The Effect of Optimal Self-Distillation in Noisy Gaussian Mixture Model

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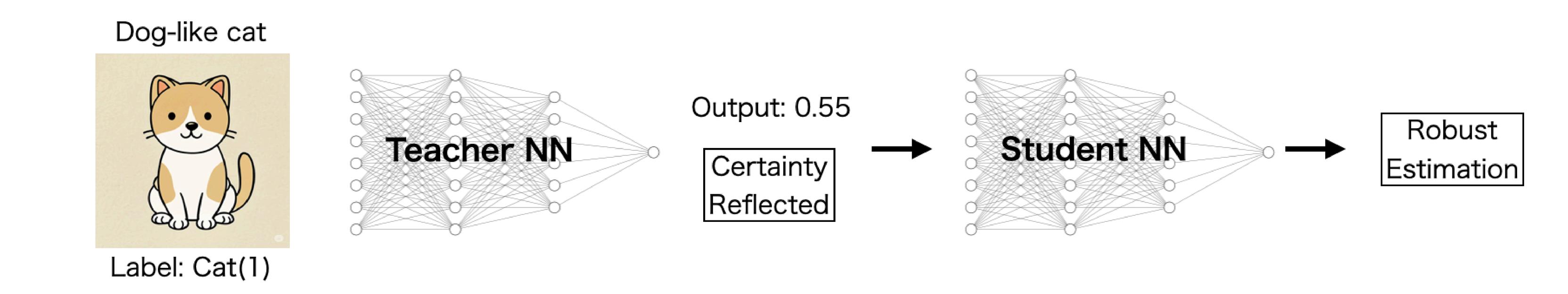
What is self-distillation (SD)?

- **Knowledge Distillation**: A large model teaches a small one for compression. (Teacher size > Student size)),
- **Self Distillation (SD)**: A model teaches itself for a performance boost. (Teacher size = Student size),

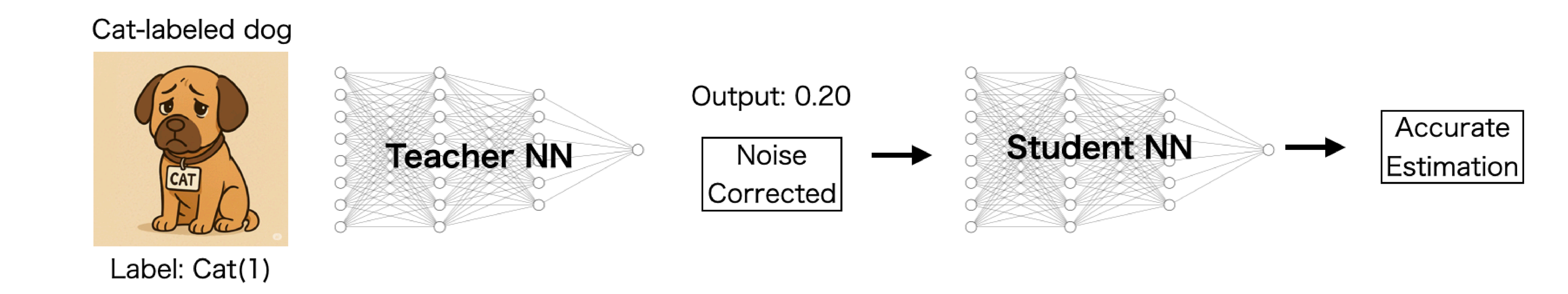
Why successful? (in classification)

- The model's output probabilities (**dark knowledge**) carry information beyond hard labels.
- SD can also remove noise from training labels (**denoising**).

Dark knowledge effect [Hinton et al., 2015]



Denoising effect [Das et al., 2023]



Question

- How do dark knowledge/denoising contribute to performance gains, and by how much?
- What improvements can multi-stage SD achieve when its hyperparameters are optimally tuned?
- What is the optimal update steps?
- What is the effect of bias learning in class-imbalanced dataset?

Approach

Statistical analysis based on inequality bounds offers only coarse insight into hyperparameter effects.
→ High-dimensional solvable models provide **precise** evaluation (exact expectaion behavior for hyperparameters).

Model

Binary classification of Gaussian mixture data with noisy labels using a single-layer neural network:

Data generation

$$\begin{cases} \alpha = (\#\text{ of Data}(=M))/(\text{Dimension}(=N)) \\ y_\mu^{\text{true}} \in \{0,1\}, \quad P(y_\mu^{\text{true}}=1)=\rho, \quad P(y_\mu \neq y_\mu^{\text{true}})=\theta \\ \boldsymbol{x}_\mu \sim (2y_\mu^{\text{true}}-1)\boldsymbol{v}/\sqrt{N} + \sqrt{\Delta}\mathcal{N}(0,1) \end{cases}$$

We learn from noisy labels $\mathcal{D} = \{(\boldsymbol{x}_\mu, y_\mu)\}_{\mu=1}^M$.

