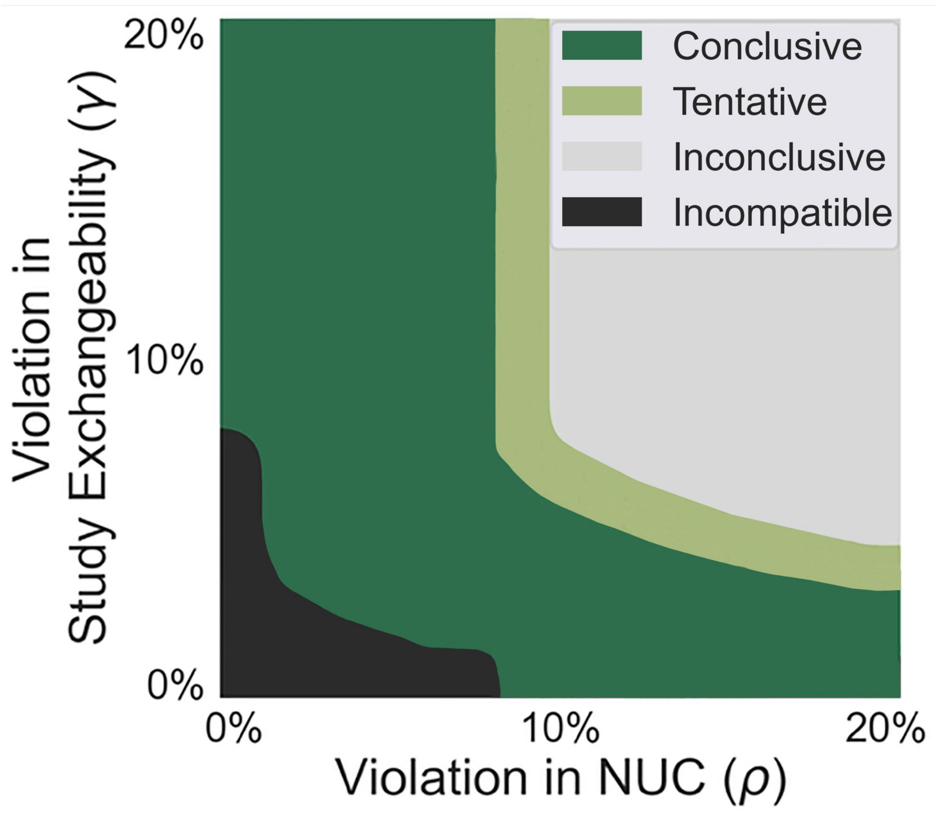
$$w(\mathbf{x},t,\rho) = e_t(\mathbf{x},0)\mu(\mathbf{x},0,t)+e_{1-t}(\mathbf{x},0)(1+\rho)\mu(\mathbf{x},0,t),$$
$$v(\mathbf{x},t,\gamma) = (1+\gamma)\mu(\mathbf{x},1,t)$$

Estimation

1. Approximate min and max using the Boltzmann operator to create smooth, differentiable approximations of these bounds.
2. Derive the efficient influence functions.
3. Construct bias corrected estimators.

Breakdown Frontier Plots

We assess how conclusions change under varying degrees of Assumptions A and B violations by estimating treatment effect (TE) bounds across a grid of (γ, ρ) values and visualizing them in a Breakdown Frontier Plot (Masten & Poirier, 2020).

