Topology-Aware Learning of Tubular Manifolds via SE(3)-Equivariant Network on Ball B-Spline Curve

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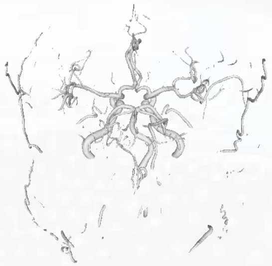
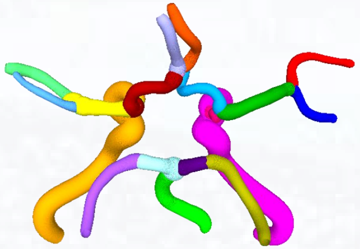
Background of Tubular Structure Analysis

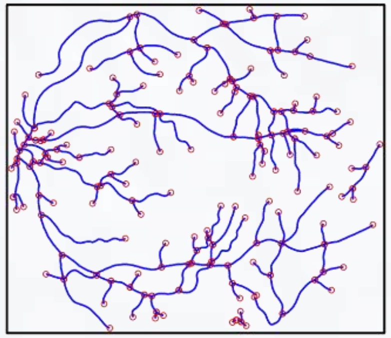
- Medical image processing (2D & 3D images)
  - Local geometric feature analysis (arc length, normal, curvature)
  - Topological relationship analysis



Geometric Analysis & Topological Analysis



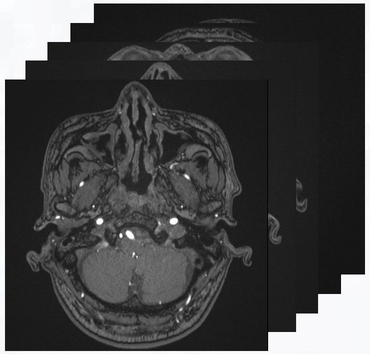
Shape Graph



Challenge & Motivation

- Data discretization (Pixels, voxels, point clouds, meshes)
  - Loss of local geometric details
  - High-precision geometric features require large scale, high-resolution data

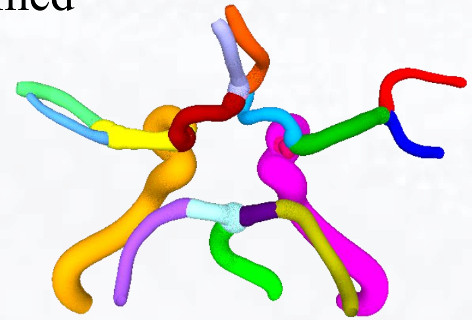
- Parameterized manifolds
  - Continuously and differentiable, allowing for the determination of precise geometric properties
  - Low storage and computing costs
  - Rigorous theoretical analysis can be performed



Voxel

Point cloud

Mesh



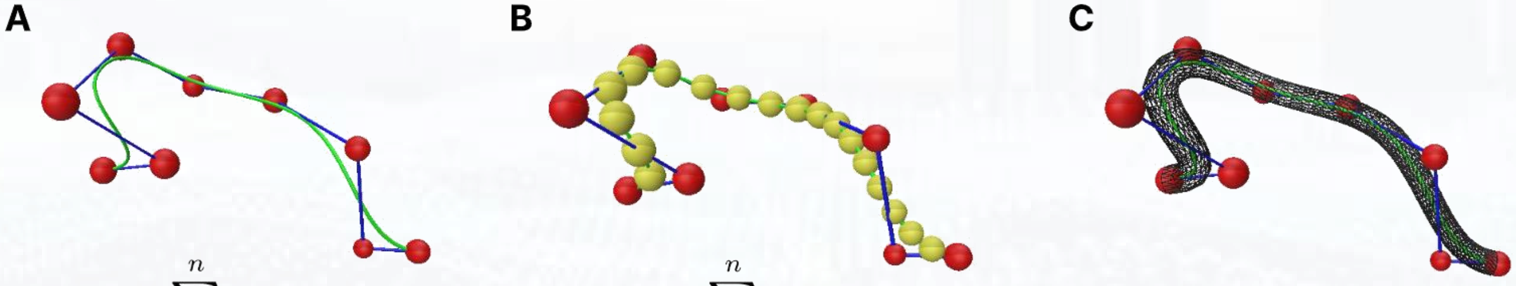
Ball B-Spline Curve

Ball B-Spline Curve

- Ball B-Spline Curve (BBSC)

