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# Topology of Reasoning

## Understanding Reasoning from Graph-Theoretic Lens

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NEURAL INFORMATION PROCESSING SYSTEMS

# Motivation

- **Reasoning models (e.g., o3) achieve success**
  They solve expert-level coding, math, and science problems by thinking.
- **How do they reason? How do they differ?**
  The *mechanisms* behind the reasoning abilities remain poorly understood.
- **Our lens: Reasoning Graphs**
  By modeling internal hidden states as structured graphs,
  we uncover what distinguishes reasoning models at the representational level.

**Question**

Joy can read 8 pages of a book in 20 minutes.
How many hours will it take her to read 120 pages?

**Generated Answer**

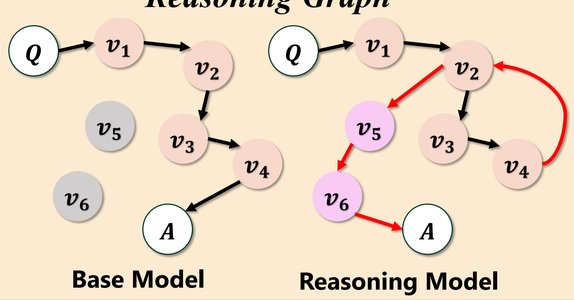
**Base Model**

In 1 hour, there are 3 sets of 20 minutes.
8 pages ÷ 20 min=0.4 pages/minute.
120 pages ÷ 0.4 pages/minute=360 minutes.
360 minutes ÷ 60 minutes/hour=6 hours.
**The answer is 6 hours.**
Incorrect

**Reasoning Model**

In 1 hour, there are 3 sets of 20 minutes.
8 pages ÷ 20 min=0.4 pages/minute.
120 pages ÷ 0.4 pages/minute=360 minutes.
360 minutes ÷ 60 minutes/hour=6 hours.
Wait, let me verify the calculations. ...
8 pages ÷ 20 min=0.4 pages/minute.
120 pages ÷ 0.4 pages/minute=300 minutes.
300 minutes ÷ 60 minutes/hour=5 hours.
**The answer is 5 hours.**
Correct!

***Reasoning Graph***




# Reasoning Graph from Internal State

- **Segment Representation**
  For each question, reasoning steps: $R = (r_1, r_2, \ldots, r_T)$
  Segment representation by averaging $h_i^\ell(t)$: $s_i^\ell = \frac{1}{L_i} \sum_{t=1}^{L_i} h_i^\ell(t)$
- **Node Definition**
  Segment $s_i^\ell$ clustered using K-means.
  Cluster center $c_k$ becomes a graph node $v_k$:
  $V = \{v_1, \ldots, v_K\}, \quad d(v_i, v_j) = \| c_i - c_j \|^2$
- **Edge Construction**
  For each question, connect nodes in order.
  $E = \{(v_i \to v_{i+1})\}_{i=1}^{T-1}$

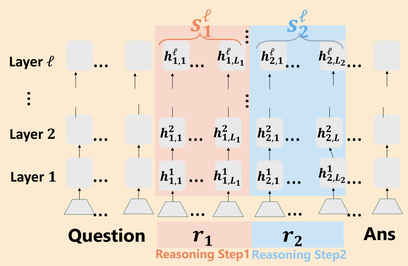
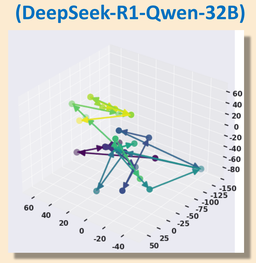



Table 1: Examples of automatically identified themes and corresponding reasoning steps.

| Theme (Node id) | Examples of Reasoning Steps |
|---|---|
| Calculations — Totals (node 83) | • 5 + 10 = 15. Then, 15 + 9 = 24. Finally, 24 + 3 = 27.<br>• Let me add them step by step. 500 + 1500 = 2000. Then, 2000 + 125 = 2125. |
| Calculations — Average (node 119) | • Average = (Sum of all values) / (Number of values).<br>• Average Speed = Total Distance / Total Time = 250 miles / 5 hours = 50 mph. |
| Calculations — Division (node 41) | • 125,000 / 20 = 6,250.<br>• 120 pieces / (15 pieces per pack) = 8 packs.<br>• 80 / 10 = 8 weeks. |
| Age Calculations (node 147) | • Sum of their ages in two years = (B + 2) + (2B + 2) = 28.<br>• C = 2 × (James's age in 8 years) − 5. |
| Cost Calculations (node 168) | • $47.00 × 5 = $235.00.<br>• Keenan's weekly cost = $160 ÷ 4 = $40. |