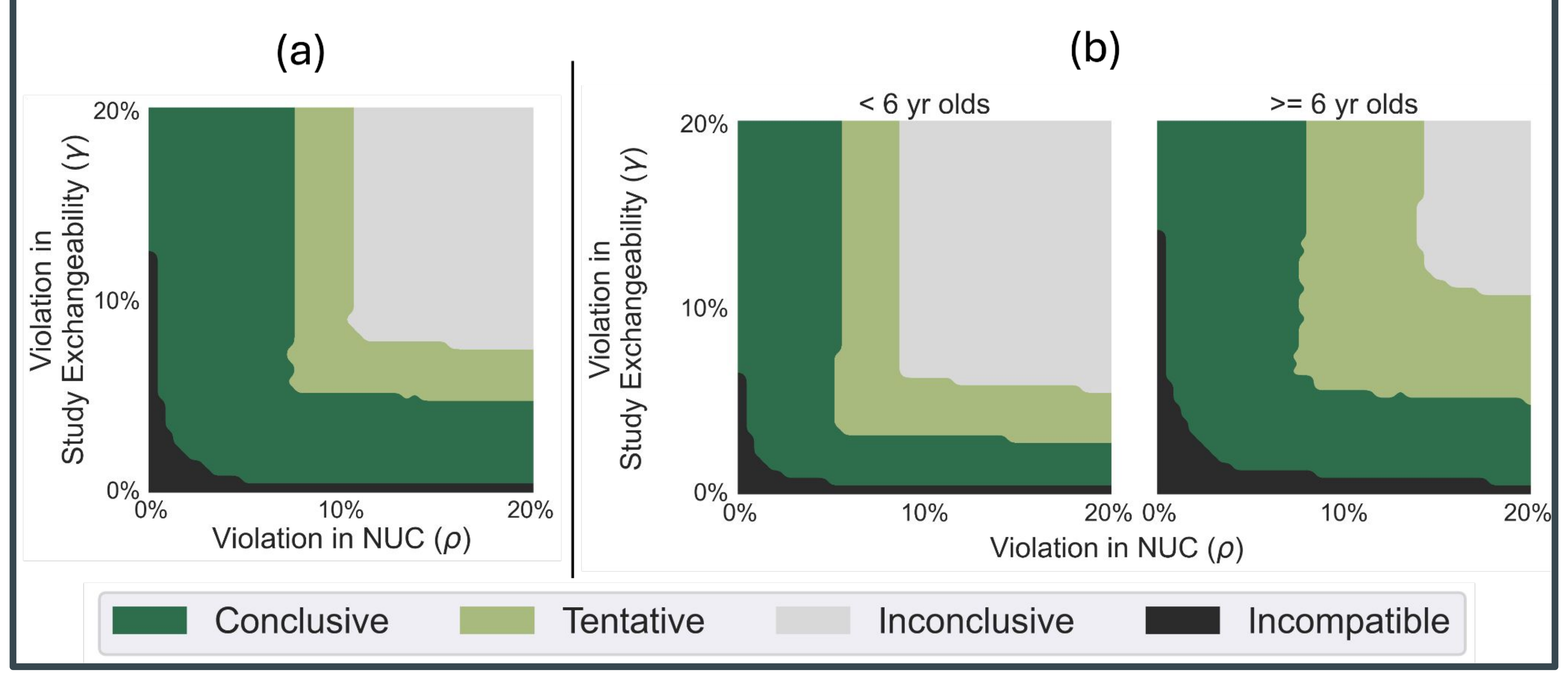


- Conclusive: TE is + (or –) with $(1-\alpha)$ confidence level.
- Tentative: TE bound point estimates are both + (or –)
- Inconclusive: TE bound point estimates intersect zero,
- Incompatible: γ and ρ imply assumptions that contradict observed discrepancies between the study group

Project STAR

We apply our framework to Project STAR, a large-scale study conducted in TN to investigate the effect of class size on student learning (Mosteller, 1995; Achilles et al., 2008). We explore:

(a) ATE and **(b)** CATE for students who enrolled in kindergarten before age six (left) and at six or older (right).



References: Achilles, C., Bain, H. P., Bellott, F., Boyd-Zaharias, J., Finn, J., Folger, J., Johnston, J., and Word, E. (2008). Tennessee's student teacher achievement ratio (star) project; Masten, M. A. and Poirier, A. (2020). Inference on breakdown frontiers. *Quantitative Economics*, 11(1):41–111; Mosteller, F. (1995). The tennessee study of class size in the earlyschool grades. *Future of Children*, 5:113–127;