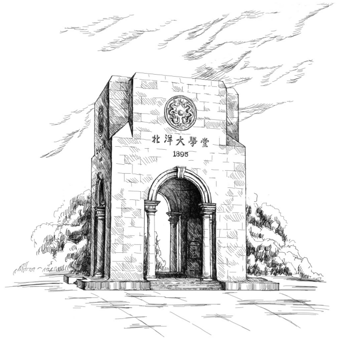
天津大学 Tianjin University
应用数学中心 Center for Applied Mathematics

NEURAL INFORMATION PROCESSING SYSTEMS

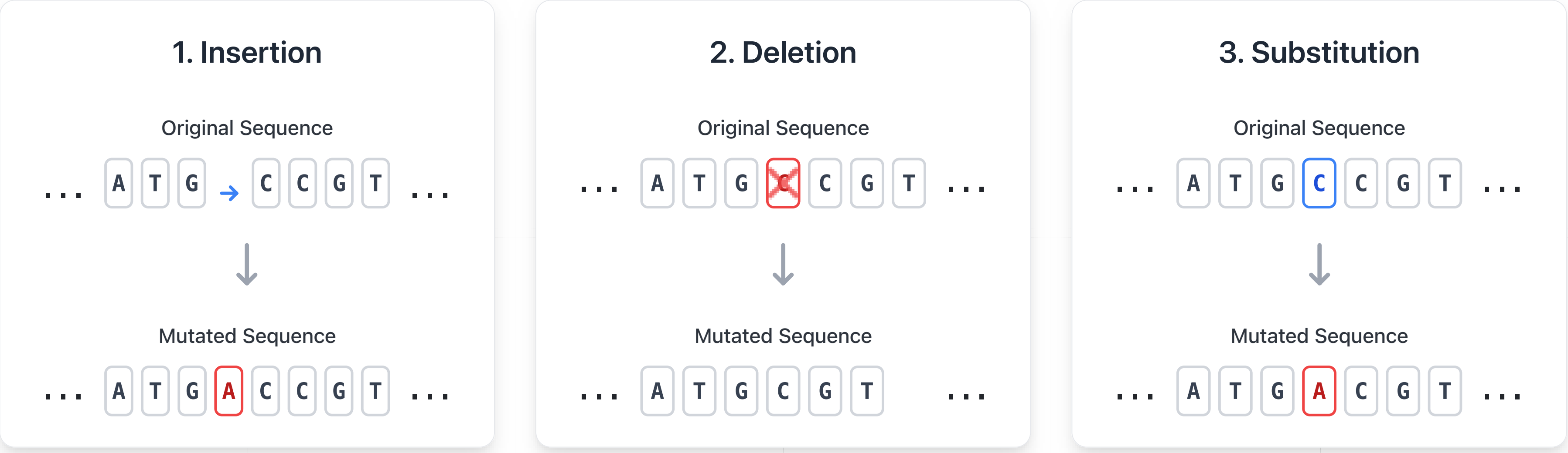
DoDo-Code: an Efficient Levenshtein Distance Embedding-based Code for 4-ary IDS Channel

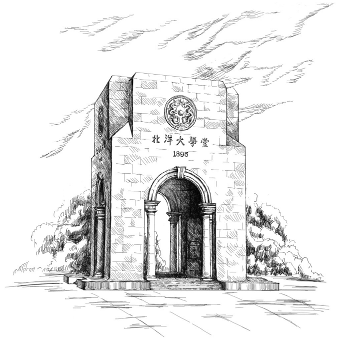
Alan J.X. Guo (郭嘉祥), Sihan Sun (孙思寒), Xiang Wei (魏祥), Mengyi Wei (魏梦怡), Xin Chen (陈鑫)



Background

Error-correcting codes for insertion, deletion, and substitution (IDS) are vital to bio-sequence information storage pipelines.





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Background

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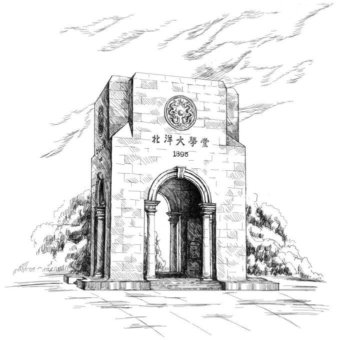
Defination: [Levenshtein Distance: Distance for IDS Operations]

- The Levenshtein distance between $\boldsymbol{s}$ and $\boldsymbol{t}$ is the minimum number of IDS operations required to transform string $\boldsymbol{s}$ into string $\boldsymbol{t}$.