$$BBSC(t) = \sum_{i=0}^{n} N_{i,p}(t) C_i = \sum_{i=0}^{n} N_{i,p}(t) \left(P_i \,;\, r_i\right) = \left(\sum_{i=0}^{n} N_{i,p}(t) P_i \,;\, \sum_{i=0}^{n} N_{i,p}(t) r_i\right)$$

(Abbreviated as β)



$$c(t) = \sum_{i=0}^{n} N_{i,p}(t) P_i \qquad r(t) = \sum_{i=0}^{n} N_{i,p}(t) r_i \qquad BBSC(t) = (c(t);\ r(t))$$

A smooth tubular surface, which can be viewed as a 2D manifold coordinated by u and v.

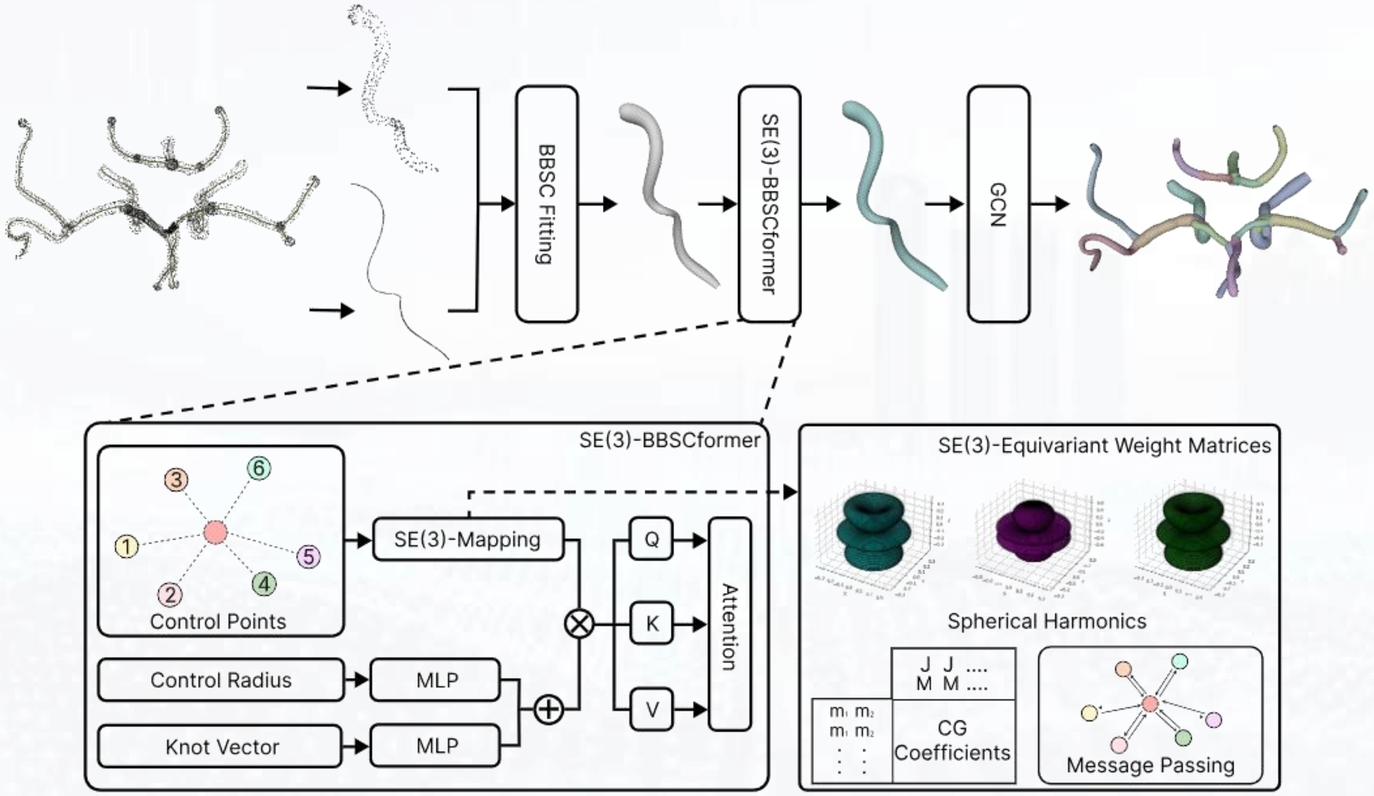
- Ball B-Spline Curve functional space

$$\mathcal{M}_p = \{\beta(P, r, \tau) \mid (P, r, \tau) \in \Theta\}, \quad \Theta = \{P \in \mathbb{R}^{3\times n},\ r \in \mathbb{R}_{>0},\ \tau \in \mathbb{T}\}$$

- Ball B-Spline Curve topological structure

$$\mathcal{GM} = \left(\bigsqcup_{i\in I} \beta_i\right) / \sim,\ \sim \subseteq \bigsqcup_{i\in I} \beta_i \times \bigsqcup_{i\in I} \beta_i$$