Multi-stage SD

The learning process at each stage $t = 0, 1, \ldots, T$ is given by

$$\hat{\boldsymbol{w}}^t, \hat{B}^t = \underset{\boldsymbol{w}^t, B^t}{\operatorname{argmin}} \left[\sum_{\mu=1}^M \ell\left(y_\mu^t, \sigma\left(\frac{\boldsymbol{w}^t \cdot \boldsymbol{x}_\mu}{\sqrt{N}} + B^t \right) \right) + \frac{\lambda^t}{2} \|\boldsymbol{w}^t\|^2 \right],$$

where y_μ^t is the label we use at stage t:

$$\begin{cases} y_\mu^0 = y_\mu \ (\text{observed label}) \\ y_\mu^t = \sigma\left(\beta^t\left((\hat{\boldsymbol{w}}^{t-1}\cdot\boldsymbol{x}_\mu)/\sqrt{N} + \hat{B}^{t-1}\right)\right)(t \geq 1) \ (\text{pseudo-label}) \end{cases}$$

Evaluation of the optimal multi-stage SD

- **Error metric**: $\mathcal{E}^t =$ (data-averaged 0-1 generalization error)
- **Hyperparameters**: $\lambda^t (t = 0, 1, \ldots)$ and $\beta^t (t = 1, 2, \ldots)$
- **Optimal SD**: $\mathcal{E}^{t*} = (\mathcal{E}^t$ at the optimal hyperparameters)
- **Optimal SD effect**: $\mathcal{E}^{0*} - \mathcal{E}^{t*}$

Technical Details: Replica method for dynamics

The dynamics of $\varphi^t = (\hat{\boldsymbol{w}}^t, \hat{B}^t)(t = 0, 1, \ldots)$ is given by

$$p(\varphi^T, \varphi^{T-1}, \ldots, \varphi^1 \mid \varphi^0, \mathcal{D}) = \lim_{\beta\to\infty} \prod_{t=1}^T \frac{\exp(-\beta \mathcal{L}(\varphi^t \mid \varphi^{t-1}, \mathcal{D}))}{Z^t(\varphi^{t-1}, \mathcal{D})},$$

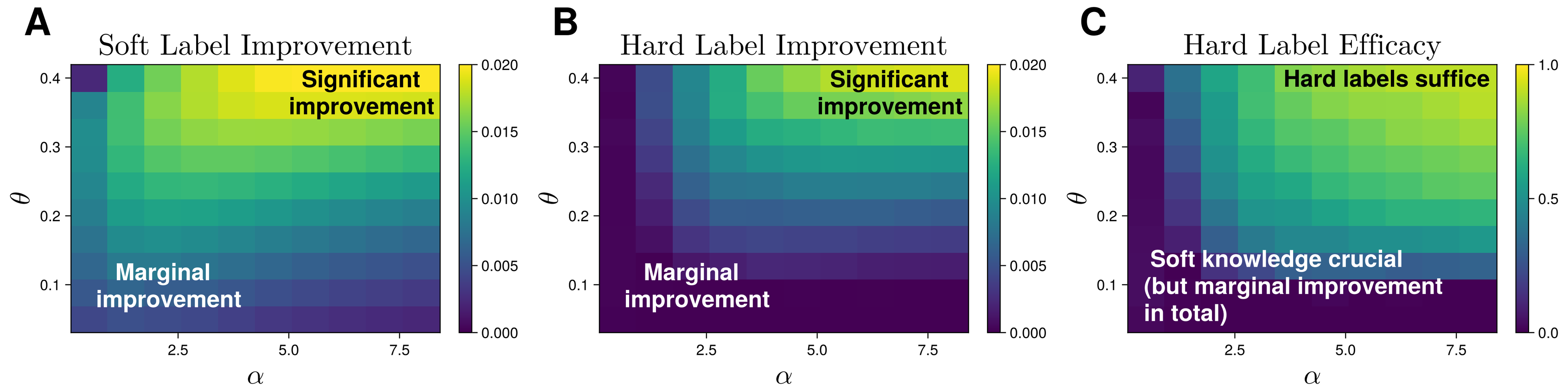
where the denominator makes data averaging difficult.

Multi-Stage Replica Method

Replicating partition function $1/(Z^t(\varphi^{t-1}, \mathcal{D})) = \lim_{n^t\to 0} (Z^t(\varphi^{t-1}, \mathcal{D}))^{n^t-1}$ for all t eliminates the averaging issue.