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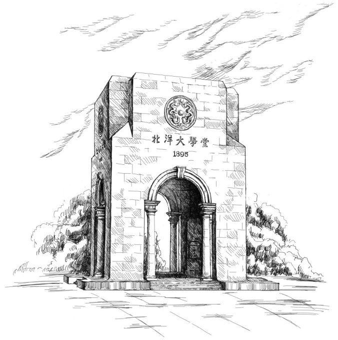
Defination: [Levenshtein Distance: Distance for IDS Operations]

- The Levenshtein distance between $\boldsymbol{s}$ and $\boldsymbol{t}$ is the minimum number of IDS operations required to transform string $\boldsymbol{s}$ into string $\boldsymbol{t}$.

Challenge: Characterizing IDS errors remains challenging due to open problems with the Levenshtein distance [1,2]:

- High computational complexity of the metric.
- The unknown structure of the "Levenshtein ball."



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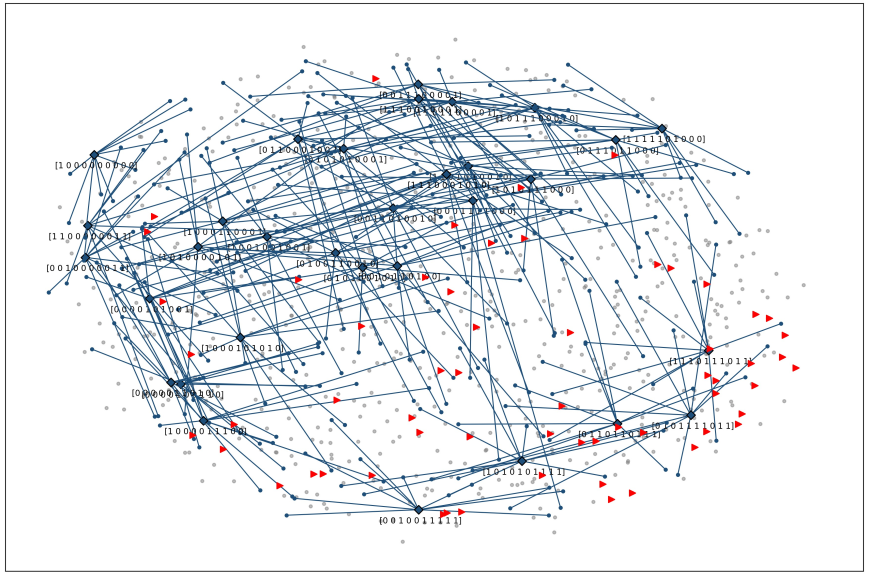
Background

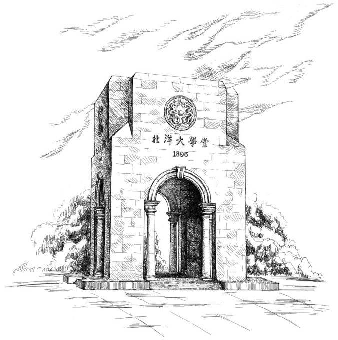
State-of-the-art 4-ary IDS-correcting codes are order optimal, requiring a redundancy of:

$$\log n + \log\log n + 7 + o(1)$$

bits [3]. This redundancy is **suboptimal** for small values of n.



Visualization of VT Code in Embedding Space.
Red marks are untapped sequences.



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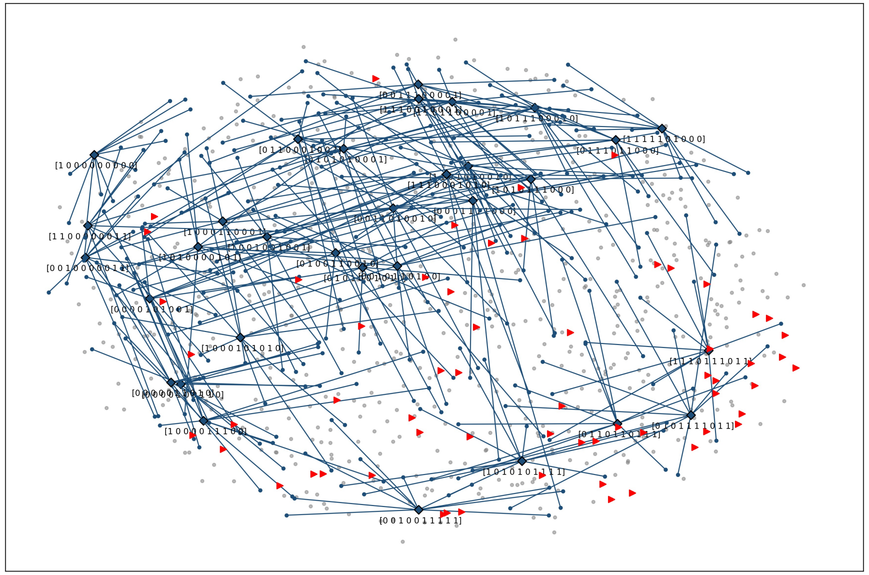

