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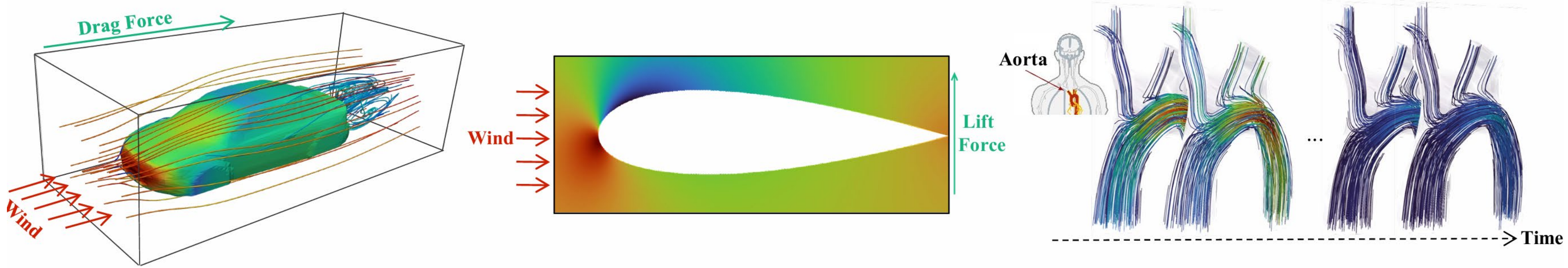
SpiderSolver: A Geometry-Aware Transformer for Solving PDEs on Complex Geometries

Kai Qi, Fan Wang, Zhewen Dong and Jian Sun

School of Mathematics and Statistics,
Xi'an Jiaotong University,
qikai1218@stu.xjtu.edu.cn,
jiansun@xjtu.edu.cn.

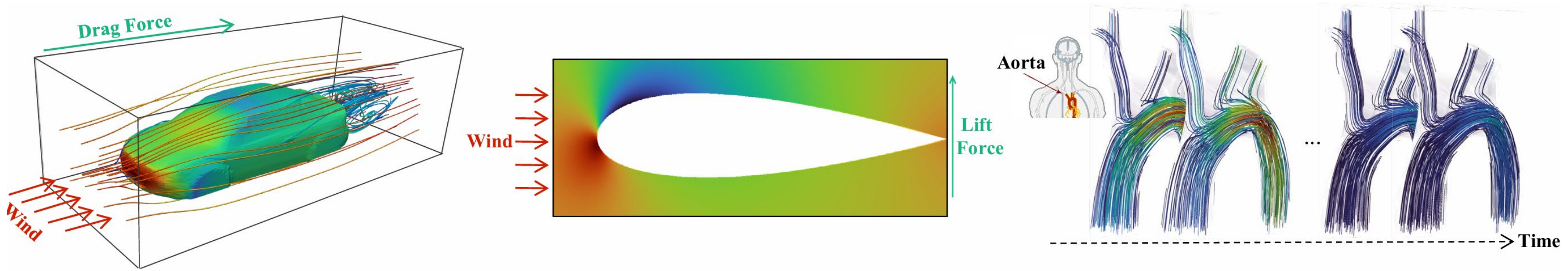
1. Background&Challenges: Solving PDEs on Complex Geometries



(a) Shape-Net Car (b) AirfRANS (c) Blood Flow

- Complex domain geometry and irregularly discretized points.
- Traditional methods struggle with mesh generation and computational cost.
- Neural operators often limited to regular domains or lack geometry awareness.

