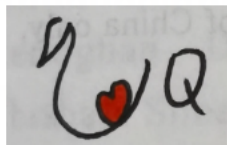




Guoqing Hu



Master student

Lab of Data Science
University of Science and Technology of China

443 Huangshan Road, Hefei, China 230027

Advisor: Prof. Xiang Wang

Prof. An Zhang

Email: HugoChinn AT mail.ustc.edu.cn

WeChat: PB15671953077

QQ: 2306106034

• [GitHub](#) • [Google Scholar](#)



Alpha Lab

I am currently in the 2nd year of my Master program at [USTC Lab for Data Science](#), supervised by [Prof. Xiang Wang](#) and [Prof. An Zhang](#). My research interests include diffusion models and generative recommendations, particularly in discrete diffusion models.



Fading to Grow:

Growing Preference Ratios via Preference Fading Discrete Diffusion for Recommendation

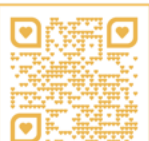
Guoqing Hu¹, An Zhang¹, Shuchang Liu², Wenyu Mao¹, Jiancan Wu¹, Xun Yang¹,
Xiang Li², Lantao Hu², Han Li², Kun Gai², Xiang Wang¹

¹ University of Science and Technology of China

² KUAISHOU



Author



Code



Paper

$$p(item|user)$$

□ User Condition:



□ Item Representation: discrete ID, continuous embedding, Semantic ID, etc

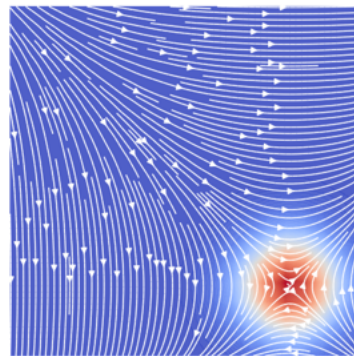
□ Data sparsity challenge: user–item interaction data is extremely sparse.

Table 3: Statistics of datasets after preprocessing.

Dataset	# users	# items	# Interactions	sparsity
Movies	6040	3883	1001456	04.27%
Steam	39795	9265	2949605	00.80%
Beauty	22,363	12,101	198,502	00.07%
Toys	19,412	11,924	138,444	00.06%
Sports	35,598	18,357	256,598	00.04%

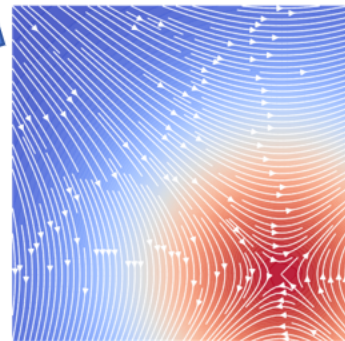
□ **Data Sparsity** leads to **Gradient Collapse**

Directly modeling sparse data distributions is inherently challenging.



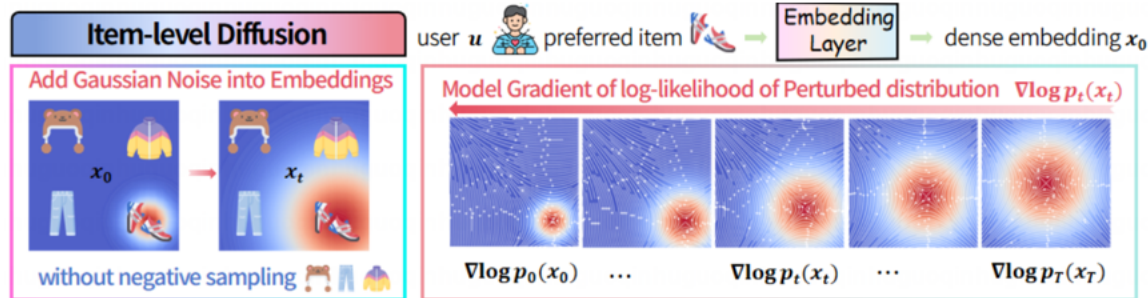
□ **Injecting Noise** smooths and densifies data

The injection of noise offers an **indirect** pathway to tackle data sparsity.

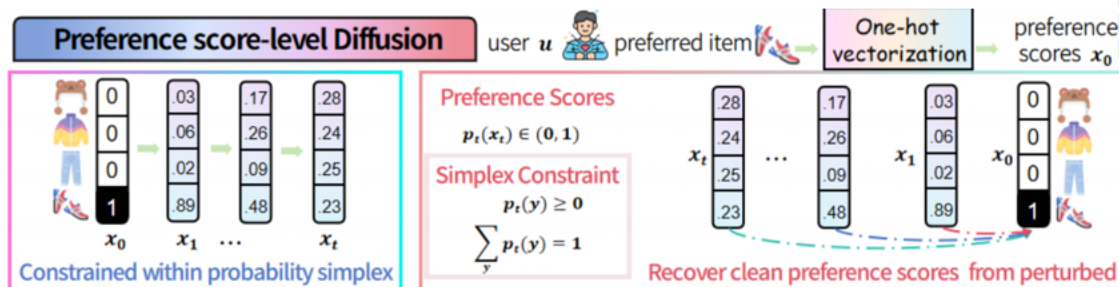


Recent advances in **generative modeling** (e.g., VAE, Flow, **Diffusion**, and Consistency models) hold promise for effectively alleviating data sparsity.

