Yinjie ${\sf Min}^{*\,1}$, Furong ${\sf Xu}^{*\,1}$, Xinyao ${\sf Li}^2$, Changliang ${\sf Zou}^{\dagger 1}$, Yongdao ${\sf Zhou}^{\dagger 1}$

1 School of Statistics and Data Science, Nankai University 2 School of Computer Science and Engineering, University of Electronic Science and Technology of China

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Active Learning Challenge in Regression

- Active Learning (AL) reduces annotation costs by selecting informative samples.
- Informativeness consists of two components:
 - Model sensitivity (measured via parameter gradients).
 - Predictive uncertainty (hard to estimate without labels).
- Regression tasks face fundamental challenge: no direct uncertainty estimation.

Key Problem

How to estimate predictive uncertainty for regression when true labels are unavailable?

Auxiliary Data: An Underutilized Resource

- Real-world scenarios often have **imperfect auxiliary data**:
 - Medical images with varying symptom manifestations.

 Automorphism data from position and an image and a series of the serie
 - Autonomous vehicle data from varied environments.
 - Industrial sensor logs with recording inaccuracies.
- These data are typically discarded due to distribution shifts.
- Our insight: Auxiliary data can provide reliable uncertainty estimation when properly weighted.

AGBAL: Core Idea

- Key innovation: Weighted loss approximation based on density ratio.
- Auxiliary data guides uncertainty estimation despite distribution shifts.
- Three-step process: density ratio estimation \rightarrow auxiliary loss computation \rightarrow weighting.

Experimental Results

Mathematical Formulation

The Decompose of the Loss Gradient $\partial R(\theta, P)/\partial \theta$

$$\frac{\partial R(\theta; P)}{\partial \theta} = \mathbb{E}_{X,Y \sim P} \frac{\partial l(Y, f(X; \theta))}{\partial \theta}
= \mathbb{E}_{X,Y \sim P} \frac{\partial l(Y, f(X; \theta))}{\partial f(X; \theta)} \frac{\partial f(X; \theta)}{\partial \theta}
= \mathbb{E}_{X \sim P_X} \phi_1(\theta; X) \Big\{ \mathbb{E}_{Y \sim P_{Y|X}} \frac{\partial l(Y, f(X; \theta))}{\partial f(X; \theta)} \Big\}
= \mathbb{E}_{X \sim P_Y} \phi_1(\theta; X) \phi_2(\theta; X).$$

Mathematical Formulation

Density Ratio Weighting

We estimate the expected loss gradient using auxiliary data:

$$\widehat{\phi}_2(\theta; x) = \frac{1}{n'} \sum_{i=1}^{n'} \widehat{r}(X_i', Y_i') \cdot \frac{\partial l(Y_i', f(X_i'; \theta))}{\partial f(X_i'; \theta)},$$

where $\widehat{r}(x,y)$ is the density ratio estimator.

Auxiliary-Guided Gradient Kernel

$$K_{\mathsf{grad-aux}}(x, x'; \theta) = \{ \widehat{\phi}_2(\theta; x) \phi_1(\theta; x) \}^{\top} \{ \widehat{\phi}_2(\theta; x') \phi_1(\theta; x') \}.$$

Theoretical Guarantees

Theorem 1 (Uncertainty Estimation Consistency)

Under Neural Tangent Kernel (NTK) theory, our auxiliary data guided estimator $\phi_2(\theta;x)$ provides a consistent surrogate for the true expected loss gradient, with variance proportional to the ridge estimator variance.

- Provides theoretical foundation for uncertainty estimation.
- Justifies use of distributionally shifted auxiliary data.
- Ensures reliability of the selection process.

Comprehensive Evaluation Setup

- Datasets: 2 synthetic (S1, S2) + 5 real-world (BIO, BIKE, DIAMOND, CT, STOCK).
- **Comparison**: 8 selection methods + random baseline.
- Metrics: Area Under Curve (AUC) of MSE learning curves, RMSE at step 10.
- **Settings**: $|\mathcal{L}_0| = 200$, batch size N = 200, 15 active learning steps.

AUC Performance Across Datasets (AGBAL vs BMDAL)

Table 1: Comparison of 8 selection methods across synthetic and real-world datasets in terms of AUC, where Avg Impro represents improvement over BMDAL averaged across 7 experiments.