2. Motivation

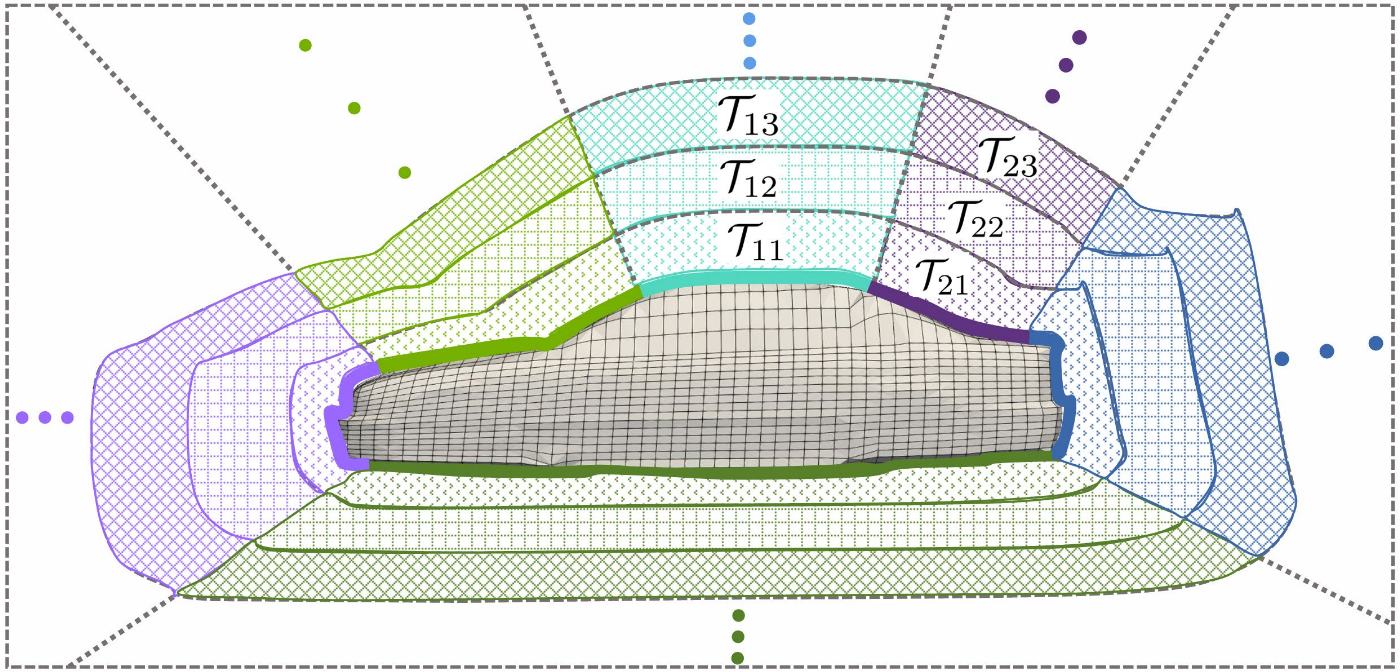


(a) Shape-Net Car (b) AirfRANS (c) Blood Flow

- Layered flow structures require geometry-aware discretization.
- Physical phenomena are geometry-dominated.
- Near-boundary effects critical for force coefficients (e.g., drag and lift).