Item-level Operation on Euclidean Space: $\mathbb{R}^d$



Preference score-level Operation on Prob. Vector: $\vec{p}$



discrete and ranking nature of recommendation

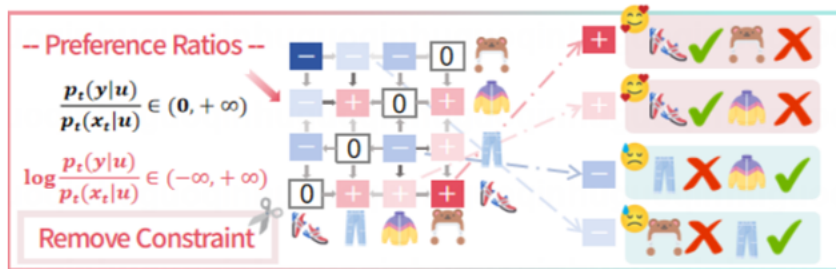
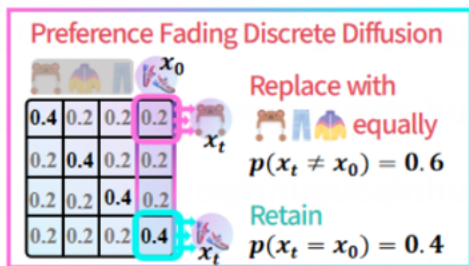
Operation on Discrete item corpus: $\mathcal{X}$

PreferGrow: Discrete Diffusion Modeling Preference Ratios

user u  preferred item x_0 

□ Preference Fading Perturbation

□ Modeling Preference ratios



- Perturb with other items to align with the **discrete nature**
- Model preference ratios to reflect the **ranking nature**

-- Preference **Fading** Forms Reference Ratios --

♡ Forward Perturbation: $\mathbf{P}_{t|0} = \alpha_t \mathbf{I} + (1 - \alpha_t) \mathbf{E}$

Retain $p(\mathbf{x}_t = \mathbf{x}_0) = \alpha_t$ or Replace $p(\mathbf{x}_t \neq \mathbf{x}_0) = 0.6$

-- Model Preference **Ratios** via Score Entropy --

♡ Preference Ratios: $\log$ concrete score $\log \frac{p_t(\mathbf{y}|\mathbf{u})}{p_t(\mathbf{x}_t|\mathbf{u})} \in (-\infty, +\infty)$

♡ Score Entropy: advanced loss for discrete diffusion

$$l_{SE}(\mathbf{x}_0, \mathbf{x}_t, \mathbf{y}) = e^{\mathbf{s}_\Theta(\mathbf{y}, \mathbf{x}_t)} - \frac{p_{t|0}(\mathbf{y}|\mathbf{x}_0)}{p_{t|0}(\mathbf{x}_t|\mathbf{x}_0)} \mathbf{s}_\Theta(\mathbf{y}, \mathbf{x}_t) + \frac{p_{t|0}(\mathbf{y}|\mathbf{x}_0)}{p_{t|0}(\mathbf{x}_t|\mathbf{x}_0)} \left[\log \frac{p_{t|0}(\mathbf{y}|\mathbf{x}_0)}{p_{t|0}(\mathbf{x}_t|\mathbf{x}_0)} - 1 \right]$$

-- Preference **Growing** from Preference Ratios --

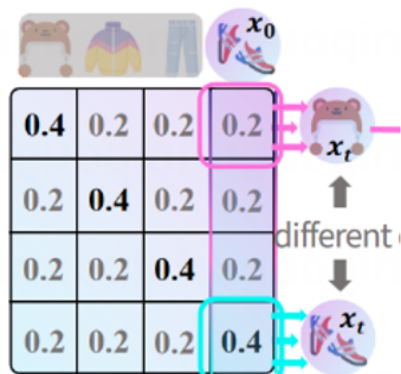
♡ Backward Generation: $\mathbf{P}_{s|t} = \mathbf{P}_{t|s}^{-1} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t} \right)^\top \right] \odot \mathbf{P}_{t|s}^\top$



PreferGrow: Step1: Preference Fading Forms Reference Ratios.