**Base Model (Qwen2.5-32B)**

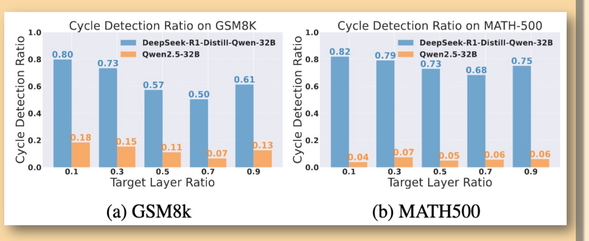
**Reasoning Model (DeepSeek-R1-Qwen-32B)**



# Key Graph Properties

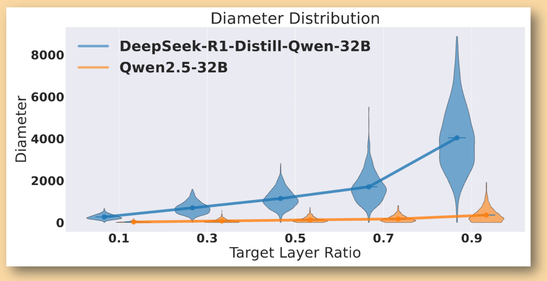
## Cycles

- **Measurement**
  Detect cycles in the reasoning graph, defined as repeated visits to the same node, excluding. self-loops.
- **Result**
  Reasoning models have *more cycles*.
  Difficult tasks induce more cycles



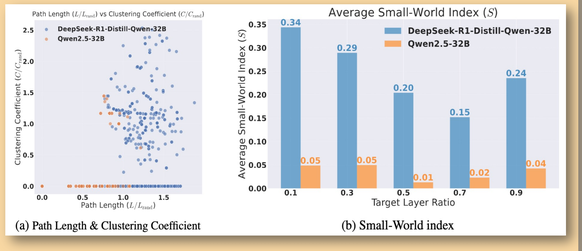

## Diameters

- **Measurement**
  Maximum shortest path distance between any two reachable node
  $\text{Diameter} = \max_u \max_{u \neq v} d(u, v)$
- **Result**
  Reasoning models *larger diameters*
  They explore broader spaces



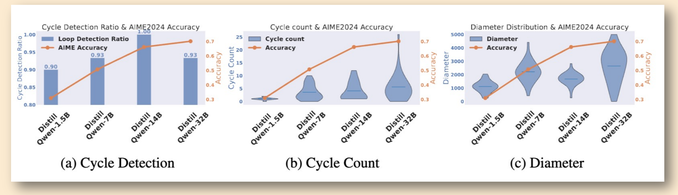

## Small-World

- **Measurement**
  Common in networks—SNS, biology, etc.
  Clustering coefficient / Path length.
  $\text{Smallworld} = \frac{C/C_{rand}}{L/L_{rand}}$
- **Result**
  Reasoning models show
  higher $C$ and longer $L$
  forming small-world graphs




# Model Size Matters

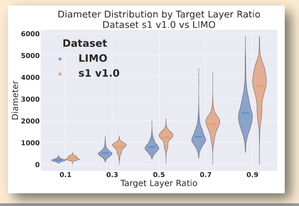
**Larger models show more cycles, broader graphs.**
As model size increase (1.5B → 32B), reasoning graph exhibit more cycles and larger diameters.




# Practical Takeaway: SFT

**Better SFT data = Better reasoning graphs**
s1 prompts yield larger reasoning graphs than LIMO.

Graph properties
could help evaluate and build
better reasoning-SFT data




[1] Niklas et. al., "s1: Simple test-time scaling" , [2] Ye et. al., "LIMO: Less is More for Reasoning"