NEURAL INFORMATION PROCESSING SYSTEMS

上海交通大學
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Conformal Prediction in The Loop: A Feedback-Based Uncertainty Model for Trajectory Optimization

Han Wang, Chao Ning*

Data-driven Optimization & Control under Uncertainty (DOCU) Lab
Shanghai Jiao Tong University, Shanghai, China

NeurIPS 2025, December 2-7, San Diego CA, USA

Background

Conformal Prediction (CP) Theory

GAP

Conformal Prediction Application

Distribution Shift

Achieved through online learning or sliding-window methods.

Coverage Efficiency

Achieved through optimizing the shape or the length of the prediction region.

Limitation: Focusing on improving the predictive performance rather than directly targeting downstream decision-making.

Conformal Prediction-based Trajectory Optimization

Applying traditional and improved CP (ACI, EndPI) methods to trajectory optimization in single-agent and multi-agent systems.

Limitation: CP is employed to sequentially generate prediction regions without utilizing the decided information.

Motivation:

- Most existing CP studies focus on improving prediction models but overlook their impact on downstream decision-making.
- Current CP-based decision pipelines are sequential, blocking feedback from decisions to CP and thus limiting future performance improvement.

Preliminaries

- **Agent Nonlinear Dynamics**

$$x_{t+1} = f\left(x_t, u_t\right)$$
$$x_0 = x_{init}$$

System States

Control Inputs

Initial State

- **Joint Obstacle Dynamics**

$$Y_{t+1} = g\left(Y_{t-h}, \ldots, Y_{t-1}\right) + \omega_t$$
$$Y := \left(Y_0, \ldots, Y_T\right)$$

Joint Obstacle States

State Predictor (e.g. LSTM)

Random Modeling Error

Entire Trajectory of Obstacles

- **Calibration Dataset**

$$D_{cal} := \left\{Y^{(1)}, \ldots, Y^{(N)}\right\}$$

Division →

$$D_{cal}^{1} := \left\{Y^{(1)}, \ldots, Y^{(K)}\right\}$$
$$D_{cal}^{2} := \left\{Y^{(K+1)}, \ldots, Y^{(K+L)}\right\}$$

The calibration dataset is divided into two subsets

Preliminaries

- **Trajectory Optimization (TO) Problem**

$$\min_{x_{t+1:T},u_{t+1:T}} J\left(x_{t+1:T},u_{t+1:T}\right)=l_T\left(x_T\right)+\sum_{\tau=t}^{T} l_\tau\left(x_\tau,u_\tau\right)$$

$$s.t.\quad x_{\tau+1}=f\left(x_\tau,u_\tau\right),$$

$$x_\tau\in\mathcal{X},$$

$$u_\tau\in\mathcal{U},$$

$$\mathbb{P}\left\{\bigcap_{\tau=1}^{T}\left\{c\left(x_\tau,Y_\tau\right)\geq 0\right\}\right\}\geq 1-\alpha$$

Minimizing the cost function (Optimizing the trajectory)

Dynamics of system

System state constraint

Control input constraint

Joint chance constraint: The joint probability of satisfying the constraint over the total mission time is no less than 1 - α

- **Joint Chance Constraint Reformulation**

$$\mathbb{P}\left\{\bigcap_{\tau=1}^{T}\left\{c\left(x_\tau,Y_\tau\right)\geq 0\right\}\right\}\geq 1-\alpha \Leftarrow \begin{cases}\mathbb{P}\left\{c\left(x_\tau,Y_\tau\right)\geq 0\right\}\geq 1-\alpha_\tau\\ \sum_{\tau=1}^{T}\alpha_\tau\leq\alpha\end{cases}$$