Results

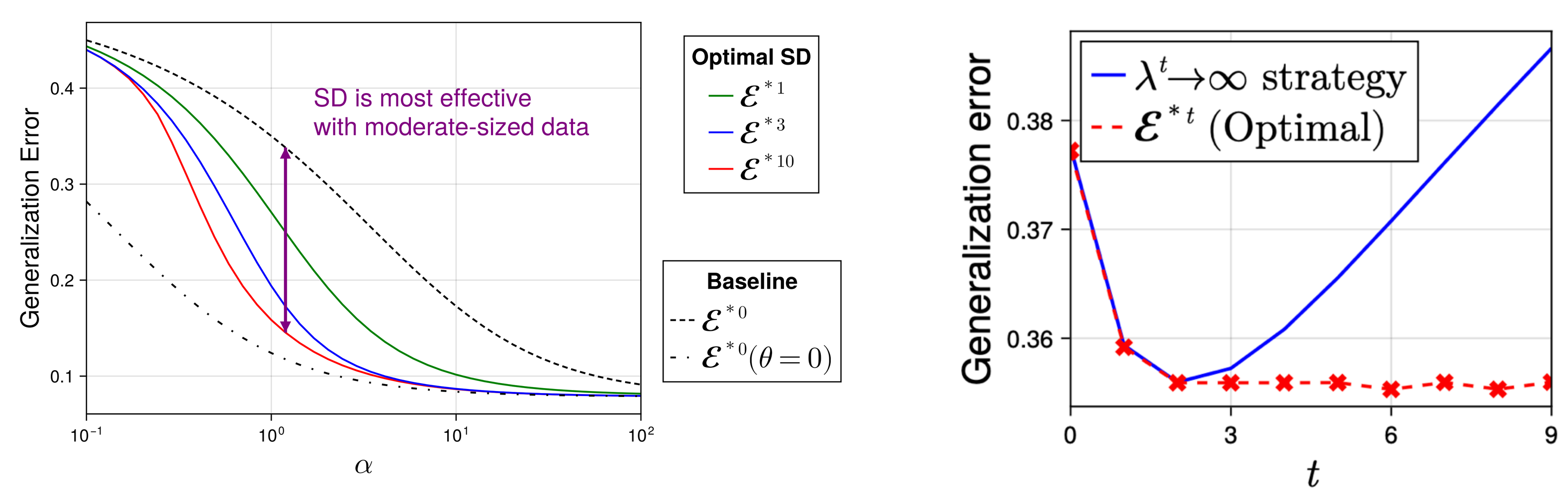
The Role of Soft Labels in SD (T=1)



Takeaways

Hard label improvement ≈ Soft error improvement
→ **Dark knowledge effect is limited**, denoising is the key factor for SD.

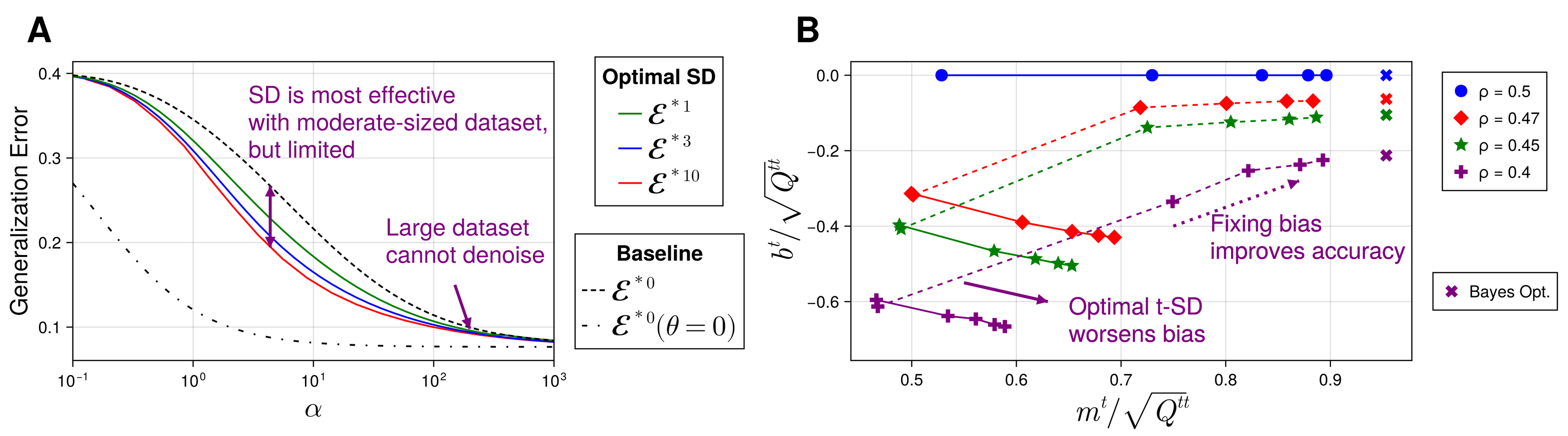
The Effect of Multi-Stage SD in Balanced Data



Takeaways

- The benefit of SD peaks at intermediate dataset size.
- Naive SD can perform as poorly as random guessing.
  → **Early stopping** heuristic matches optimal SD.

Hardness of learning bias



Takeaways

aligning both the bias and the decision-boundary orientation at once is hard under imbalanced dataset
→ **Bias fixing** heuristic approaches Bayes opt.

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