SE(3)-BBSCformerGCN



- BBSC fitting
  - Input: surface point, centerline
  - Output: control parameters, knot vectors

(Fitting scattered data points with ball b-spline curves using particle swarm optimization)

- SE(3)-BBSCformer
  - Input: $f_{in}^{c_{i0}} = (MLP(r),\ MLP(\tilde{\tau}))\ ,\ \tilde{\tau} = ((t_1,...,t_{p+1}),...,(t_n,...t_{n+p+1}) \in \mathbb{R}^{n\times(p+1)}$
  - Output: $\vec{q}_i = \bigoplus_{c_o}\sum_{c_i} W_Q^{c_oc_i} f_{in,i}^{c_i},\ \vec{k}_j = \bigoplus_{c_o}\sum_{c_i} W_K^{c_oc_i}(P_j-\overline{P}) f_{in,j}^{c_i},\ \alpha_{ij} = \frac{\exp(\vec{q}_i^{\,T}\vec{k}_j)}{\sum_{j=1}^{n} \exp(\vec{q}_i^{\,T}\vec{k}_j)}$

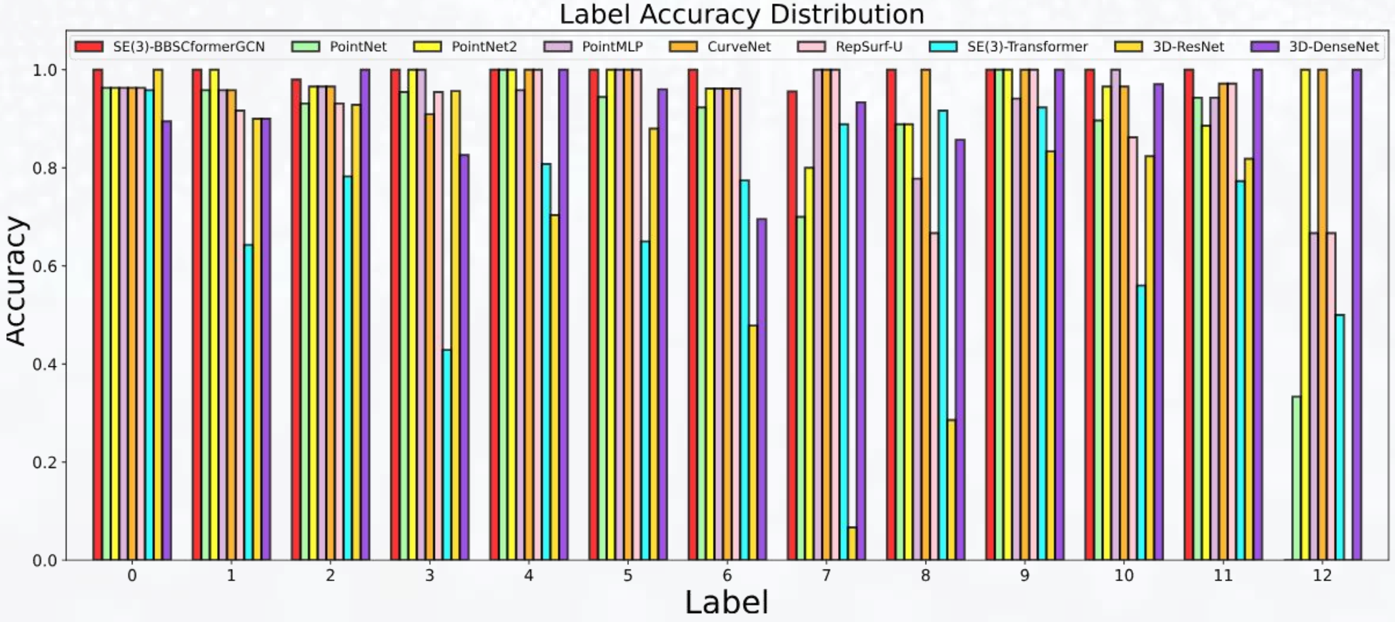
$$f_{out,i}^{c_o} = W_V^{c_oc_o} f_{in,i}^{c_o} + \sum_k \alpha_{i,j} W_V^{c_oc_i}(P_i-\overline{P}) f_{in,j}^{c_i}$$

