Cyber-Physical Simulation
TU Darmstadt

TECHNISCHE UNIVERSITÄT DARMSTADT

NEURAL INFORMATION PROCESSING SYSTEMS

Stable Port-Hamiltonian Neural Networks

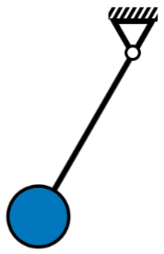
Fabian J. Roth, Dominik K. Klein, Maximilian Kannapinn, Jan Peters and Oliver Weeger

NEURAL INFORMATION PROCESSING SYSTEMS

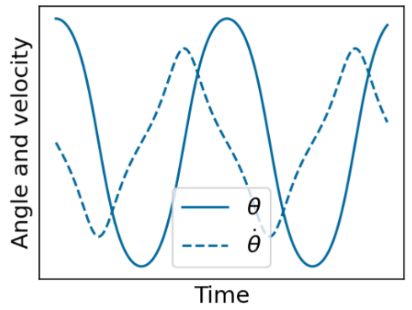
SYSTEM IDENTIFICATION

Problem: Obtain mathematical model of dynamic systems from data

Physical System



Measurement / Simulation Data



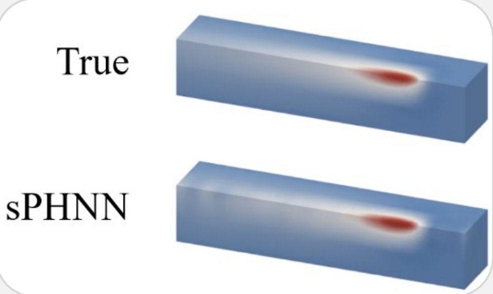
Model

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_{NN}(t, \boldsymbol{x}, \boldsymbol{u}; \boldsymbol{\theta})$$

$\boldsymbol{x}$ – state
t – time
$\boldsymbol{u}$ – inputs
$\boldsymbol{\theta}$ – parameters

