Place Cells as Multi-Scale Position Embeddings: Random Walk Transition Kernels for Path Planning

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Key Ideas

- Hippocampus supports spatial navigation by encoding **cognitive maps** via collective place-cell activity.
- We model the place-cell population as **non-negative spatial embeddings** from **spectral decomposition** of **multi-step random-walk transition kernels**. Inner products (or Euclidean distances) between embeddings approximate **normalized transition probabilities** across scales.
- **Non-negativity + inner-product structure ⇒ emergent sparsity**, explaining **localized place fields** without explicit regularization.

Method

- **Random walk kernel:**
  - Symmetric transitions on a lattice
  - τ -step kernel $P(y|x,\tau)$ measures adjacency at scale τ
- **Heat-diffusion:**
  - Discrete random walk → heat equation with reflecting boundaries;
  - Small- τ behavior ties to **geodesic distance**:

$$p(y|x,\tau) \approx \frac{1}{4\pi\alpha\tau} \exp(\frac{d_g^2(x,y)}{4\alpha\tau})$$

- **Spectral embeddings:**
  - We earn place cell population as vector embeddings $\mathrm{h}(x,\tau) \geq 0$:

$$< h(x,\tau), h(y,\tau) > \approx q(y|x,\tau) ,$$

the inner product approximates the normalized transition probability

Method

- **Emergent sparsity from Non-negativity + Orthogonality :**
  - If two positions x and y are non-adjacent, $q(y|x,\tau) = 0$, the corresponding vectors must be orthogonal.
  - Since all components of $h(x,\tau)$ are non-negative, orthogonality implies **disjoint support**
  - This forces most components to be zero, yielding sparse, localized activity patterns; sparsity increases as τ decreases
- **Matrix squaring:**
  - $P_{2\tau} = P_\tau^2$ composes **global** from **local** transitions; supports **preplay-like shortcut detection**.

Method

- **Adaptive scale path planning:**
  - Dynamically adjust $\sqrt{\tau^*} \propto d(x, y)$ for precise navigation
  - Near obstacles, $\nabla_x q(x|y, \tau)$ flows parallel to boundaries, preventing collisions, akin to hippocampal obstacle avoidance

Experiments

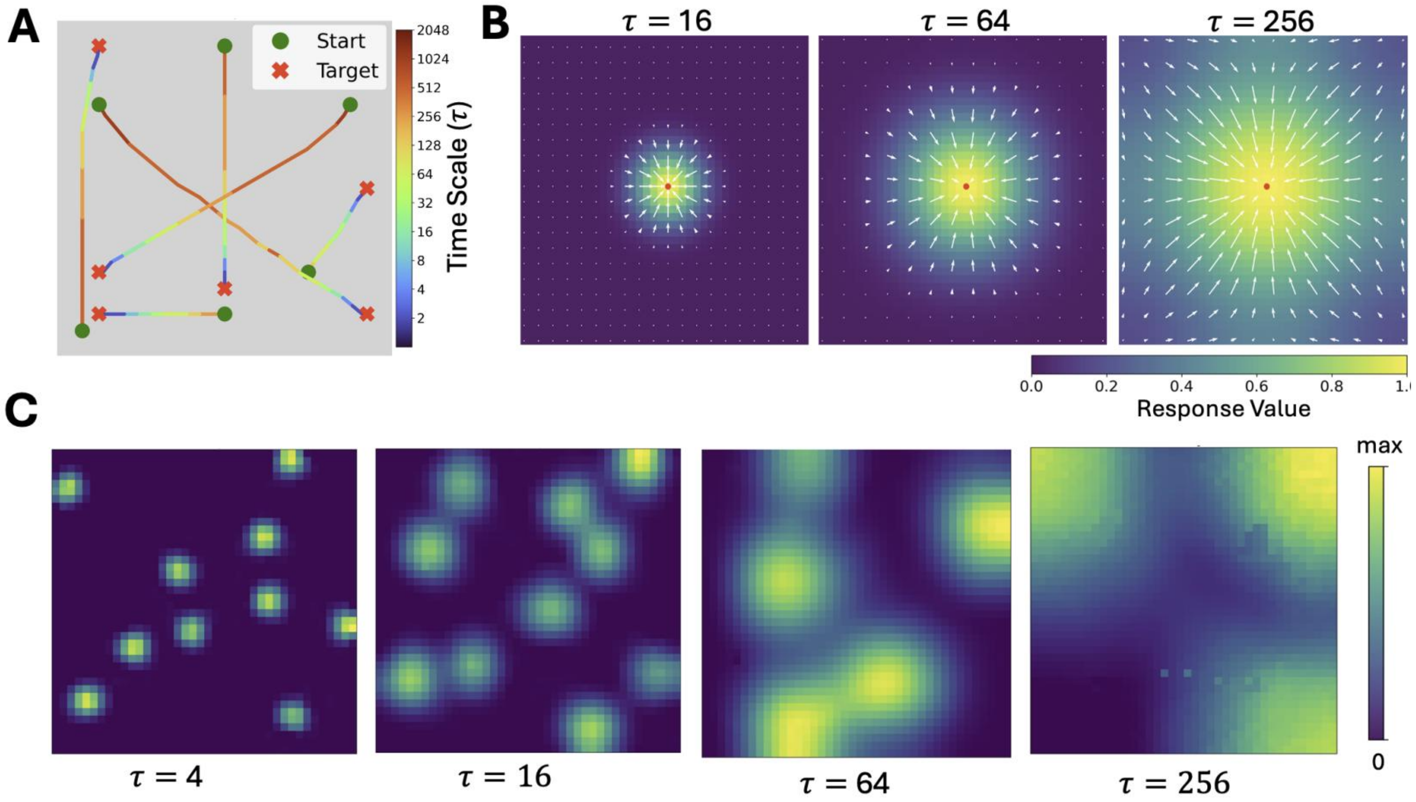

Figure 1: Place Cell Representations and Navigation in **Open Field Environment.**
(A) Goal-directed path planning trajectories with adaptive scale selection
(B) Normalized transition probability kernels $q(y|x,\tau)$ at multiple scales with gradient vector fields
(C) Learned activation patterns of $h(x,\tau)$ at different scales across randomly chosen cells