Visualization of VT Code in Embedding Space.
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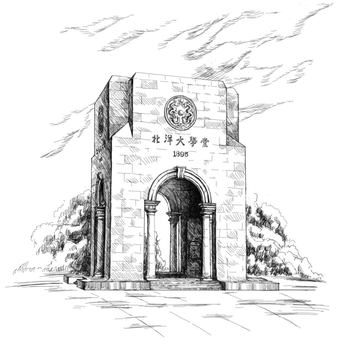
There is room for improvement in coderate. Unfortunately, we are not experts in designing combinatorial codes.



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Idea

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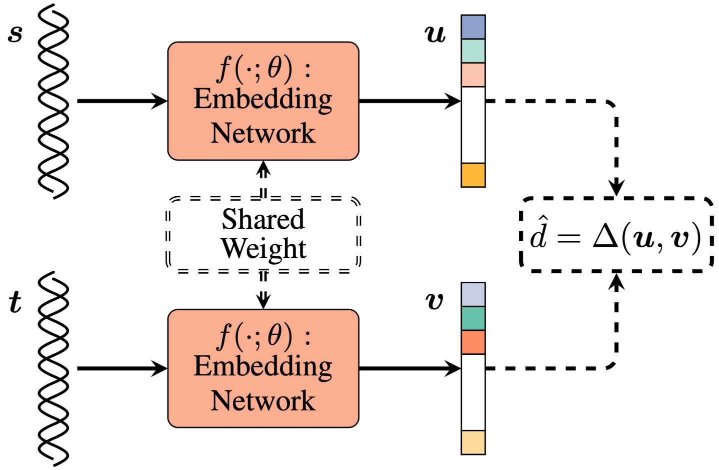
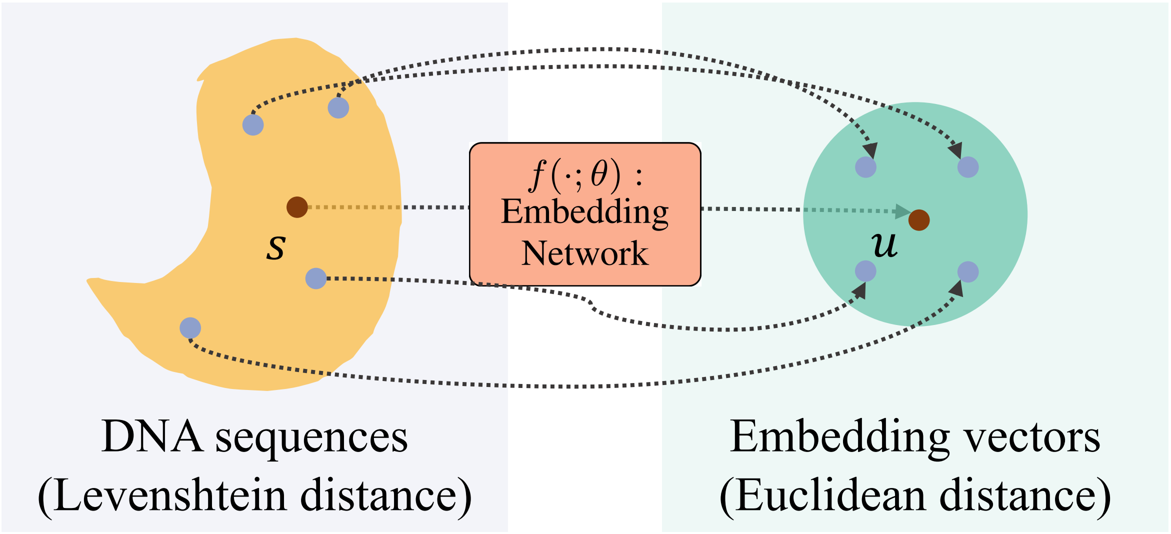
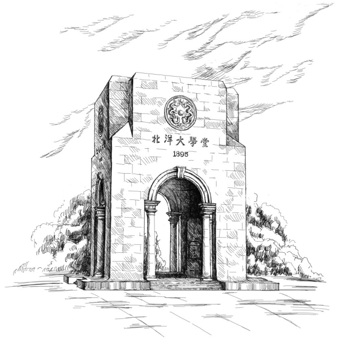
Leverage a deep embedding space[4] as a geometric proxy for the Levenshtein domain to guide code design.



