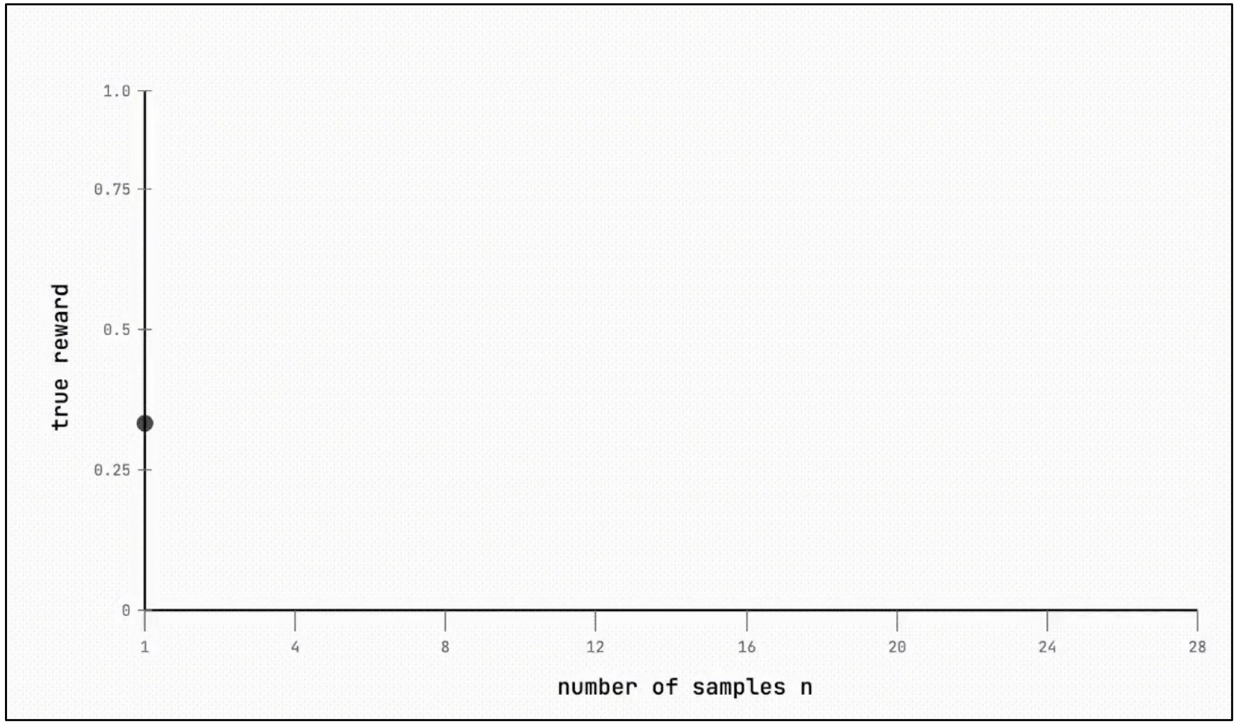

Inference-Time Reward Hacking in Large Language Models

Spotlight Award! 🌟

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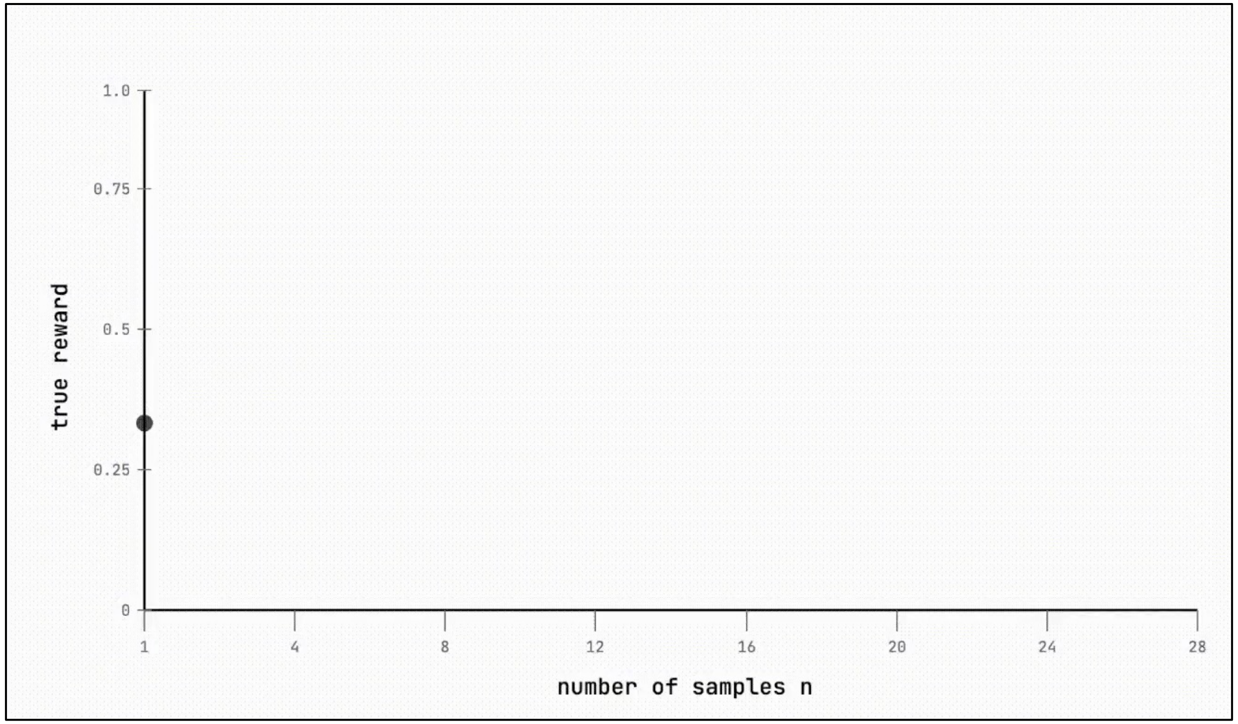
1. **Proxy rewards** are the computable signals we can use
   Example: Scores from a trained reward model

2. **True rewards** are the quality of an output according to a desired objective
   Example: An output's helpfulness or toxicity

All proxy reward models are bad!

We are maximizing for the wrong reward.

This mismatch can cause **reward hacking**!



Inference-time Alignment

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One example is **Best-of-n**.

For a given question,

1. Sample *n* responses
2. Score them using a reward model
3. Choose the one with the highest reward

Inference-time Alignment

We leverage a reward model to improve our responses without any training.

Another example is **Soft Best-of-n.**

For a given question,

1. Sample *n* responses
2. Score them using a reward model
3. Sample a response using a temperature-scaled softmax over rewards

What should you do with a proxy reward?

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Don’t fully trust it. Instead, you should **hedge**!

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Our contributions:

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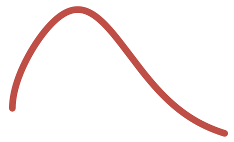
1. We characterize reward hacking in inference-time settings;

2. We propose **Best-of-Poisson** that provides an efficient, near-exact approximation of the optimal policy at inference;

3. We introduce **HedgeTune**, a lightweight method to find the best inference-time parameter. We show that **HedgeTune** mitigates hacking on math, reasoning, and human-preference setups.

Why does reward hacking happen?



Consider an inference-time method with parameter θ .

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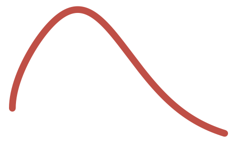

Consider an inference-time method with parameter θ .

An inference-time method is ***greedy*** if as we increase θ, it becomes more likely to choose a response with higher proxy reward.

Example: Best-of-n!

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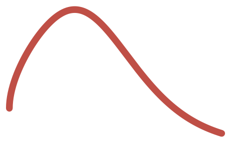

Theorem (Informal): If the *inference-time method* is *greedy*, then the expected true reward attains *at most one extremum* *w.r.t.* θ .

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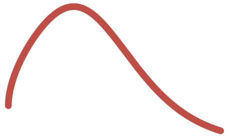

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Consequence: We explain hacking in Best-of-*n*.

If there is at most one maxima, search for it! This is the idea behind **HedgeTune**.