Experiments

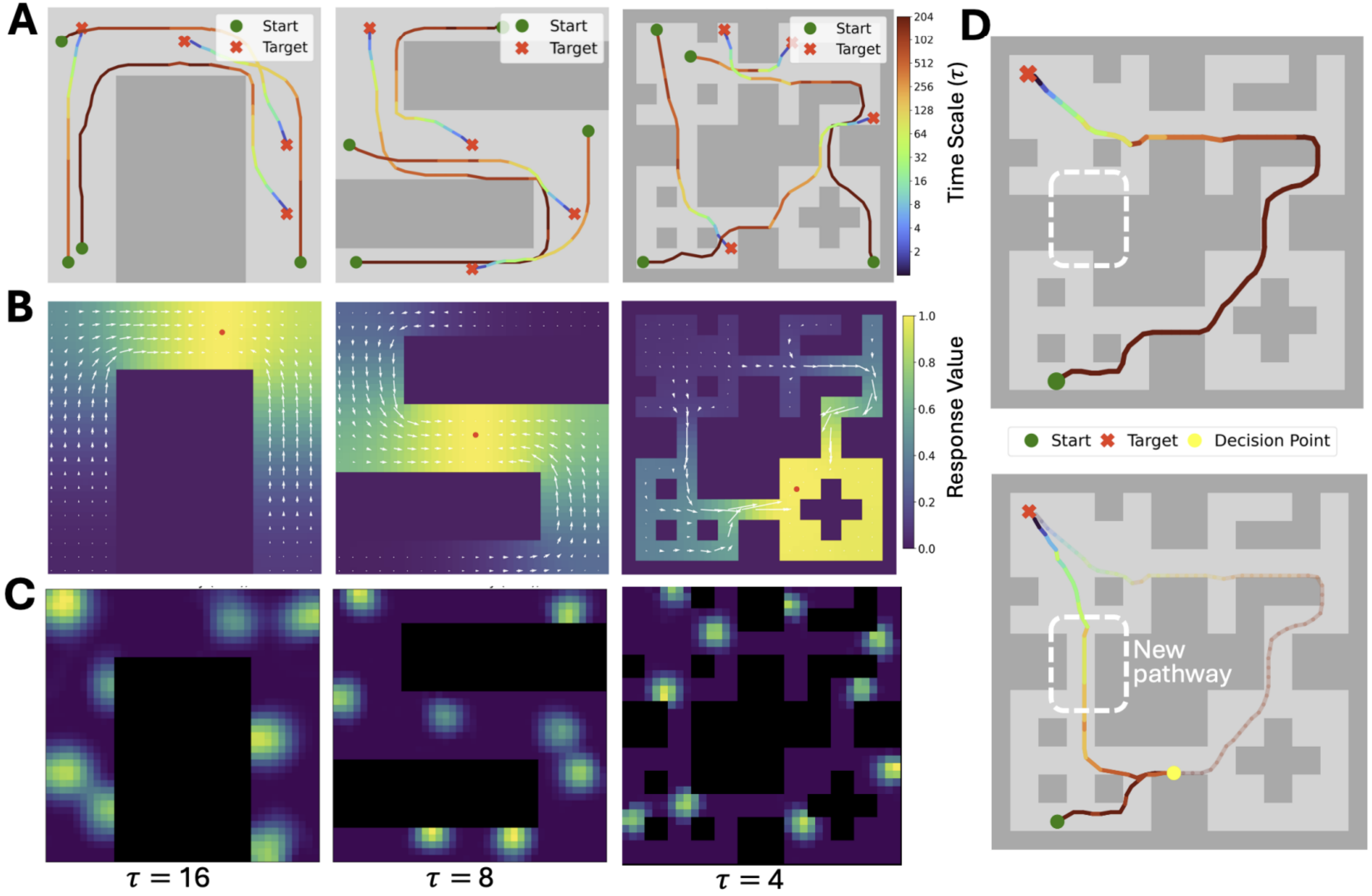

Figure 2: Place Cells in **Complex Maze Environments**
(A) Path planning through obstacle environments.
(B) Transition kernels $q(y|x, \tau)$ with gradient fields.
(C) Sampled place cell profiles at multiple spatial scales.
(D) Remapping with environmental modification.

Summary

- We reconceptualize hippocampal place cells as **population embeddings** that reconstruct **transition probabilities.**
- It yields **sparse, multi-scale cognitive maps** and **trap-free, shortcut-seeking navigation**—with a single representational geometry that also accounts for **theta-phase** structure.

Thank you!

- Paper link: https://arxiv.org/pdf/2505.14806
- Project page: https://sites.google.com/view/place-cells
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