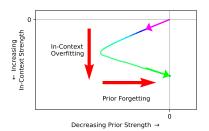
# Prior Forgetting and In-Context Overfitting

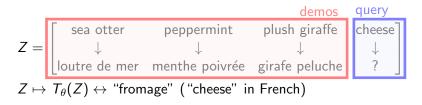
### Sungyoon Lee

Department of Computer Science, Hanyang University



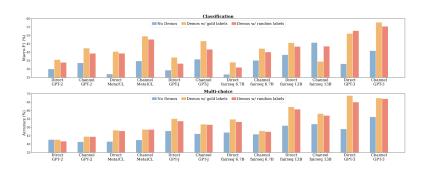
#### NeurIPS 2025

# What Makes LLMs Successful (In-Context Learning; ICL)



In-Context Learning (ICL): a model's ability to learn (without updating any model parameter) and perform a new unseen task from a few demonstrations of input-output pairs given at test time.

### Do Demos Matter?



No Demos < Good Demos ≈ Bad Demos [Min+22]

[Min+22; Lyu+23; Yoo+22; Wei+23; Shi+23]

### ICL = TR + TL

-	pre)training	inference (forward pass)
classical(?) DNN Transformer		feature extraction ICL ("weight(?)" update)

- ➤ TR (Task Recognition): the model recalls similar functions and concepts learned (priorly) in the pretraining phase
- ► TL (Task Learning): the model smoothly adapts to and implicitly learns the (observed) in-context task.

How do ICL abilities (TR+TL) emerge (and disappear) during pretraining?

### Overview

- Simple Dynamics
  - ▶ in-context linear regression
  - lacktriangle single-layer linear self-attention model ightarrow two-parameter transformer
- ► TR-TL Decomposition
  - demonstration-query task independence
  - noncentral task distribution
- New Phenomena
  - Prior Forgetting
  - In-Context Overfitting

### In-Context "Linear Regression"

### Task: English-to-French Translation

$$Z = \begin{bmatrix} \text{sea otter} & \text{peppermint} & \text{plush giraffe} & \text{cheese} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \text{loutre de mer} & \text{menthe poivrée} & \text{girafe peluche} & ? \end{bmatrix}$$

$$Z \mapsto T_{\bullet}(Z) \leftrightarrow \text{"from age"} \text{ ("cheese" in French)}$$

$$Z\mapsto \mathcal{T}_{\theta}(Z)\leftrightarrow$$
 "fromage" ("cheese" in French)

Task: 
$$x \mapsto \mathbf{w}^{\top} x$$

$$Z = \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(n)} & x^{(n+1)} \\ \downarrow & \downarrow & & \downarrow & \downarrow \\ y^{(1)}_{(=\mathbf{w}^{\top} x^{(1)})} & y^{(2)} & \dots & y^{(n)} & 0 \end{bmatrix}$$

$$Z \mapsto T_{\theta}(Z) \leftrightarrow \mathbf{w}^{\top} x^{(n+1)} \text{ (target)}$$

# Single-Layer Transformer w/ "Linear" Self-Attention (LSA)

$$Z \mapsto T_{\theta}(Z) = -\left[Z + \frac{1}{n}\mathsf{LSA}_{\theta}(Z)\right]_{-1,-1}$$

$$\mathsf{LSA}_{\theta}(Z) = \underbrace{W_{V}Z}_{\mathsf{Value}} \underbrace{M}_{\mathsf{Mask}} \underbrace{\mathsf{softmax}}_{\mathsf{Query}} (\underbrace{W_{Q}Z}_{\mathsf{Query}})^{\top} \underbrace{W_{K}Z}_{\mathsf{Key}})$$

$$= \underbrace{W_{V}ZMZ^{\top}}_{P} \underbrace{W_{Q}^{\top}W_{K}Z}_{Q}$$

$$P = \begin{bmatrix} 0_{d \times d} & 0_{d} \\ p^{\top} & \kappa \end{bmatrix}, Q = \begin{bmatrix} \bar{Q} & 0_{d} \\ q^{\top} & 0 \end{bmatrix}$$

Here, we put the first d rows  $P_{1:d,:}$  of P and the last column  $Q_{:,-1}$  of Q as 0 as they do not affect the output.

### Two-Parameter Transformer

$$LSA_{\theta}(Z) = \underbrace{W_{V}}_{P} ZMZ^{\top} \underbrace{W_{Q}^{\top} W_{K}}_{Q} Z$$

$$P = \begin{bmatrix} 0_{d \times d} & 0_{d} \\ p^{\top} & \kappa \end{bmatrix} = \begin{bmatrix} 0_{d \times d} & 0_{d} \\ \alpha \mu^{\top} & \kappa \end{bmatrix} \qquad (\underbrace{\alpha}_{TR}, \underbrace{\kappa}_{TL} \in \mathbb{R})$$

$$Q = \begin{bmatrix} \bar{Q} & 0_{d} \\ q^{\top} & 0 \end{bmatrix} = \begin{bmatrix} I_{d} & 0_{d} \\ 0_{d}^{\top} & 0 \end{bmatrix}$$