Siamese neural network



Embedding network maps sequences to embedding vectors

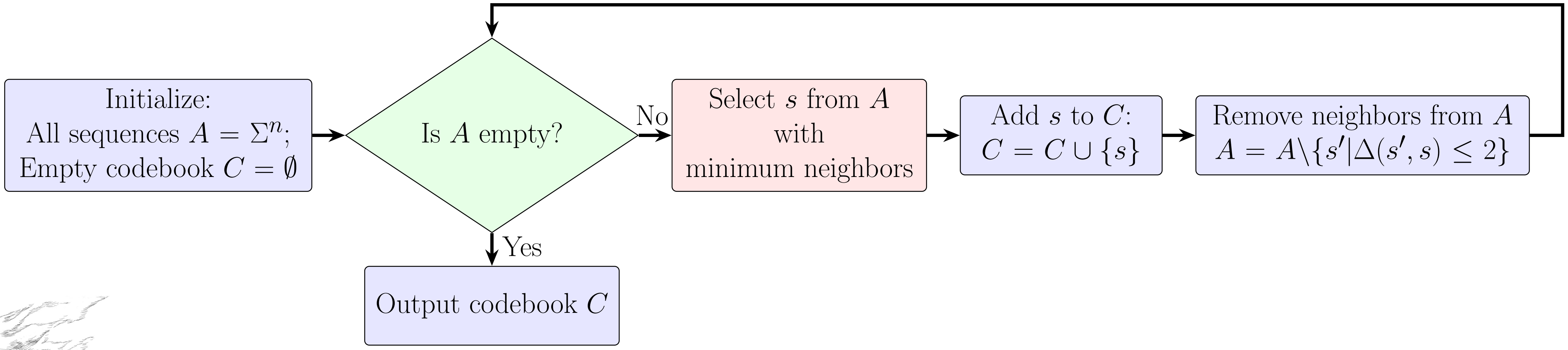


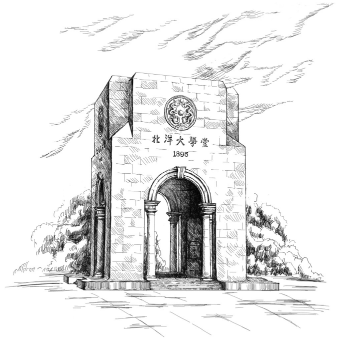
[4] Wei, Xiang, et al. “Levenshtein distance embedding with poisson regression for DNA storage.” AAAI 2024.

Method – Codebook design

Proposition: Generate a codebook C where the minimum Levenshtein distance between any two distinct codewords c_1, $c_2 \in C$ is at least 3. This codebook C can correct a single IDS error.

Greedy search of the codebook:





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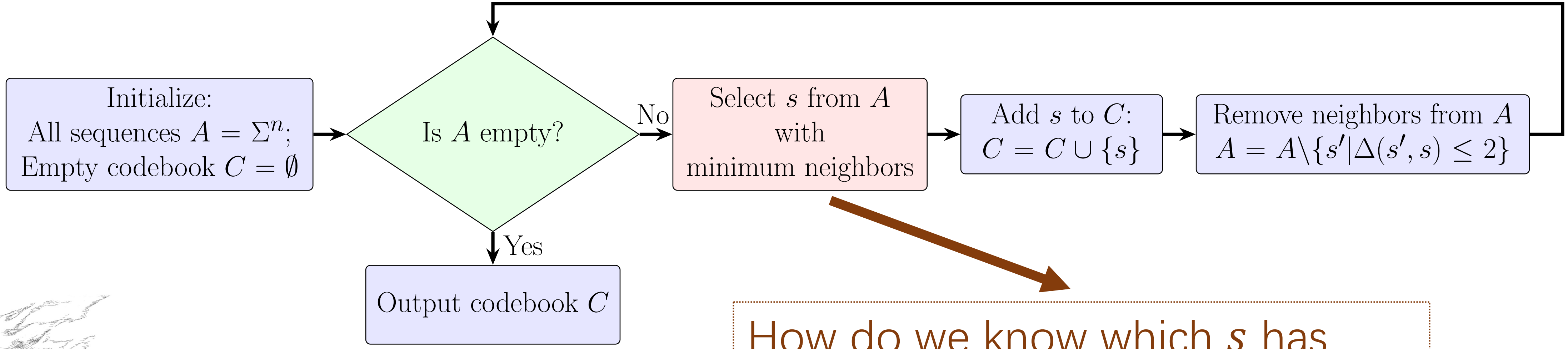
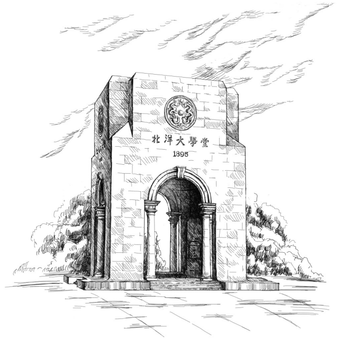


How do we know which s has minimum neighbors from A?