Method	S1	S2	BIO	BIKE	DIAMOND	CT	STOCK	Avg Impro
random	0.928	1.421	0.451	0.459	21.009	0.380	0.392	_
lcmd	1.011	1.517	0.417	0.394	19.714	0.255	0.370	
Icmd (AGBAL)	0.846	1.279	0.420	0.435	20.687	0.291	0.363	+0.6%
maxdist	0.863	1.310	0.428	0.439	20.643	0.270	0.389	_
maxdist (AGBAL)	0.834	1.266	0.417	0.401	21.179	0.298	0.361	+1.8%
kmeanspp	0.894	1.378	0.414	0.404	19.691	0.271	0.372	
kmeanspp (AGBAL)	0.842	1.294	0.406	0.364	19.613	0.264	0.355	+4.5%
fw	0.953	1.448	0.434	0.455	21.115	0.347	0.388	_
fw (AGBAL)	0.899	1.341	0.418	0.404	21.840	0.310	0.377	+5.5%
bait	0.853	1.340	0.441	0.481	22.057	0.435	0.392	_
bait (AGBAL)	0.835	1.293	0.431	0.398	22.134	0.374	0.371	+6.3%
maxdet	0.876	1.318	0.418	0.486	19.702	0.320	0.378	_
maxdet (AGBAL)	0.833	1.254	0.409	0.362	19.846	0.254	0.359	+8.9%
maxdiag	0.903	1.401	0.451	0.597	24.594	0.526	0.415	_
maxdiag (AGBAL)	0.836	1.270	0.420	0.410	20.745	0.304	0.361	+18.0%

AGBAL Outperforms Across Datasets

- AGBAL consistently outperforms BMDAL (no auxiliary data).
- Significant improvements in both synthetic and real-world datasets.
- Best performance with maxdet and kmeanspp selection methods.

Visualization: Better Uncertainty Estimation

- AGBAL identifies truly high-uncertainty points.
- BMDAL selects well-trained points.

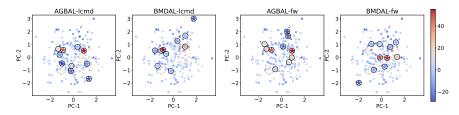


Figure 1: Visualization of the loss of selected points across four AL configurations. Left, right panels display lcmd, fw results of AGBAL and BMDAL, respectively.

Auxiliary Data Quality Analysis

Table 2: Worst case AUC comparison between AGBAL and BMDAL with distributional shift parameter $\zeta=64$.

		Selection Methods								
Dataset	Method	maxdiag	maxdet	bait	fw	maxdist	kmeanspp	lcmd		
S1	BMDAL	0.956	0.952	0.914	1.038	0.933	0.975	1.131		
	AGBAL	0.942	0.918	0.904	1.038	0.947	0.940	0.975		
	Improv.	1.5%	3.6%	1.1%	0.0%	-1.5%	3.6%	13.8%		
S2	BMDAL	1.501	1.430	1.417	1.583	1.406	1.479	1.647		
	AGBAL	1.437	1.398	1.390	1.543	1.436	1.426	1.468		
	Improv.	4.3%	2.2%	1.9%	2.5%	-2.1%	3.6%	10.9%		

AUC with Varying $N_{ m aux}$

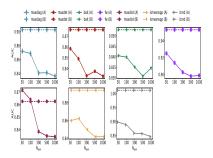


Figure 2: AUC for S1 ($N_{
m aux}$ variation).

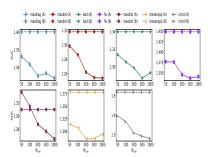


Figure 3: AUC for S2 ($N_{
m aux}$ variation).

Contributions Summary

- Theoretical: Formal decomposition of informativeness into sensitivity and uncertainty.
- Methodological: AGBAL framework for leveraging imperfect auxiliary data.
- **Empirical**: Consistent improvements across diverse datasets and selection strategies.
- **Practical**: Lightweight implementation with minimal computational overhead.

Broader Impact & Future Directions

Positive Impacts

- Reduces annotation costs in resource-constrained domains.
- Enables use of otherwise discarded imperfect data.
- Applicable to healthcare, autonomous driving, industrial monitoring.

Future Work

- Extend to high-dimensional structured data (images, time series).
- Investigate privacy-preserving variants.
- Explore cross-modal auxiliary data utilization.