TopCoW 2024 Experiment

Classification performance

Method	Accuracy%	AUC-ROC%	Precision%	Recall%	F1 Score%
3D DenseNet [53]	97.02 ± 0.57	99.63 ± 1.94	95.86 ± 0.51	95.42 ± 0.41	97.47 ± 0.48
3D ResNet [54]	96.54 ± 0.54	99.61 ± 0.11	96.79 ± 0.76	96.44 ± 0.83	96.52 ± 0.80
PointNet [24]	94.10 ± 0.88	99.46 ± 0.20	94.81 ± 1.36	94.69 ± 0.92	94.57 ± 1.10
PointNet++ [55]	96.61 ± **0.53**	99.61 ± 0.09	96.72 ± **0.49**	96.52 ± 0.55	96.45 ± 0.60
CurveNet [56]	97.14 ± 1.16	99.72 ± 0.20	97.31 ± 1.16	97.07 ± 1.19	97.09 ± 1.22
RepSurf-U [57]	97.41 ± 0.68	99.86 ± 0.07	96.83 ± 0.52	96.61 ± 0.54	96.50 ± 0.55
PointMLP [58]	94.10 ± 2.27	99.03 ± 0.56	94.41 ± 2.35	94.05 ± 2.31	93.94 ± 2.40
SE(3)-Transformer [30]	61.35 ± 5.80	90.93 ± 1.31	62.36 ± 5.28	60.77 ± 5.70	60.58 ± 5.69
SE(3)-BBSCformer	87.91 ± 1.35	98.24 ± 0.31	88.97 ± 1.66	88.01 ± 1.32	87.98 ± 1.26
BBSCformerGCN	**97.95** ± 1.22	99.89 ± 0.02	97.25 ± 0.79	98.34 ± 0.60	97.78 ± 0.70
SE(3)-BBSCformerGCN	97.53 ± 0.83	**99.99** ± **0.01**	**97.36** ± 0.53	**98.44** ± **0.31**	**97.87** ± **0.41**

