

MOBO-OSD: Batch Multi-Objective Bayesian Optimization via Orthogonal Search Directions

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Paper



Code



Multi-Objective Bayesian Optimization (MOBO)

Goal: MOBO finds the **Pareto optimal solutions** of expensive black-box **vector-valued** objective functions $\mathbf{f} = [f_1 \quad \dots \quad f_M]$,

$$\mathcal{P}_f = \min_{x \in \mathcal{X}} (f_1(x) \quad \dots \quad f_M(x)),$$

where \mathcal{X} is the search space, M is the number of objective.

- In MOBO, no single best solutions, find the set of Pareto optimal $\mathbf{f}(x)$ (*Pareto front*) and x (*Pareto set*) in a **sample-efficient** manner.

Motivation

Drawbacks of current MOBO solutions:

- Scalarization (ParEGO [1]): bad **diversity** on concave Pareto Front
 - DGEMO [2] (NeurIPS 2020): cannot **scale** to $M > 3$
 - EHVI [3] (NeurIPS 2020): computationally **expensive**
- Focus on **Pareto Front diversity** while maintaining **scalability** and **feasibility**

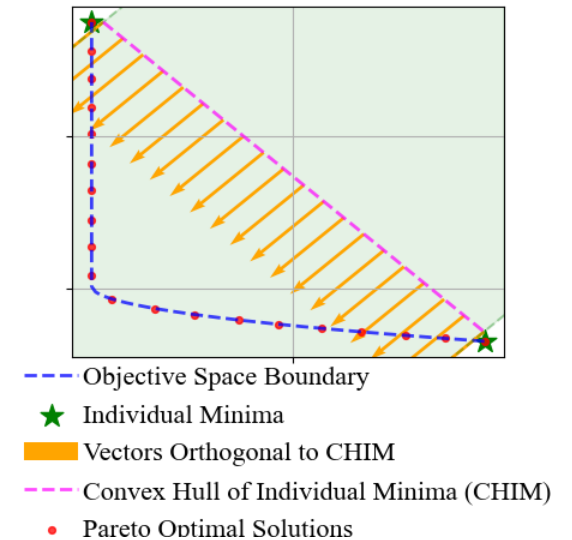
Key Ideas

Inspiration:

- Insight from the Normal Boundary Intersection (NBI) technique [4]
- The *intersection points* between **the boundary of the objective space** and **the vectors orthogonal to the CHIM** → Pareto optimal solutions

Proposed Solutions MOBO-OSD:

1. Propose the *Approximated CHIM* and the well-distributed *Orthogonal Search Directions* (OSD) → The MOBO-OSD Components
2. Find the intersection points (Pareto optimal solutions) using the proposed OSD → The MOBO-OSD Subproblems
3. Enrich the set of Pareto optimal solutions → Pareto Front Estimation
4. Select next data points for evaluations → Batch Selection Policy

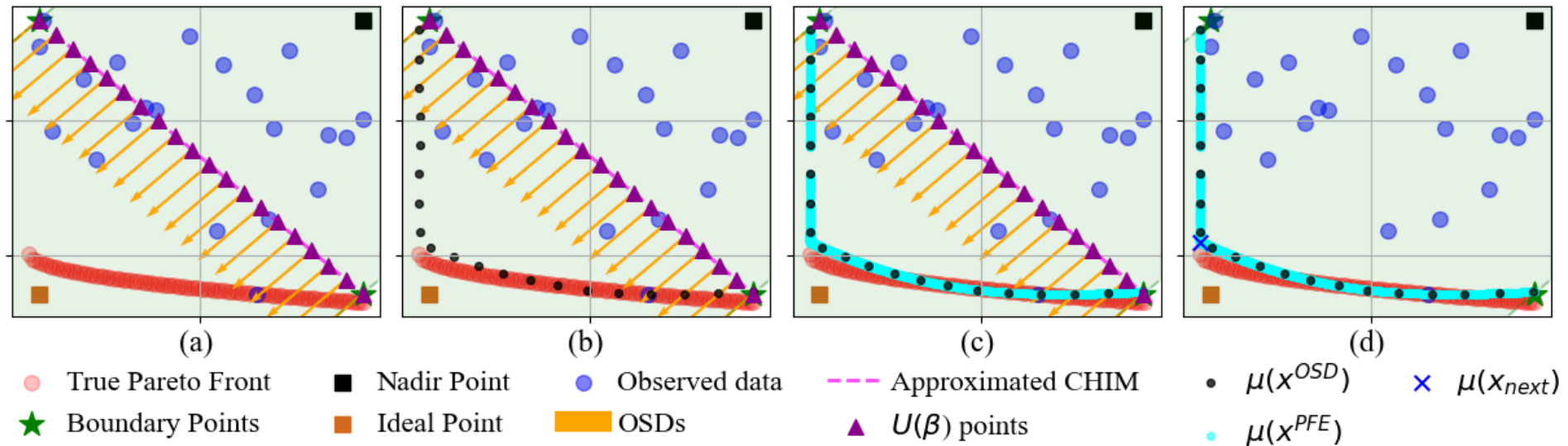


Batch Multi-Objective Bayesian Optimization via Orthogonal Search Directions (**MOBO-OSD**)