Boole's inequality

Preliminaries

- **Vanilla Conformal Prediction**

Nonconformity Score

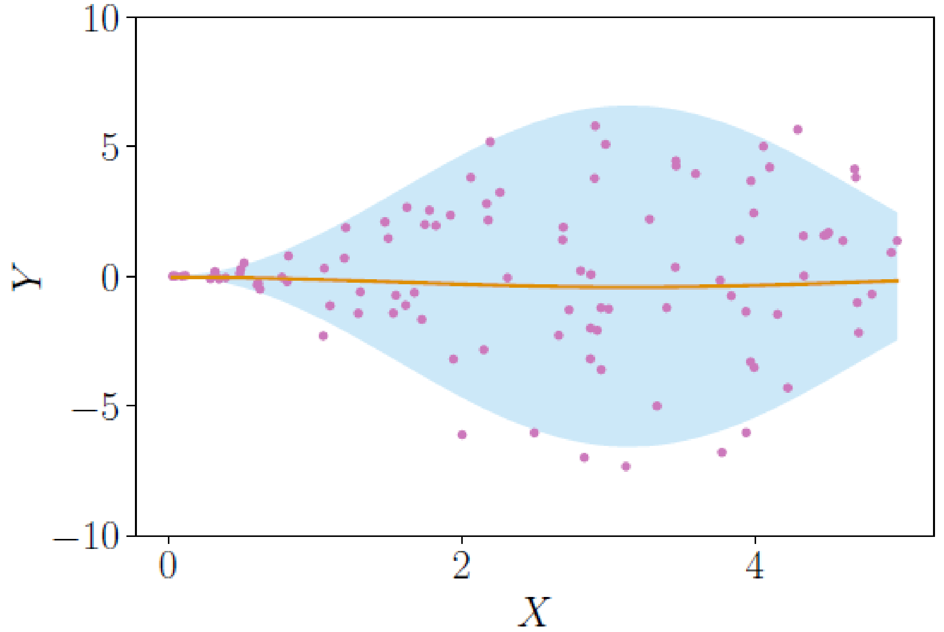
$R^{(0)}, R^{(1)}, \ldots, R^{(N)}$

N + 1 **Exchangeable** Random Variable

Coverage Guarantee

$$\mathbb{P}\left\{R^{(0)} \leq 1 - C^{1-\alpha}\right\} \geq 1-\alpha$$

$$C^{1-\alpha} = Quantile_{1-\alpha}\left(R^{(1)}, \ldots, R^{(N)}, \infty\right)$$



If $R^{(0)}, R^{(1)}, \ldots, R^{(N)}$ are N+1 **exchangeable** random variables, then for a failure probability $\alpha \in (0,1)$, it holds that

$$\mathbb{P}\left\{R^{(0)} \leq 1 - Quantile_{1-\alpha}\left(R^{(1)}, \ldots, R^{(N)}, \infty\right)\right\} \geq 1-\alpha$$

Feedback-based Conformal Prediction

- **Individual Chance Constraints Reformulation**

Define the nonconformity score

$$R_{\tau|t} = \left\| Y_\tau - \hat{Y}_{\tau|t} \right\|, R_{\tau|t}^{(i)} = \left\| Y_\tau^{(i)} - \hat{Y}_{\tau|t}^{(i)} \right\|, \forall i = 1, \ldots, K$$

Vanilla Conformal Prediction Theory

Obtain coverage guarantee

$$\mathbb{P}\left\{ \left\| Y_\tau - \hat{Y}_{\tau|t} \right\| \le Quantile_{1-\alpha_\tau}\left(R_{\tau|t}^{(1)}, \ldots, R_{\tau|t}^{(K)}, \infty \right) \right\} \ge 1 - \alpha_\tau$$

Lem. 4.1.

$$\mathbb{P}\{c(x_\tau, Y_\tau) \ge 0\} \ge 1 - \alpha_\tau$$

Reformulation →

$$c(x_\tau, \hat{Y}_{\tau|t}) \ge LQuantile_{1-\alpha_\tau}(R_{\tau|t}^{(1)}, \ldots, R_{\tau|t}^{(K)}, \infty)$$

Feedback-based Conformal Prediction

- **Posterior Probability Computation**

Define the nonconformity score

$$S_\tau = c\left(x_\tau^*, Y_\tau\right), S_\tau^{(i)} = c\left(x_\tau^*, \hat{Y}_{\tau|\tau-1} + \omega_\tau^{(i)}\right), \forall i = K+1, \ldots, L$$

Vanilla Conformal Prediction Theory

Obtain coverage guarantee

$$\mathbb{P}\left\{c\left(x_\tau^*, Y_\tau\right) \le Quantile_{1-\alpha_\tau}\left(S_\tau^{(K+1)}, \ldots, S_\tau^{(K+L)}, \infty\right)\right\} \ge 1 - \beta_\tau$$

Lem. 4.2. With the true state x_τ^* , the upper bound of the posterior violation probability is calculated as $\mathbb{P}\{c(x_\tau^*, Y_\tau) < 0\} \le \beta_\tau = \left(1 + \sum_{i=1}^{L} \mathbb{I}\left(S_\tau^{(K+i)} < 0\right)\right) / (1+L)$