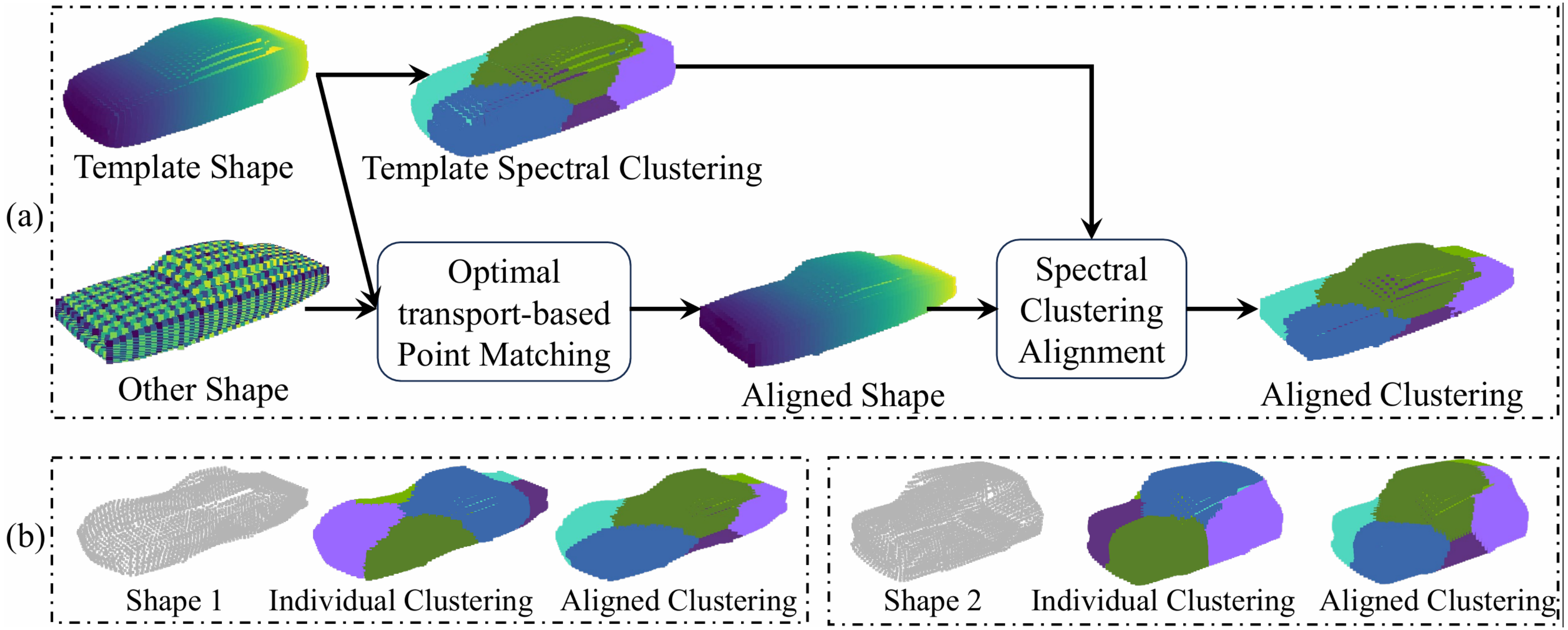
3. Method: Spiderweb Tokenization

- Spectral clustering for the domain boundary.
- Inner space is partitioned based on the boundary clustering and SDF intervals.



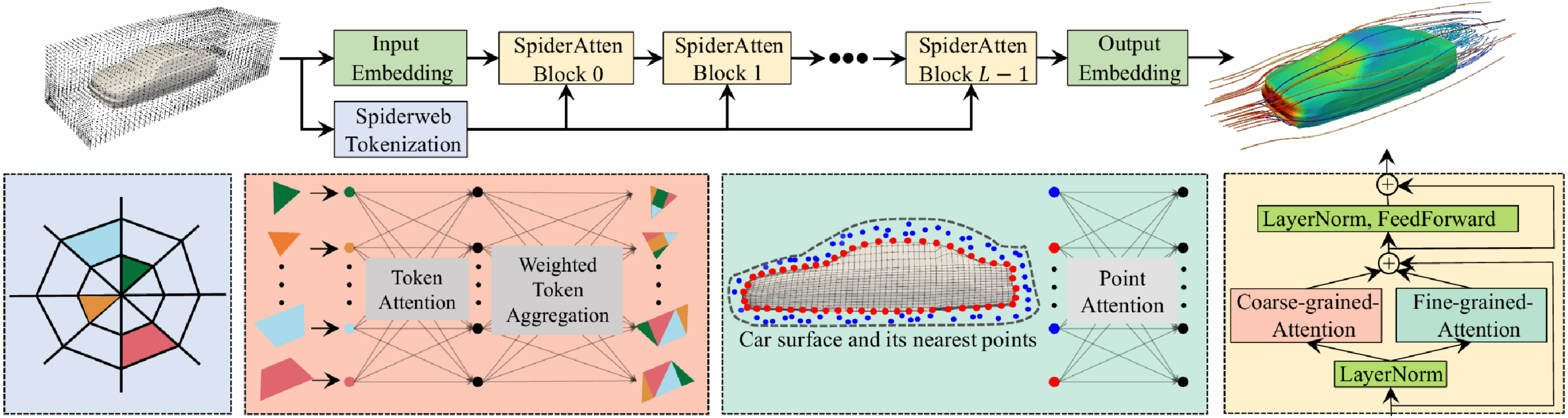
3. Method: Aligned Boundary Clustering

- Avoids inconsistent clustering across shapes via optimal transport.
- Transfers labels from a template shape to ensure semantic alignment of patches.



3. Model Architecture: SpiderSolver

- Coarse-grained Attention interacts the features over the spiderweb tokens.
- Fine-grained Attention enables interactions between points of boundary and near boundary.
- T2P aggregation: updates point features from token representations.