The Proposed MOBO-OSD Algorithm

Overall Process: MOBO-OSD iteration:

- (a) Compute the components (Approximated CHIM + OSDs)
- (b) Solve MOBO-OSD subproblems for candidates
- (c) Approximate the PF for additional candidates
- (d) Select optimal candidates for evaluations.



Experimental Results

Baselines

qParEGO [1]

USEMO [7] – AAAI 2020

PDBO [8] – AAAI 2024

DGEMO [2] – NeurIPS 2020

qEHVI [3] – NeurIPS 2020

JES [9] – NeurIPS 2022

NSGA-II [10]

NBI [4]



State-of-the-art MOBO methods



A well-known Multi-Objective Evolutionary Algorithm



The original Normal Boundary Intersection method

Experimental Results

Benchmark Problems

Synthetic Functions:

- DTLZ (D=5, M=2,3,4)
- ZDT (D=5, M=2)
- VLMOP2 (D=5, M=2)

Real-world Problems:

- Speed Reducer (D=7, M=3)
- Car Side Design (D=7, M=4)
- Marine Design (D=6, M=4)
- Water Planning (D=3, M=6)

Comparison metrics:

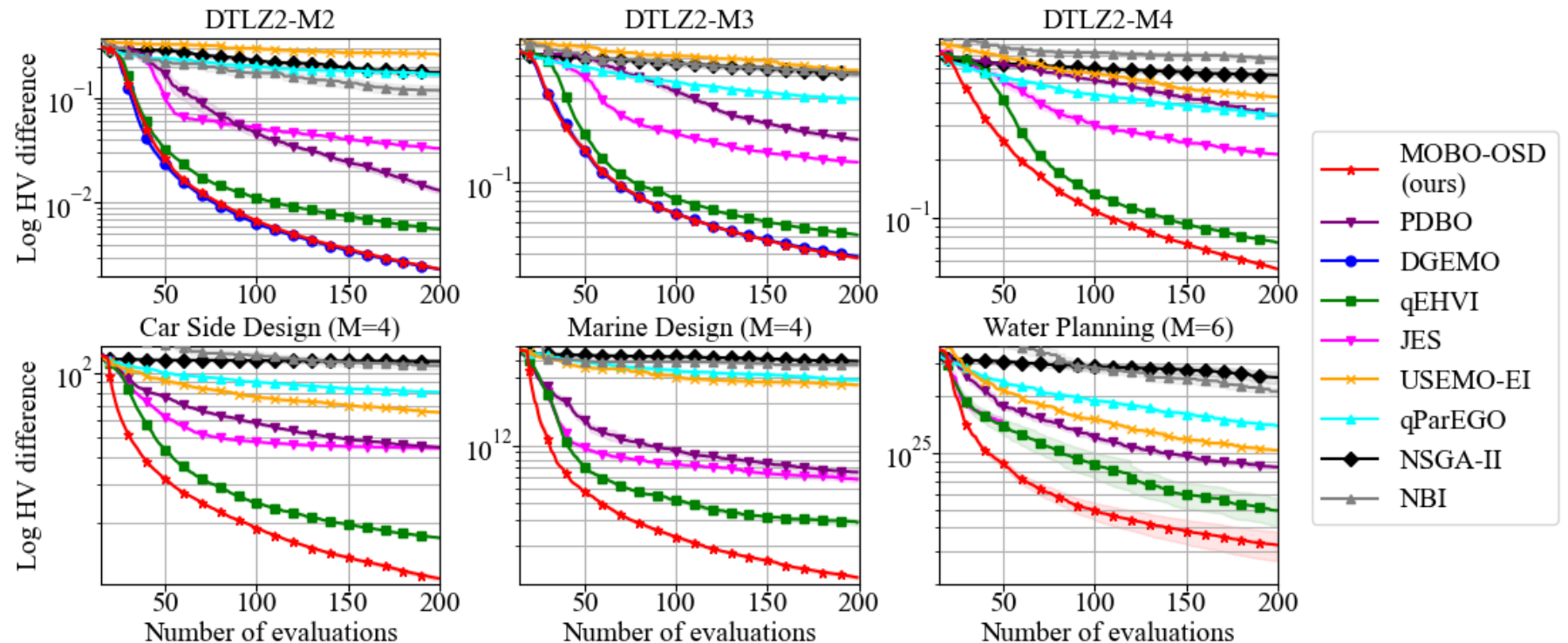
- Hypervolume (the most common in MOBO)
- We run each problem 10 times and report the accumulated best results (mean and standard errors).