Feedback-based Conformal Prediction

- **Optimization Problem Reformulation**

$$\min_{x_{t+1:T},u_{t+1:T}} \quad J(x_{t+1:T},u_{t+1:T}) = l_T(x_T) + \sum_{\tau=t}^{T} l_\tau(x_\tau,u_\tau)$$

$$\begin{aligned} s.t. \quad & x_{\tau+1} = f(x_\tau,u_\tau), \\ & x_\tau \in \mathcal{X}, \\ & u_\tau \in \mathcal{U}, \\ & \mathbb{P}\left\{\bigcap_{\tau=1}^{T}\{c(x_\tau,Y_\tau)\geq 0\}\right\} \geq 1-\alpha \end{aligned}$$

However, the reformulated problem requires treating the allocation risk $\alpha_{t+1:T}$ as a decision variable jointly optimized with $x_{t+1:T}$ and $u_{t:T-1}$, which makes the problem **challenging to solve**.

Boole's inequality

$$\begin{cases} \mathbb{P}\{c(x_\tau,Y_\tau)\geq 0\} \geq 1-\alpha_\tau \\ \sum_{\tau=1}^{T}\alpha_\tau \leq \alpha \end{cases}$$

Lem. 4.1.

$$c(x_\tau,\hat{Y}_{\tau|t}) \geq LQuantile_{1-\alpha_\tau}(R_{\tau|t}^{(1)},\ldots,R_{\tau|t}^{(K)},\infty)$$

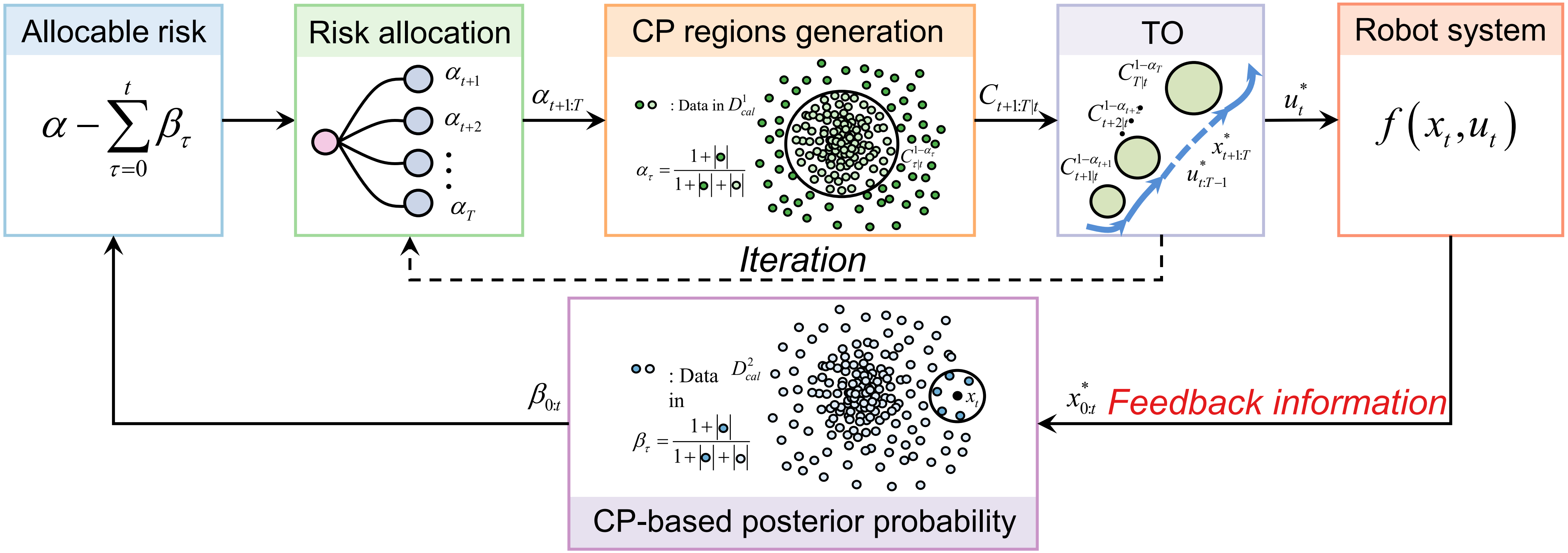
Lem. 4.2.