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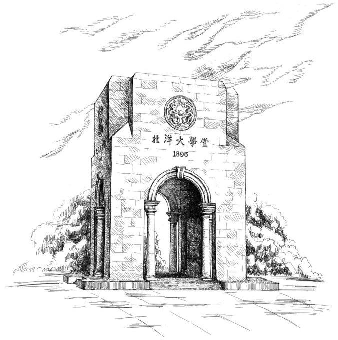

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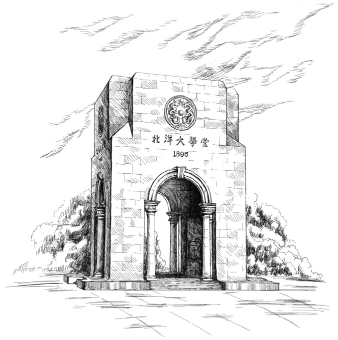
How do we know which s has minimum neighbors from A?

Hypothesis:
The geometric density of embedding vectors serves as a **proxy** for the combinatorial density of neighbors in the Levenshtein domain.



Method – Codebook design

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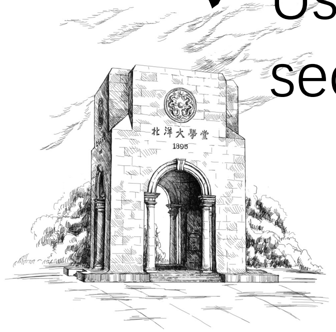
The embedding vectors follow a multivariate normal distribution $N(\mathbf{0}, \boldsymbol{\Sigma})$, as long as a normalization layer is applied to the embedding model.

✓ Use the estimated PDF of embedding vectors as a criterion to identify sequences with fewer neighbors in Levenshtein distance.



Method – Codebook design

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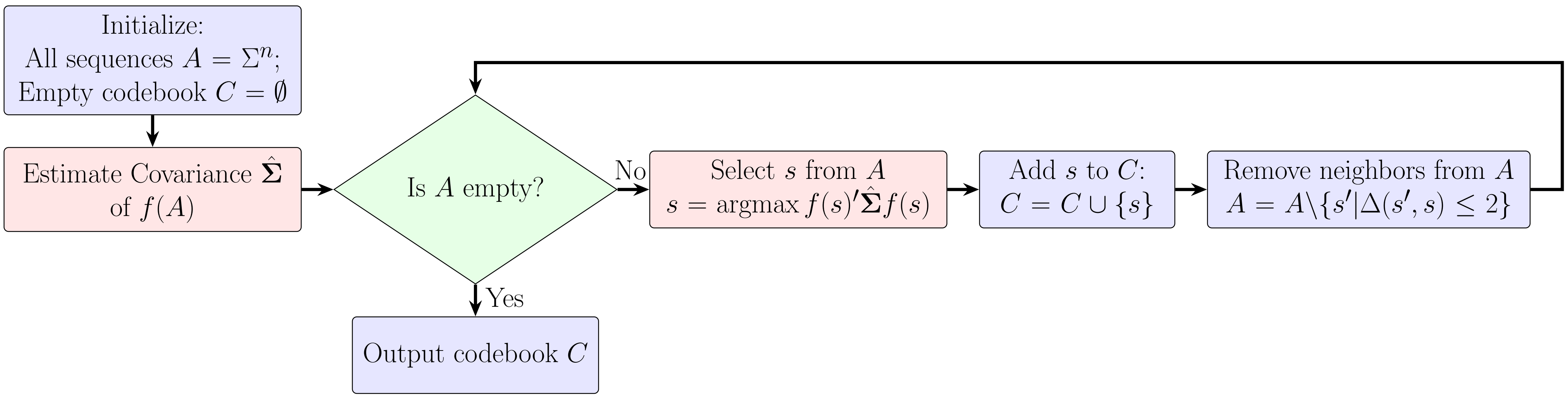
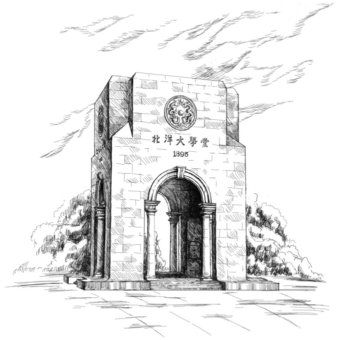
Implementable greedy search of the codebook:





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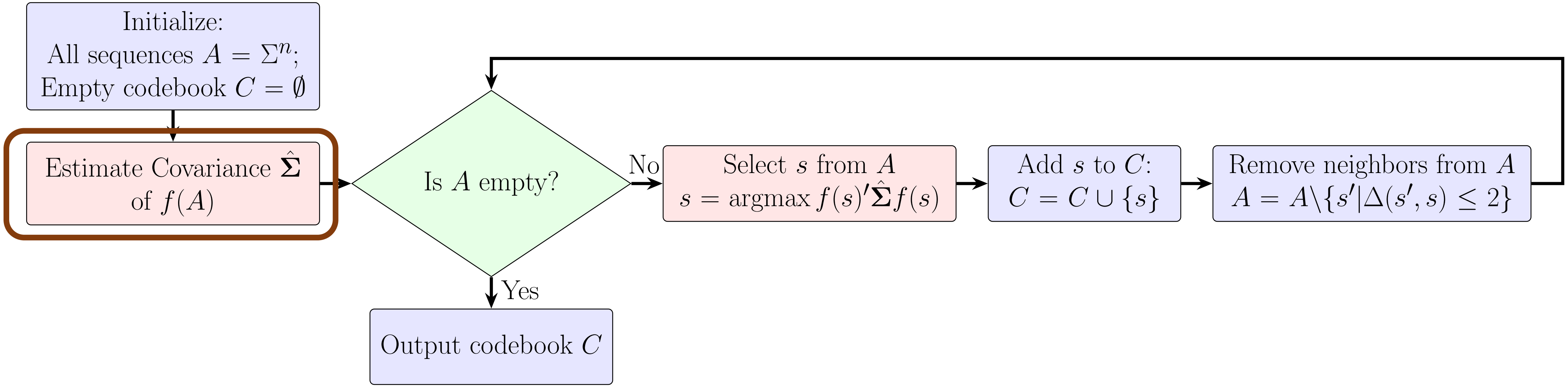

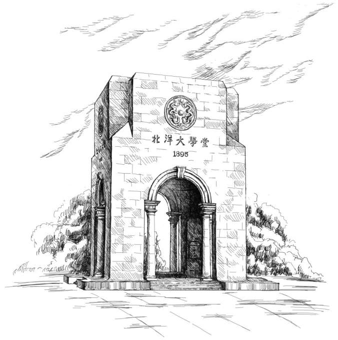

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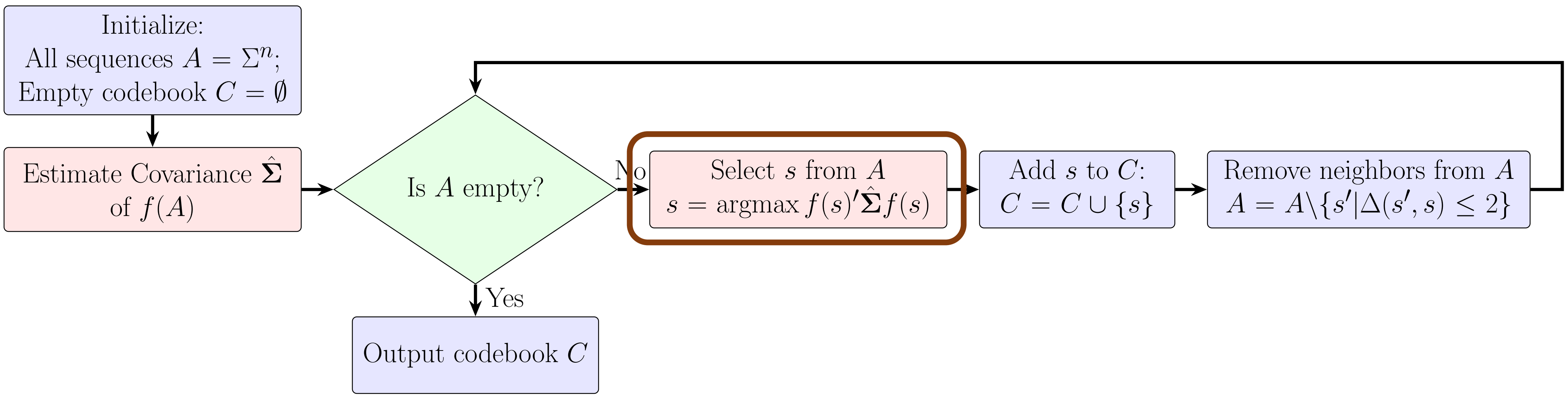
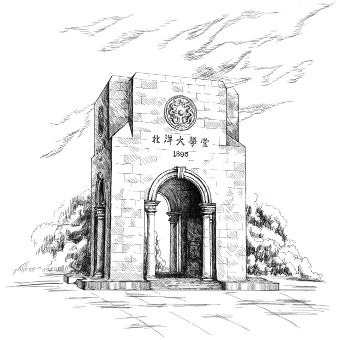