Best-of-Poisson

All alignment methods try to solve the following problem:

$$\pi^*(x) = \arg\max_{\pi_x \in \Delta_{\mathcal{X}}} \mathbb{E}_{\pi_x} \left[r_p(X) \right] - \frac{1}{\lambda} D_{\mathsf{KL}}(\pi_x \| \pi_{\mathsf{ref}})$$

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We can estimate it through:

- RLHF: Expensive, need to rerun training for every penalty
- Best-of-n: Cheap but coarse control over the KL divergence
- Soft Best-of-n: Need to set two parameters (n, temperature)

Best-of-Poisson

For a given question:

1. Sample *n* from Poisson distribution
2. Sample *n* responses from the LLM
3. Score them using a reward model
4. Choose the one with the highest reward

We randomize 🎲 our *n.* This gives us continuous control over the KL divergence.

We show that the resulting **BoP** distribution is close to the optimal one!

HedgeTune

We propose **HedgeTune** as a one-time offline calibration of your inference-time parameter.

You can apply it to any LLM and any proxy reward with black-box access!

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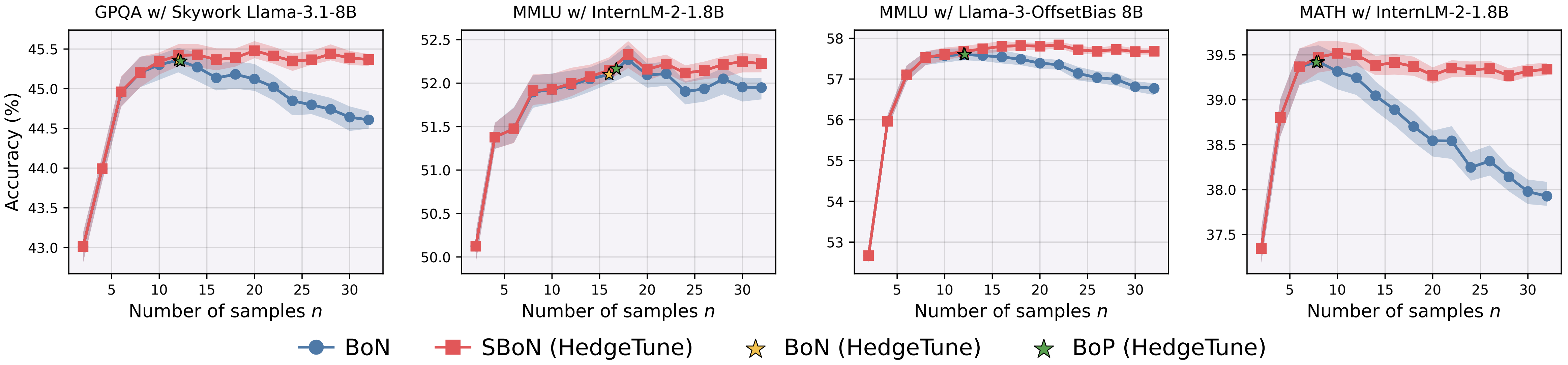
We require a small set of proxy and true reward pairs.