Per-class classification accuracy



Clinical Experiment

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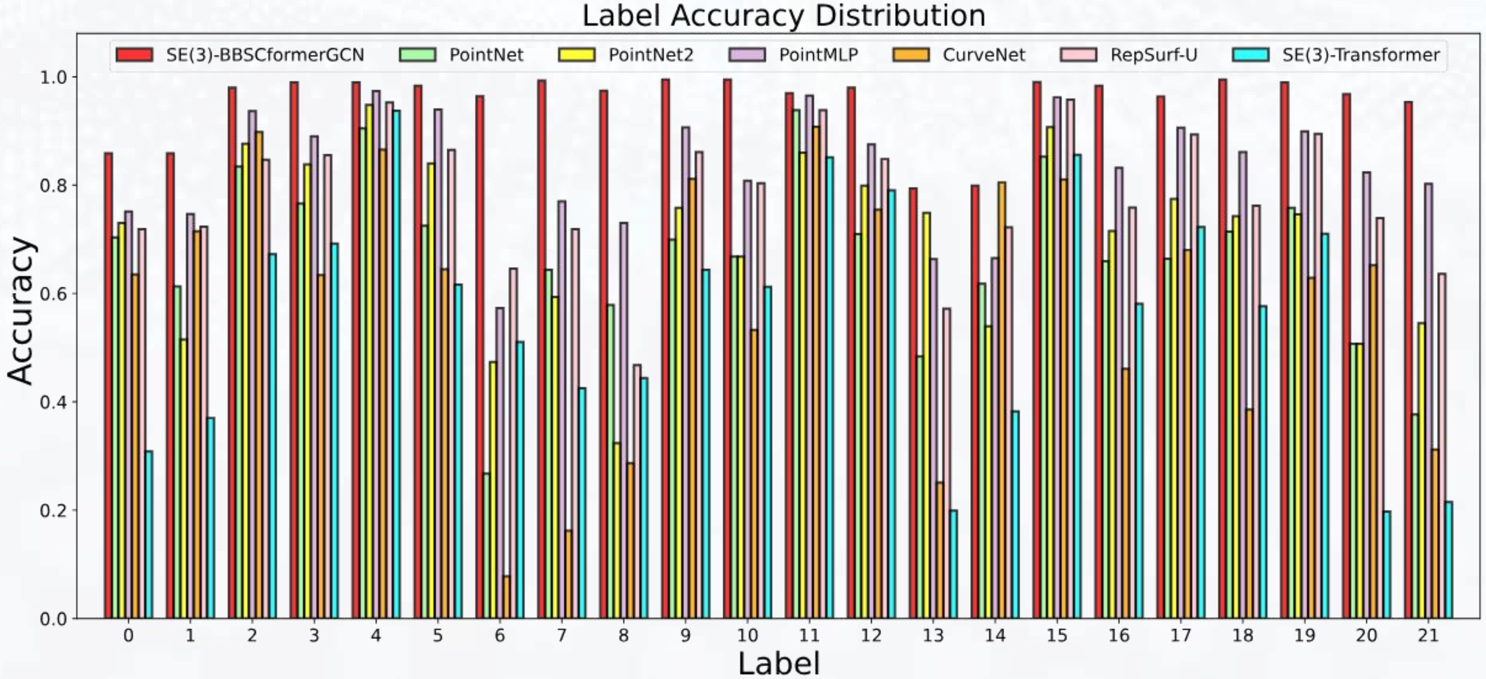
Classification performance