Experimental Results

Comparisons against Baselines – Sequential Optimization (batch = 1)

MOBO-OSD overall
outperforms all
baselines:

- Best HV performance
- Scalability to any number of objective



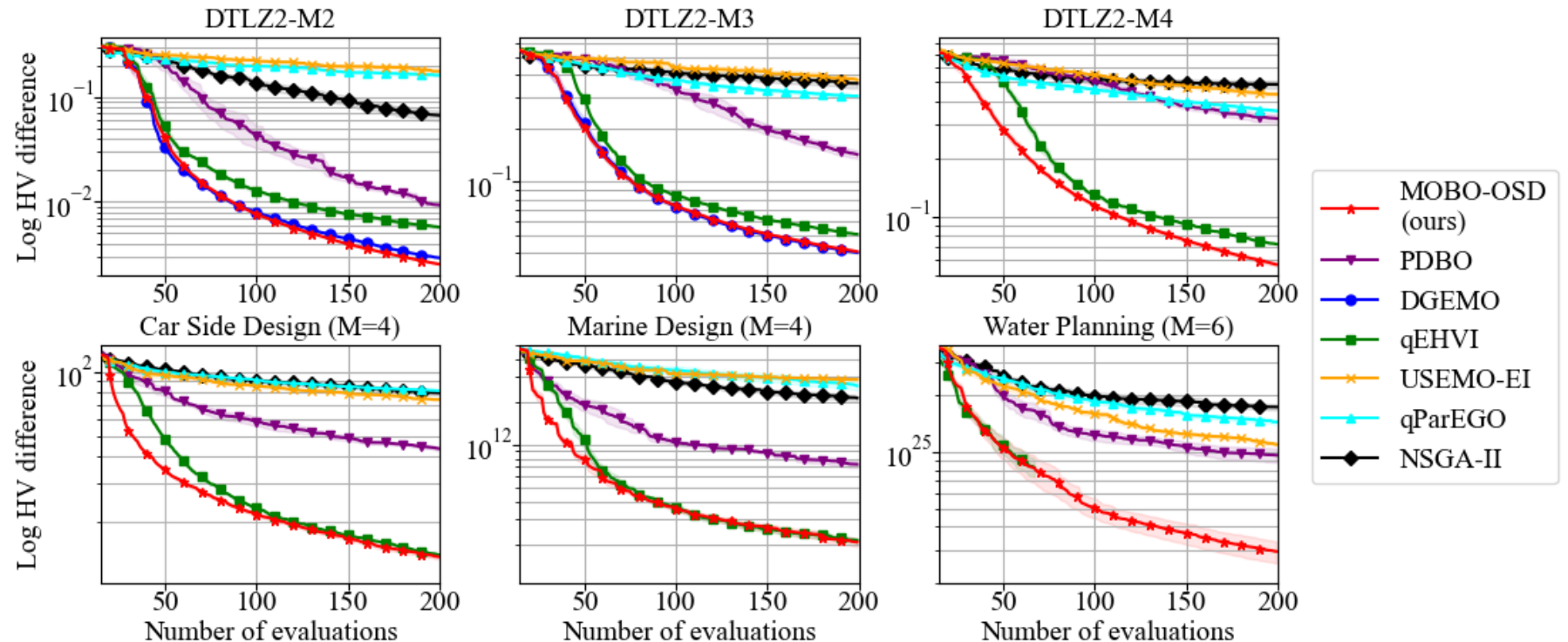
Smaller values indicate better methods

Experimental Results

Comparisons against Baselines – Batch Optimization (batch = 8)

MOBO-OSD overall outperforms all baselines :

- qEHVI: scales expensively with number of objectives and batch size → cannot finish Water Planning (M=6).



Smaller values indicate better methods

Summary

MOBO-OSD: Batch Multi-objective BO via Orthogonal Search Directions (OSD)

- Methodology:
 - Compute the MOBO-OSD components
 - Solve MOBO-OSD subproblems for the candidates (intersection points)
 - Generate additional candidates
 - Select next observations via HV-based AF
- Validate with extensive experiments (HV performance, scalability and cost)

Thank You

Poster Session: Wed 3 Dec (Exhibit Hall C,D,E)

References

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