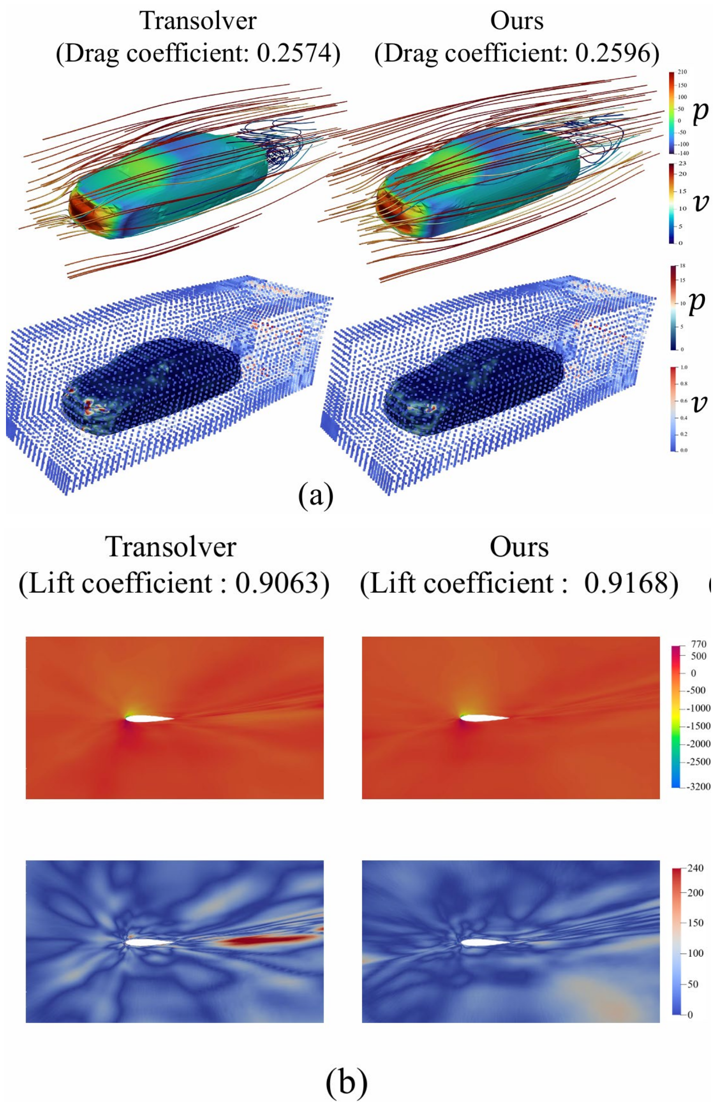


(a) Spiderweb Tokenization (b) Coarse-grained Attention (c) Fine-grained Attention (d) SpiderAtten Block

4. Experiments: Shape-Net Car and AirfRANS Datasets

Table 1: Results on Shape-Net Car and AirfRANS datasets. Vol: error of surrounding physics field; Surf: error of surface physics field. C_D, C_L: error of drag and lift coefficients; ρ_D, ρ_L: Spearman's rank correlation of drag and lift coefficients.