$$\sum_{\tau=t+1}^{T}\alpha_\tau \leq \alpha - \sum_{\tau=1}^{t}\beta_\tau$$

$$\mathbb{P}\{c(x_\tau^*,Y_\tau)<0\} \leq \beta_\tau = \left(1+\sum_{i=1}^{L}\mathbb{I}\left(S_\tau^{(K+i)}<0\right)\right)\Big/(1+L)$$

Trajectory Optimization using Fb-CP

- **Trajectory Optimization Framework using Fb-CP**



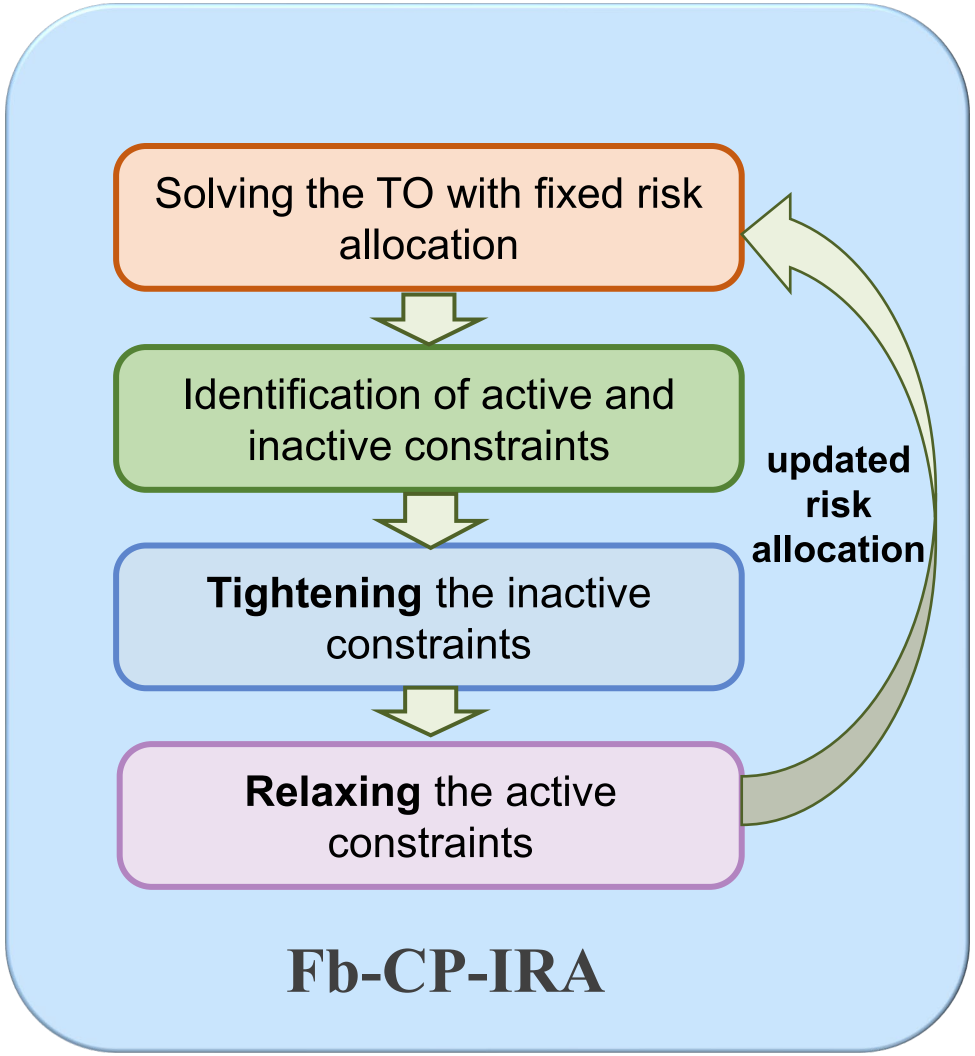
- The information in $x^*_{0:t}$ guides the feedback-based adjustments of the size of the prediction regions $C^{1-\alpha_\tau}_{\tau|t}$ through posterior probability calculations
- TO problem is solved iteratively by alternating between two steps: **1) risk allocation and 2) TO with the fixed** $\alpha_{t+1:T}$