Application

- Surrogate models
- Reduced order models



Our Method: Neural networks + Physics

Flexibility of neural networks + Physical biases + Stability guarantees

PORT-HAMILTONIAN SYSTEMS

NEURAL INFORMATION PROCESSING SYSTEMS

Structure matrix

$J(x) = -J^T(x)$

Resistive matrix

$R(x) = R^T(x)$,
$R(x) \succcurlyeq 0$

Hamiltonian

$\mathcal{H}(x)\colon \mathbb{R}^n \mapsto \mathbb{R}$

Input matrix

$G(x)$

$$\dot{x} = (J - R)\frac{\partial \mathcal{H}}{\partial x} + Gu$$

Physical bias

If $u = 0$, Energy is…

- conserved $(R(x) = 0)$
- dissipated $(R(x) \succcurlyeq 0)$

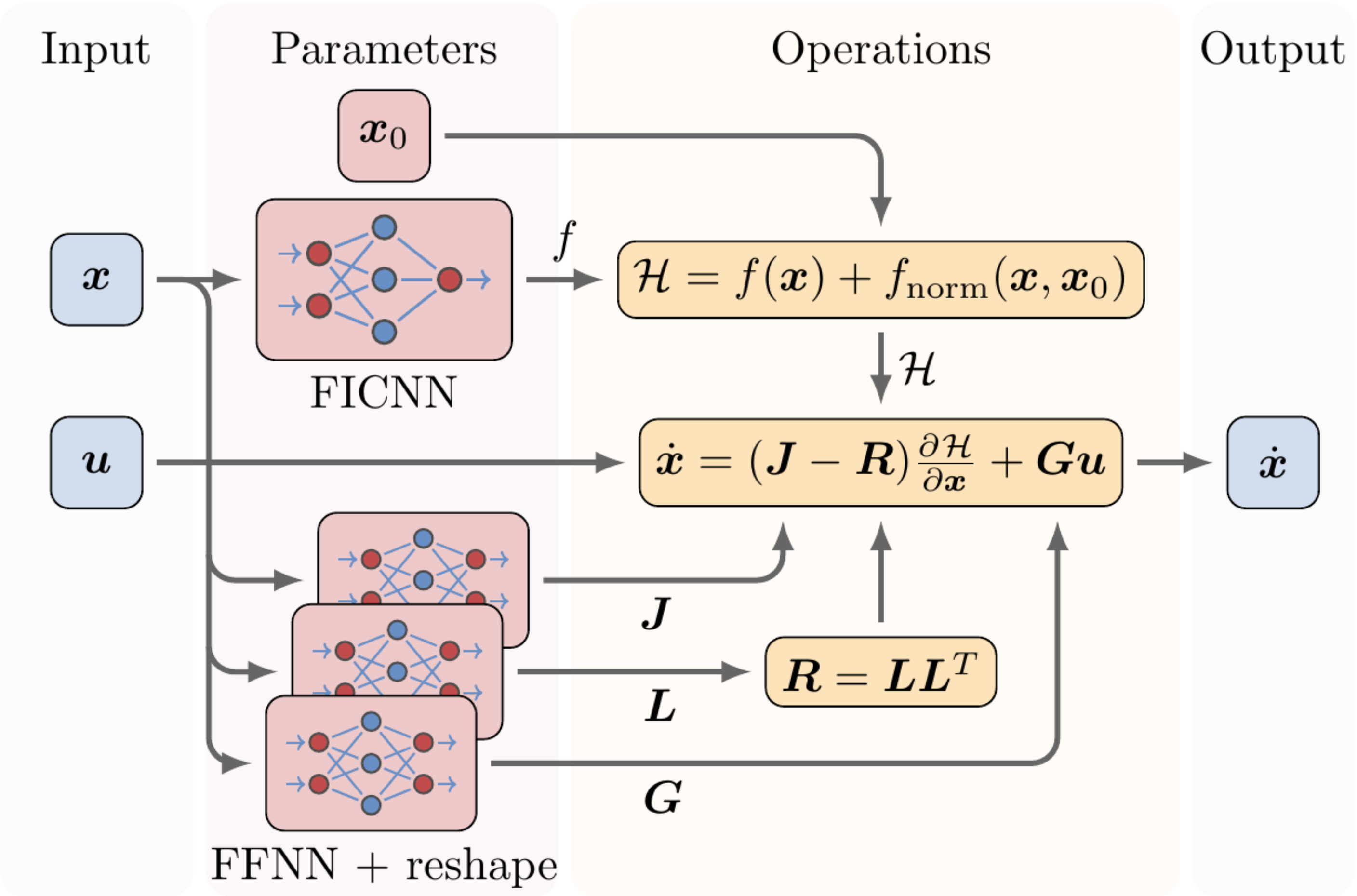
Energy balance

$$\dot{\mathcal{H}} = \underbrace{\frac{\partial \mathcal{H}}{\partial x} J \frac{\partial \mathcal{H}}{\partial x}}_{\substack{=0 \\ \textit{conservation}}} \underbrace{- \frac{\partial \mathcal{H}}{\partial x} R \frac{\partial \mathcal{H}}{\partial x}}_{\substack{\leq 0 \\ \textit{dissipation}}} \underbrace{+ \frac{\partial \mathcal{H}}{\partial x} Gu}_{\substack{s(x,u) \\ \textit{supply}}}$$

Stability implication

If $u = 0$, then $\dot{\mathcal{H}} \leq 0$
$\rightarrow$ *Minima* in $\mathcal{H}$ are (asymptotically, $R(x) \succ 0$) *stable equilibria.*

SPHNN ARCHITECTURE



Parameterization

- **Hamiltonian $\mathcal{H}$**
  - *Convex NN* [1] + normalization
  - Stable equilibrium x_0 fixed or learned
- **Matrices J, R, G**
  - FFNN, constant or known beforehand
  - Transformed to skew-symmetric, positive definite and rectangular, respectively