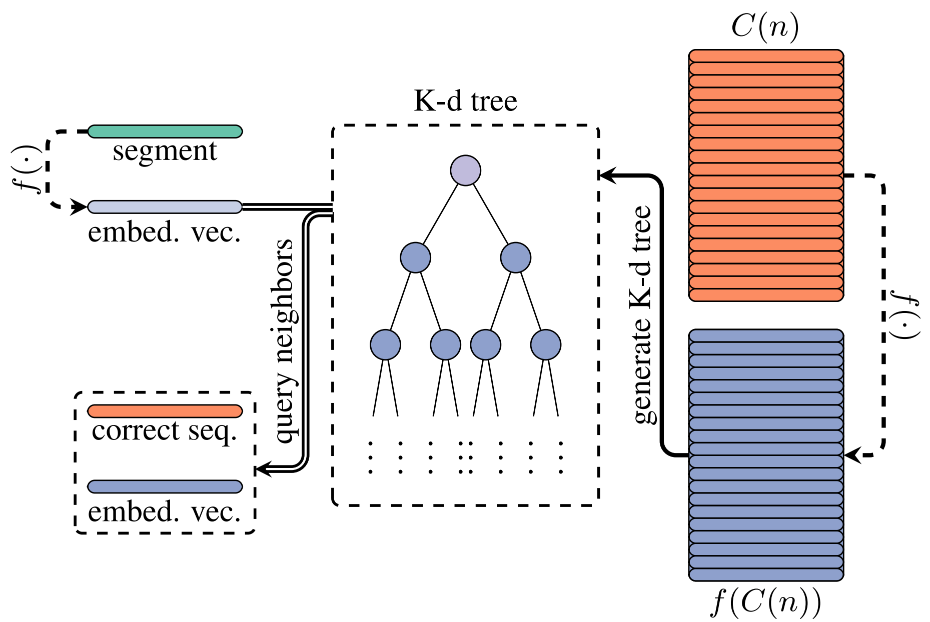
Method – Decoding in embedding space

Challenge:

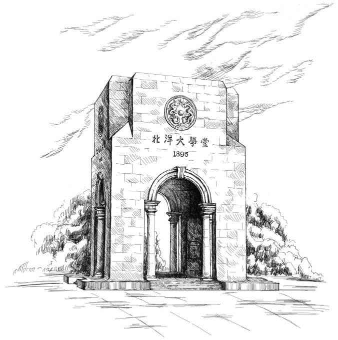
Decoding (error correcting) for this codebook is complex, driven by the high time complexity of calculating Levenshtein distance.

✓ Perform decoding in the embedding space using Euclidean distance.

✓ Mature neighbor searching methods can be leveraged.



Leverage a K-d tree in the embedding space for neighbor searching (error correcting).



Visualization of codeword selection in embedding space

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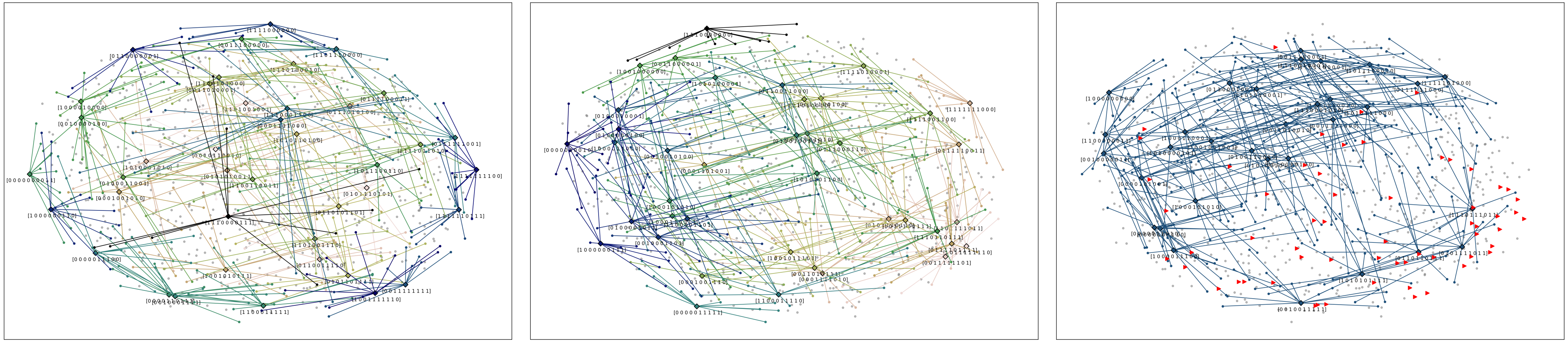
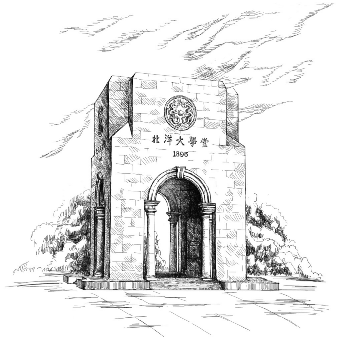