# Demonstration-Query Task Independence (Concept Shift)

### During pretraining, we assume

$$\mathbf{w} = \mathbf{w_q} \sim_{iid} \mathcal{D}_{\mathcal{W}}.$$

Task: 
$$x \mapsto w_q^\top x$$
 Good Demos
$$Z = \begin{bmatrix} x^{(1)} & x^{(2)} & \cdots & x^{(n)} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ w^\top x^{(1)} & w^\top x^{(2)} & \cdots & w^\top x^{(n)} \end{bmatrix}$$

$$Z \mapsto T_{\theta}(Z) \leftrightarrow w_q^\top x^{(n+1)}$$

# Demonstration-Query Task Independence (Concept Shift)

#### At test time, we assume

$$\mathbf{w}, \mathbf{w_q} \sim_{iid} \mathcal{D}_{\mathcal{W}}.$$

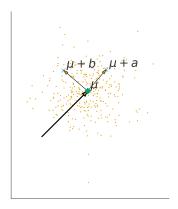
Task: 
$$x \mapsto w_q^\top x$$
 Bad Demos
$$Z = \begin{bmatrix} x^{(1)} & x^{(2)} & \cdots & x^{(n)} \\ \downarrow & \downarrow & & \downarrow \\ w^\top x^{(1)} & w^\top x^{(2)} & \cdots & w^\top x^{(n)} \end{bmatrix}$$

$$Z \mapsto T_{\theta}(Z) \leftrightarrow w_q^\top x^{(n+1)}$$

### Noncentral Task Distribution

The task center  $\mu$  explains the prior task distribution in the sense that  $\mathbf{w} = \mu + \mathbf{a}$  and  $\mathbf{w}_q = \mu + \mathbf{b}$  share the (non-zero) prior knowledge  $\mu$  and they have their own knowledge  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\mathbf{w}, \mathbf{w}_{q} \sim_{iid} \mathcal{N}(\mu, \sigma^{2}I)$$



# Quadratic Training Objective w/ Two Parameters

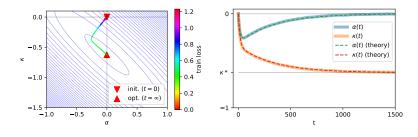
Under the two-parameter model with  $\theta = [\alpha, \kappa]^{\top}$ , we have

$$T_{ heta}(Z) = \hat{w}^{ op} x^{(n+1)} \leftrightarrow w_q^{ op} x^{(n+1)}$$
 $\hat{w} \propto \alpha \mu + \kappa w$ 
shared context-specific

The training objective

$$\begin{aligned} L_{\mathsf{train}}(\theta) &\equiv \mathbb{E}_{w,X} \left[ \left( w^{\top} x^{(n+1)} - T_{\theta}(Z) \right)^{2} \right] \\ &= \mathbb{E}_{w,X} \left[ \left( (w - \hat{w})^{\top} x^{(n+1)} \right)^{2} \right] \end{aligned}$$

is quadratic wrt  $\theta = [\alpha, \kappa]^{\top}$ .



$$\hat{w} \propto \frac{\alpha \mu}{\sin^2 \theta} + \frac{\kappa w}{\cos^2 \theta}$$

- ▶ initial phase
  - ightharpoonup in-context strength  $|\kappa| \uparrow$
  - ightharpoonup prior strength  $|\alpha| \uparrow$
- later phase
  - in-context strength  $|\kappa| \uparrow$ : in-context overfitting
  - ▶ prior strength  $|\alpha| \downarrow 0$ : prior forgetting

# Prior Forgetting and In-Context Overfitting

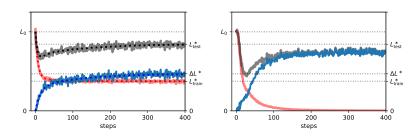
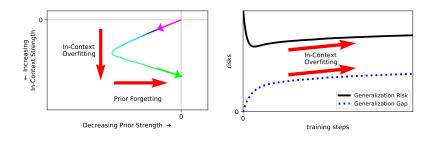


Figure: Left: two-parameter model, Right: practical Transformer

Training loss  $(w = w_q)$  monotonically decreases, but the gap increases and test loss  $(w \neq w_q)$  shows a u-shape curve.

# Summary



$$\hat{\mathbf{w}} \propto \alpha \mu + \kappa \mathbf{w}$$

- $ightharpoonup \alpha \mu$ : to learn a shared concept (TR)
- ► κw: to learn a given task (TL)

# Sungyoon Lee

# Generalization (Memorization)

Robust Emergent Feature **Abilities** Learning To Drive Al as We Desire

ΑI

Safety & Alignment

Super Controllable intelligence (LLM/GenAI)

### References

- [Lyu+23] Xinxi Lyu et al. "Z-ICL: Zero-Shot In-Context Learning with Pseudo-Demonstrations". In: The 61st Annual Meeting Of The Association For Computational Linguistics. 2023.
- [Min+22] Sewon Min et al. "Rethinking the Role of Demonstrations: What Makes In-Context Learning Work?" In: Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing. Ed. by Yoav Goldberg, Zornitsa Kozareva, and Yue Zhang. Abu Dhabi, United Arab Emirates: Association for Computational Linguistics, Dec. 2022, pp. 11048–11064. DOI: 10.18653/v1/2022.emnlp-main.759. URL: https://aclanthology.org/2022.emnlp-main.759.
- [Shi+23] Freda Shi et al. "Large language models can be easily distracted by irrelevant context". In: International Conference on Machine Learning. PMLR. 2023, pp. 31210–31227.
- [Wei+23] Jerry Wei et al. "Larger language models do in-context learning differently". In: arXiv preprint arXiv:2303.03846 (2023).
- [Yoo+22] Kang Min Yoo et al. "Ground-Truth Labels Matter: A Deeper Look into Input-Label Demonstrations". In: Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing, 2022, pp. 2422–2437.