→ Guarantees globally stable dynamics

→ Interpretable and easy to integrate prior knowledge

[1] Amos et al., 'Input convex neural networks', ICML 2017

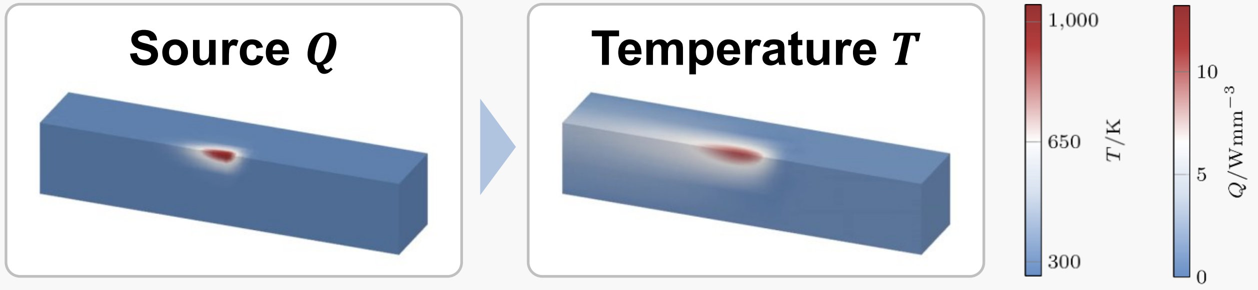
3D PRINTING SURROGATE

NEURAL INFORMATION PROCESSING SYSTEMS

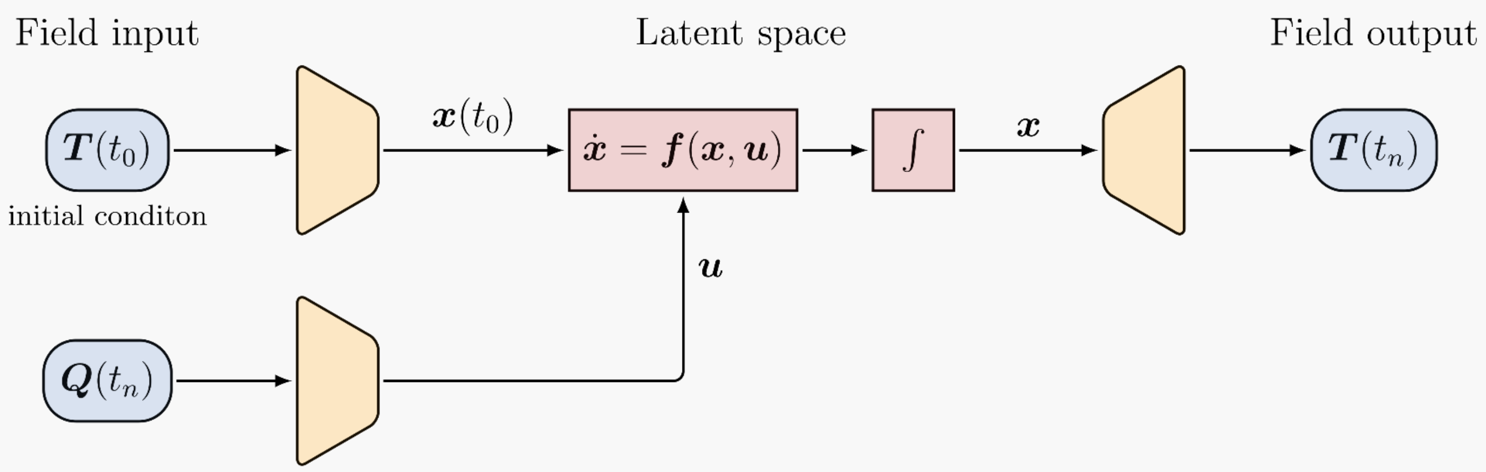
Data set

Numerically solved heat equation for moving heat source; modeling metal AM process