(a) deep embedding-based search (b) random search (c) VT code

Darker colors indicate codewords selected earlier. (a) Dodo-Code: tends to select codewords from the periphery of the embedding distribution. (b) Random Search: selects codewords without any discernible pattern. (c) VT Code: leaves untapped potential codewords within the distribution.



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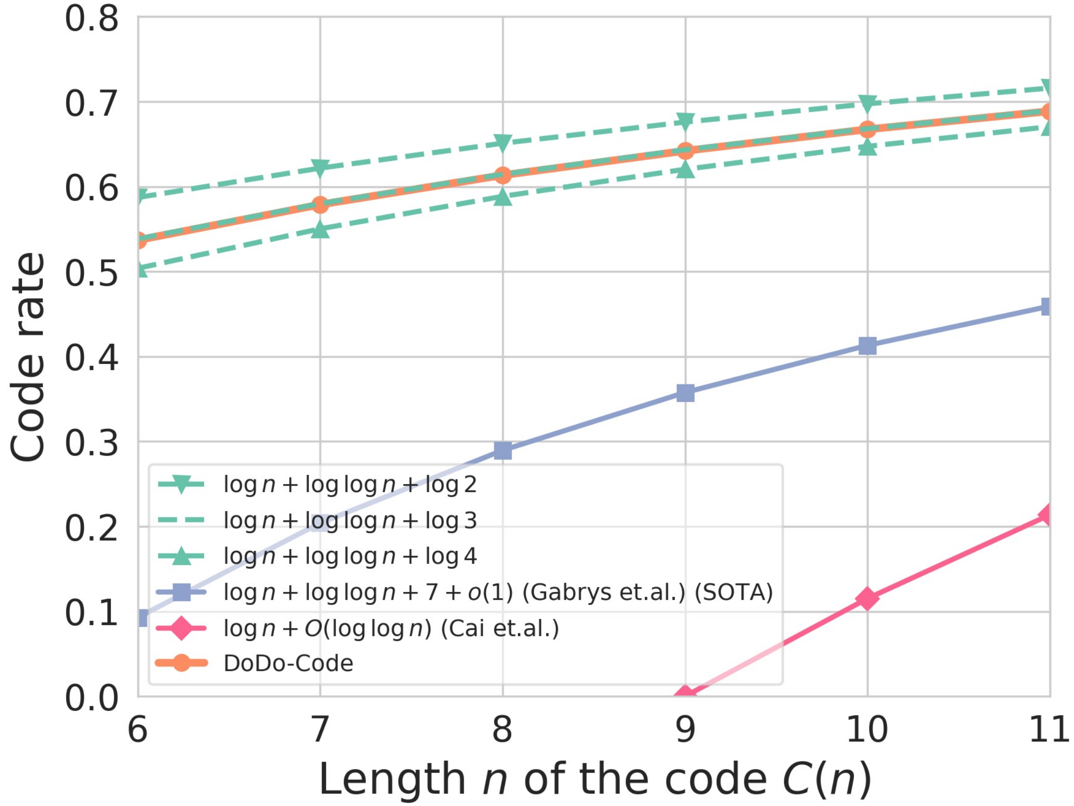
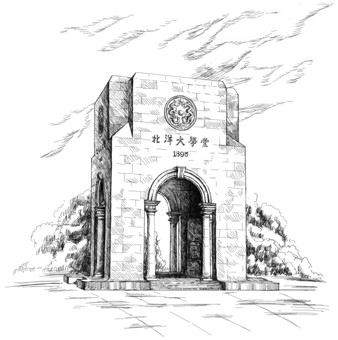
Outperforms SOTA and aligns with optimal coderate

- ✓ DoDo-Code outperforms SOTA combinatorial codes [3, 5] in coderate, when n is small.
- ✓ DoDo-Code aligns with order optimal coderate

$$\log 3n + \log\log n.$$

Note: Correcting a single substitution requires at least $\log 3n$ redundancy bits.





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[5] Cai, Kui, et al. "Correcting a single indel/edit for DNA-based data storage: Linear-time encoders and order-optimality." IEEE TIT, (2021).

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Thanks for listening!

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