Iterative Risk Allocation

Algorithm 1 Fb-CP using IRA at time t

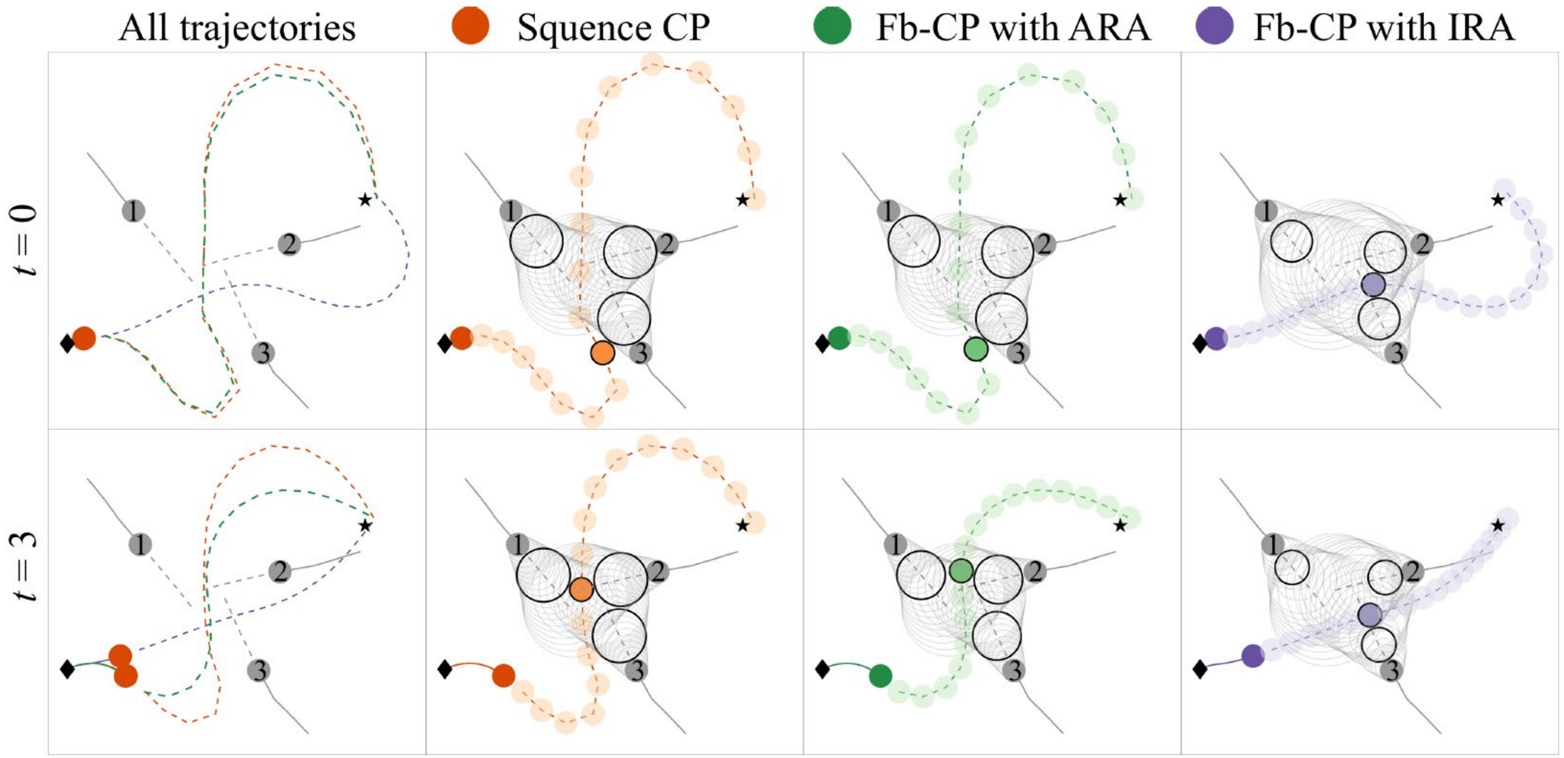
Input: α, $\alpha_{t:T}$, $\beta_{0:t-1}$, ϵ, η, D^1_{cal}, D^2_{cal}
Observe the system state x_t and joint obstacle states Y_t
$\hat{Y}_{t+1|t}, \ldots, \hat{Y}_{T|t} \leftarrow$ Trajectory prediction using LSTMs based on $Y_0, \ldots, Y_t$
$\beta_t \leftarrow$ Posterior probability calculation (8) {Using x_t and D^2_{cal}}
$J^*(\alpha^{-1}_{t+1:T}) \leftarrow \infty$, $\alpha^0_{t+1:T} \leftarrow \alpha_{t+1:T}$, $n \leftarrow 0$ {Initialization of IRA}
repeat
  $J^*(\alpha^n_{t+1:T})$, $x^n_{t+1:T}$, $u^n_{t:T-1} \leftarrow$ Solve the lower-stage problem (10) with $\alpha^n_{t+1:T}$
  $\mathcal{I}_{act}$, $\mathcal{I}_{ina}$, $N_{act} \leftarrow$ Identification of active and inactive constraints
  $\widetilde{\alpha}^n_{t+1:T} \leftarrow$ Transitional risk allocation calculation (15)
  $\alpha^{n+1}_{t+1:T} \leftarrow$ New risk allocation calculation (17)
  $n \leftarrow n+1$
until $|J^*(\alpha^{n-1}_{t+1:T}) - J^*(\alpha^{n-2}_{t+1:T})| < \epsilon$
Output: $\beta_{0:t}$, $u^{n-1}_{t:T-1}$, $\alpha_{t+1:T} = \alpha^{n-1}_{t+1:T}$