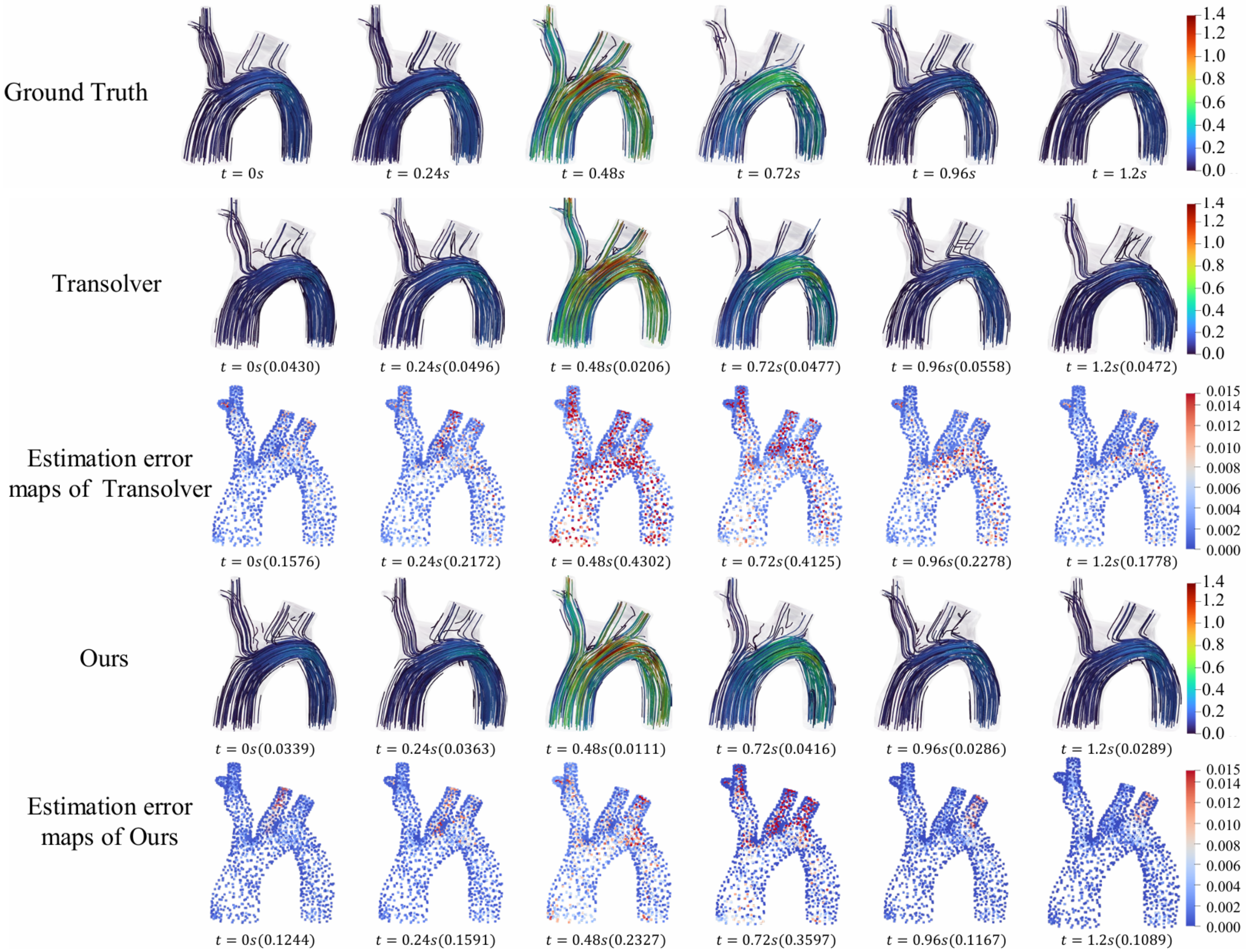
Methods	Shape-Net Car				AirfRANS			
	Vol ↓	Surf ↓	C_D ↓	ρ_D ↑	Vol ↓	Surf ↓	C_L ↓	ρ_L ↑
Simple MLP	0.0512	0.1304	0.0307	0.9496	0.0081	0.0200	0.2108	0.9932
G-SAGE [28]	0.0461	0.1050	0.0270	0.9695	0.0087	0.0184	0.1476	0.9964
PointNet [29]	0.0494	0.1104	0.0298	0.9583	0.0253	0.0996	0.1973	0.9949
G-U-Net [30]	0.0471	0.1102	0.0226	0.9725	0.0076	0.0146	0.1677	0.9944
MG-Net [31]	0.0354	0.0781	0.0168	0.9840	0.0214	0.0387	0.2252	0.9945
GNO [2]	0.0383	0.0815	0.0172	0.9834	0.0269	0.0405	0.2016	0.9934
Galerkin [13]	0.0339	0.0878	0.0179	0.9764	0.0074	0.0159	0.2336	0.9957
Geo-FNO [6]	0.1670	0.2378	0.0664	0.8280	0.0361	0.0820	0.6614	0.9257
GNOT [32]	0.0329	0.0798	0.0178	0.9833	0.0049	0.0152	0.1992	0.9942
GINO [5]	0.0386	0.0810	0.0184	0.9826	0.0297	0.0482	0.1821	0.9958
3D-GeoCA [8]	0.0319	0.0779	0.0159	0.9842	/	/	/	/
Transolver [14]	0.0228	0.0793	0.0129	0.9916	0.0025	0.0080	0.1026	0.9977
SpiderSolver	**0.0210**	**0.0738**	**0.0100**	**0.9928**	**0.0017**	**0.0043**	**0.0741**	**0.9988**



4. Experiments: BloodFlow Dataset

Table 2: Results on Blood Flow dataset.