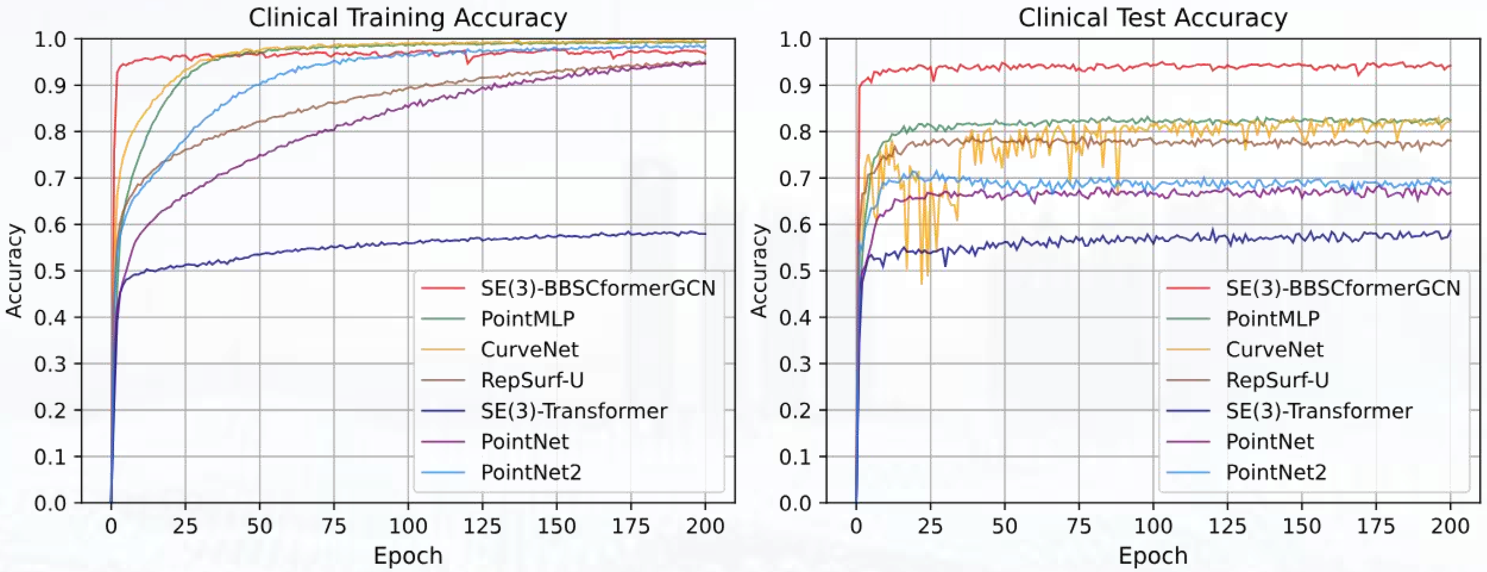
Method	Accuracy%	AUC-ROC%	Precision%	Recall%	F1 Score%
PointNet [24]	69.20 ± 0.98	96.50 ± 0.24	70.02 ± 0.77	69.17 ± 0.96	68.90 ± 1.02
PointNet++ [55]	70.27 ± 0.16	96.60 ± 0.17	70.82 ± 0.43	70.28 ± 0.20	70.05 ± **0.14**
CurveNet [56]	81.91 ± 0.14	93.80 ± 1.13	62.92 ± 3.14	60.13 ± 4.25	58.51 ± 4.58
RepSurf-U [57]	78.22 ± 0.17	98.40 ± 0.11	78.34 ± 0.28	77.99 ± 0.29	78.00 ± 0.25
PointMLP [58]	82.70 ± **0.10**	98.66 ± **0.06**	82.86 ± **0.14**	82.70 ± **0.10**	82.67 ± **0.14**
SE(3)-Transformer [30]	61.96 ± 1.45	92.32 ± 0.96	61.05 ± 2.07	61.94 ± 1.47	60.77 ± 2.57
SE(3)-BBSCformer	71.06 ± 1.39	93.25 ± 0.60	70.25 ± 1.16	71.04 ± 1.39	70.45 ± 1.21
BBSCformerGCN	95.45 ± 1.09	99.34 ± 0.21	95.10 ± 1.48	95.38 ± 1.13	95.23 ± 1.29
SE(3)-BBSCformerGCN	**96.11** ± 1.01	**99.38** ± 0.19	**96.11** ± 0.93	**96.08** ± 0.97	**96.02** ± 1.05