It iterates between risk allocation and trajectory optimization.



Simulation Result (Trajectory)

Trajectories of different control methods



Owing to posterior probability updates, Fb-CP-ARA and Fb-CP-IRA can dynamically tighten the prediction regions as more vehicle positions become available, thereby achieving less conservative trajectories than S-CP.

Through iterative risk allocation, Fb-CP-IRA further enhances flexibility in distributing risks across future times, resulting in a more efficient and less conservative trajectory.

Simulation Result (Cost)

Average cost, computation time, and collision avoidance rate with different methods

		CC		ACI-MP	RF-CP	S-CP	Fb-CP ARA	Fb-CP IRA
Average cost	$\eta = 1000$	59.25	$\alpha = 0.05$	17.970	15.794	17.321	15.356	7.189
	$\eta = 500$	47.50	$\alpha = 0.10$	17.263	14.378	16.17	14.228	6.798
	$\eta = 100$	22.46	$\alpha = 0.15$	16.096	11.922	14.83	12.354	6.191
	$\eta = 50$	21.34	$\alpha = 0.20$	15.310	10.032	13.217	10.22	5.398
Average computation time	$\eta = 1000$	0.019	$\alpha = 0.05$	0.022	0.487	0.022	0.027	0.038
	$\eta = 500$	0.019	$\alpha = 0.10$	0.026	0.494	0.020	0.021	0.039
	$\eta = 100$	0.021	$\alpha = 0.15$	0.021	0.545	0.021	0.020	0.037
	$\eta = 50$	0.022	$\alpha = 0.20$	0.022	0.500	0.020	0.019	0.036
Collision avoidance rate	$\eta = 1000$	97.0%	$\alpha = 0.05$	98.6%	98.7%	98.8%	98.2%	96.3%
	$\eta = 500$	92.8%	$\alpha = 0.10$	93.3%	96.9%	93.5%	94.6%	94.1%
	$\eta = 100$	82.5%	$\alpha = 0.15$	91.5%	92.4%	92.0%	90.2%	91.9%
	$\eta = 50$	79.1%	$\alpha = 0.20$	87.9%	90.0%	88.2%	86.7%	88.2%

The methods used in the simulation

- Conformal Control **(CC)**
- ACI for Motion Planning **(ACI-MP)**
- Recursively Feasible MPC using CP **(RF-CP)**
- Sequential Conformal Prediction **(S-CP)**
- Fb-CP with Average Risk Allocation **(Fb-CP-ARA)**
- Fb-CP with Iterative Risk Allocation **(Fb-CP-IRA)**

- The Fb-CP-ARA reduces the cost by an average of 11.34% compared with S-CP, thanks to the feedback information of posterior probabilities, with a negligible additional computational burden.
- By flexibly allocating the additional allowable risk provided by posterior probabilities, Fb-CP-IRA achieves a 58.50% reduction in average cost compared with S-CP.

Conclusions

- We propose a novel framework that integrates CP with decision-making, enabling feedback-driven adjustment of prediction regions.
- Fb-CP provably maintains prediction validity while improving decision performance.
- An Iterative Risk Allocation method is developed, providing convergence guarantees and enhanced trajectory optimization.
- Fb-CP is extended to handle distribution shifts via a weighted calibration strategy, ensuring robustness under changing environments.

Thank you very much!

Code repository:

https://github.com/DOCU-Lab/Feedback-based_Conformal_Prediction

PDF of our paper:

https://arxiv.org/abs/2510.16376