Methods	Velocity ↓
Simple MLP	0.3080
DeepONet [4]	0.8926
POD-D [38]	0.3742
Geo-FNO [6]	0.1209
GNOT [32]	0.0411
NORM [36]	0.0453
Geo-FNO [6]	0.1209
3D-GeoCA [8]	0.2863
GINO [5]	0.1864
Transolver [14]	0.0438
SpiderSolver	**0.0322**



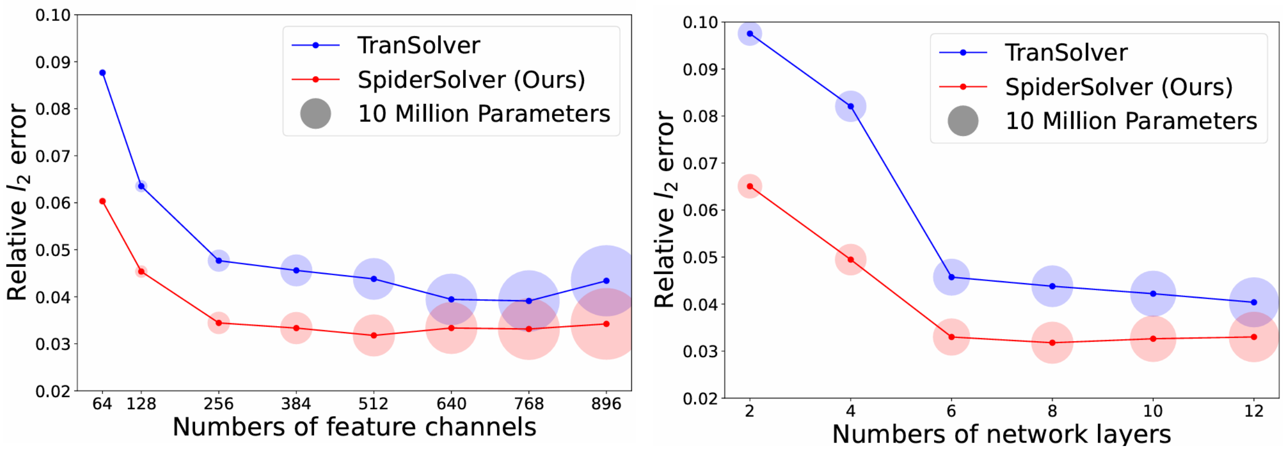
4. Experiments: More Results

Table 3: Generalization in OOD on AirfRANS.

Methods	Reynolds OOD $C_L \downarrow$	Reynolds OOD $\rho_L \uparrow$	Angles OOD $C_L \downarrow$	Angles OOD $\rho_L \uparrow$
Simple MLP	0.6205	0.9578	0.4128	0.9572
G-SAGE [28]	0.4333	0.9707	0.2538	0.9894
PointNet [29]	0.3836	0.9806	0.4425	0.9784
G-U-Net [30]	0.4664	0.9645	0.3756	0.9186
GNO [2]	0.4408	0.9878	0.3038	0.9836
Galerkin [13]	0.4615	0.9862	0.3814	0.9821
GNOT [32]	0.3268	0.9865	0.3497	0.9863
GINO [5]	0.4180	0.9645	0.2583	0.9923
Transolver [14]	0.3889	0.9911	0.2490	0.9940
SpiderSolver	**0.2291**	**0.9922**	**0.1062**	**0.9941**

Table 4: Generalization to shape variations of cars on Shape-Net Car dataset.

Methods	Shape-Net Car Vol $\downarrow$	Surf $\downarrow$	$C_D \downarrow$	$\rho_D \uparrow$
Transolver [14]	0.0660	0.191	0.0735	0.9142
SpiderSolver	**0.0510**	**0.161**	**0.0550**	**0.9222**



Thanks for your attention !