- 25 simulations, varied speed and power
- *Two training* and 23 evaluation parameter-pairs



Reduced order model



Latent compression:

Singular value decomposition
Latent dimensions: $x, u \in \mathbb{R}^{40}$

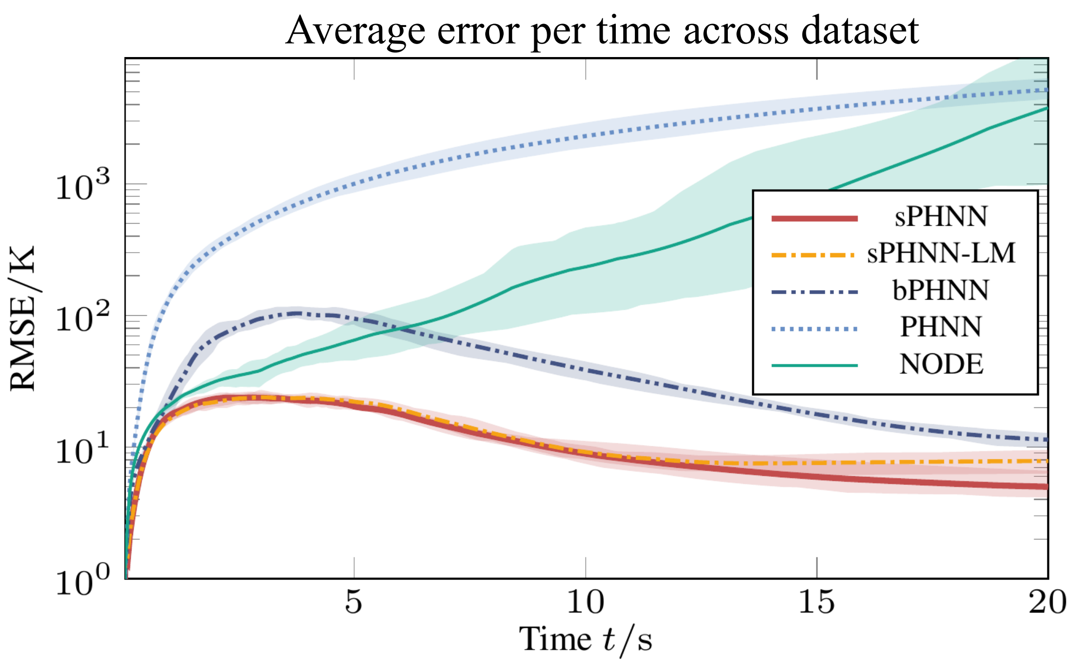
Latent dynamics model:

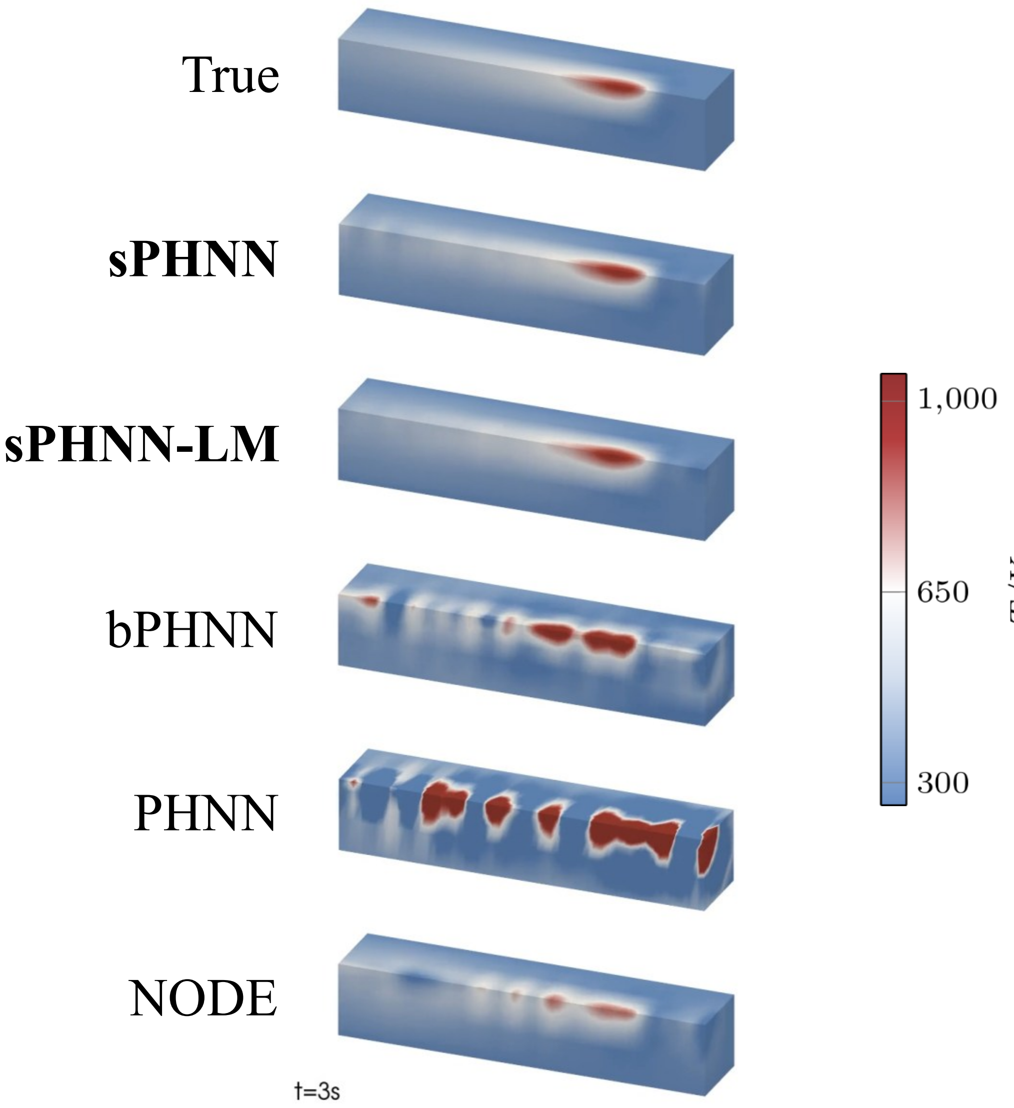
sPHNN, sPHNN-LM
bPHNN[2], PHNN, NODE[3]

[2] Adapted from Yu et al., 'OnsagerNet: Learning Stable and Interpretable Dynamics using a Generalized Onsager Principle', Physical Review Fluids 2023

[3] Chen et at., 'Neural Ordinary Differnetial Equations', Advances in Neural Information Processing Systems 2018

3D PRINTING SURROGATE





Successfully learns dynamics in **higher-dimensional state spaces** from **sparse data** both with and without prescribed equilibrium $\boldsymbol{x}_0$.

NEURAL INFORMATION PROCESSING SYSTEMS

SUMMARY

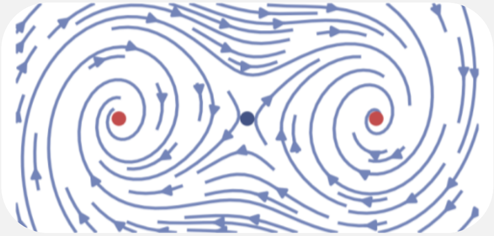
Stable port-Hamiltonian neural networks

Physics-guided system identification with explicit *stability guarantees*.

- When applicable, global stability provides a strong inductive bias
- Robust and noise-resistant
- Generalization from sparse training data

Future extensions

- Multiple equilibria
- Extend to input-state-output systems with suitable stability properties
- Incorporate uncertainties



GitHub

https://github.com/CPShub/sphnn-publication