Per-class classification accuracy



Training Efficiency, Stability, and Computational Cost

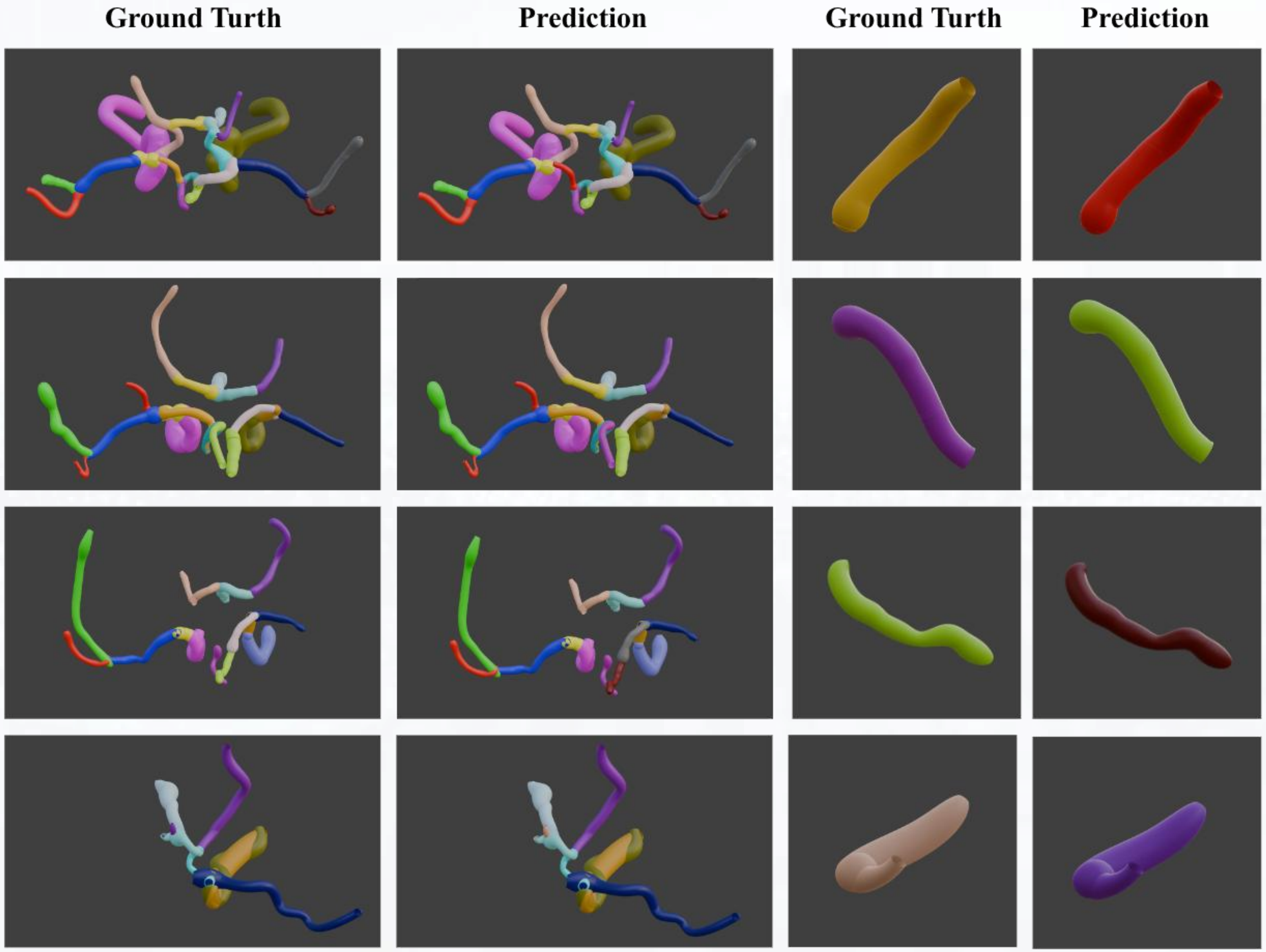
- Training
  - Convergence speed
  - Training stability
  - Resistance to overfitting



- Computational Cost
  - Evaluation time (ms)
  - FLOPS
  - Model parameters

Method	Evaluation Time(ms)	FLOPs(M)	Parameters(M)
3D DenseNet [53]	8.03	9743.76	18.56
3D ResNet [54]	10.95	11463.08	85.23
PointNet [24]	0.88	450.38	3.46
PointNet++ [55]	1.54	4067.53	1.74
CurveNet [56]	81.95	269.56	2.13
RepSurf-U [57]	59.84	911.32	1.48
PointMLP [58]	38.39	15733.95	13.23
SE(3)-Transformer [30]	19.03	456.33	0.12
SE(3)-BBSCformer	1.92	1.72	0.35
BBSCformerGCN	1.14	88.98	3.60
SE(3)-BBSCformerGCN	3.02	59.01	2.70

Case Study and Failure Case



Conclusion & Acknowledgements

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- Conclusion
  - Using BBSC to analyze tubular structure, and a functional space is constructed for this type of tubular manifold.
  - An analytical method is constructed that uses SE(3) isovariant mapping to analyze BBSC and graph convolution to process the topology of tubular topological manifolds.
  - Achieves superior accuracy in Circle of Willis branch classification.

- Acknowledgements

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Thanks