HedgeTune

Algorithm 4 `HedgeTune`: Parameter Optimization for Hedging

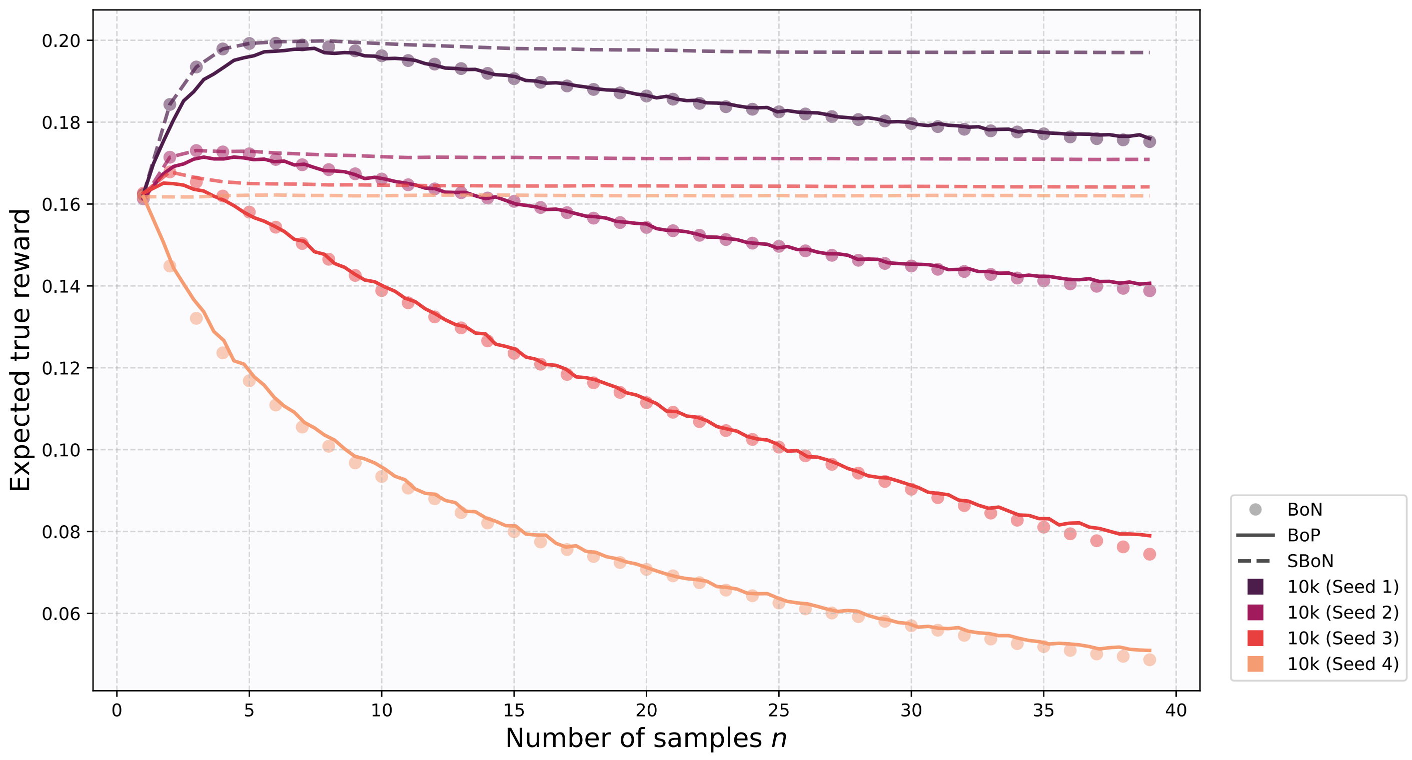
1: **Inputs:** Proxy and true rewards $\{s_{t,k}, r_{t,k}\}$ per prompt t; parameter domain Θ
2: **Output:** Optimal hedge parameter $\theta^\star$
3: STEP 1. For each prompt t, sort responses by their proxy scores and map their ranks to empirical quantiles $u_{t,k} \in (0, 1)$.
4: STEP 2. Specify the score function $\psi(u, \theta)$ and density $p_\theta(u)$ according to the inference-time method (e.g., BoN, SBoN, BoP; see Appendix D).
5: STEP 3. For a given t and $\theta \in \Theta$, define the residual $R_t(\theta) = \mathbb{E}_{u\sim p_\theta}[r_t(u)\, \psi(u, \theta)]$. This can be estimated from the empirical pairs $\{(u_{t,k}, r_t(u_{t,k}))\}$.
6: STEP 4. Find $\theta^\star \in \Theta$ such that the average residual $\bar{R}(\theta^\star) = \frac{1}{|T|}\sum_t \hat{R}_t(\theta) = 0$ via one-dimensional root-finding.

Hedging in verifiable setups



Result: Hedging mitigates reward hacking and achieves superior reward-distortion tradeoffs on standard verifiable benchmarks such as MMLU Pro and GPQA, even with large proxy rewards (8B)!

Hedging with human preferences



Result: Hedging mitigates reward hacking in a realistic RLHF setup

Conclusion

We offer a cheap and lightweight method to improve performance and mitigate reward hacking at inference.

We show that hedging is a promising framework to leverage proxy rewards and build safer, more reliable AI systems!