Input: target item ID x_0



Output: perturbed item ID x_t

Fading Matrix E:

$$E = \frac{\vec{p}_T \vec{1}^\top}{\vec{1}^\top \vec{p}_T}$$

$$E = \begin{pmatrix} 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 \\ 1 & \dots & 1 & 1 \end{pmatrix}$$

Pointwise

$$E = \begin{pmatrix} \frac{1}{N} & \dots & \frac{1}{N} \\ \vdots & \ddots & \vdots \\ \frac{1}{N} & \dots & \frac{1}{N} \end{pmatrix}$$

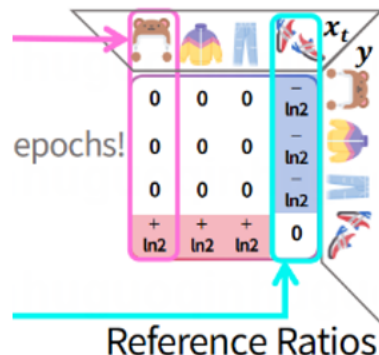
Pairwise

$$E = \begin{pmatrix} \frac{\lambda}{N} & \dots & \frac{\lambda}{N} & \frac{\lambda}{N} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\lambda}{N} & \dots & \frac{\lambda}{N} & \frac{\lambda}{N} \\ 1 - \lambda & \dots & 1 - \lambda & 1 - \lambda \end{pmatrix}$$

Hybridwise

Input: item ID x_0, x_t, y

$$r_t = \log \frac{p_{t|0}(y|x_0, u)}{p_{t|0}(x_t|x_0, u)}$$



Output: preference $y \succ x_t|x_0$

- ♡ Simplex Constrains $\Rightarrow$ Non-negativity & Normalization
- ♡ Invertibility & Markov Property $\Rightarrow$ Idempotence

Rank-1 Solution: $\mathbf{E} = \frac{\vec{p}_T \vec{1}^\top}{\vec{1}^\top \vec{p}_T}$.

- ♡ **Example 1: Pointwise Masked Diffusion**
- ♡ **Example 2: Pairwise Uniform Diffusion**

$$\vec{p}_T = \vec{e}_{-1} \in \mathbb{R}^{N+1} \quad \mathbf{E} = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 1 & \cdots & 1 & 1 \end{pmatrix}$$

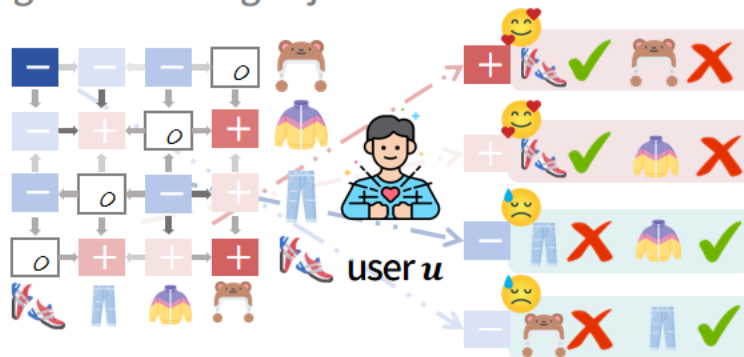
$$\vec{p}_T = \vec{1} \in \mathbb{R}^N \quad \mathbf{E} = \begin{pmatrix} \frac{1}{N} & \cdots & \frac{1}{N} \\ \vdots & \ddots & \vdots \\ \frac{1}{N} & \cdots & \frac{1}{N} \end{pmatrix}$$

Rank-r Solution: $\mathbf{E} = \sum_{i=1}^r \frac{\vec{p}^i \vec{1}_{\text{supp}(\vec{p}^i)}^\top}{\vec{1}_{\text{supp}(\vec{p}^i)}^\top \vec{p}^i}$

- ♡ Partition N items into r clusters
- ♡ Independent rank-1 solution for each cluster

♡ Preference Ratios: $\log \frac{p_t(y|u)}{p_t(x_t|u)} \in (-\infty, +\infty)$

♡ Illustration: the finest-grained ranking objective



♡ Bradley-Terry model: Direct Preference Optimization

$$p(i_p \succ i_d | u) = \frac{p(i_p | u)}{p(i_p | u) + p(i_d | u)} = \frac{1}{1 + \exp\left(-\log \frac{p(i_p | u)}{p(i_d | u)}\right)} = \sigma\left(\log \frac{p(i_p | u)}{p(i_d | u)}\right).$$

Input: reference ratios r_t

const

$$\text{Loss: } \mathcal{L}_{SE}(x_0, x_t, y) = e^{s_{\Theta}(x_t, t, u)_y} - s_{\Theta}(x_t, t, u)_y \cdot e^{r_t(x_0, x_t, y)} + e^{r_t(x_0, x_t, y)} (r_t(x_0, x_t, y) - 1).$$

compute $s_{\Theta}(x_t, t, u)_y = y^{\top} \text{MLP}(\text{concat}(x_t, t, u)),$

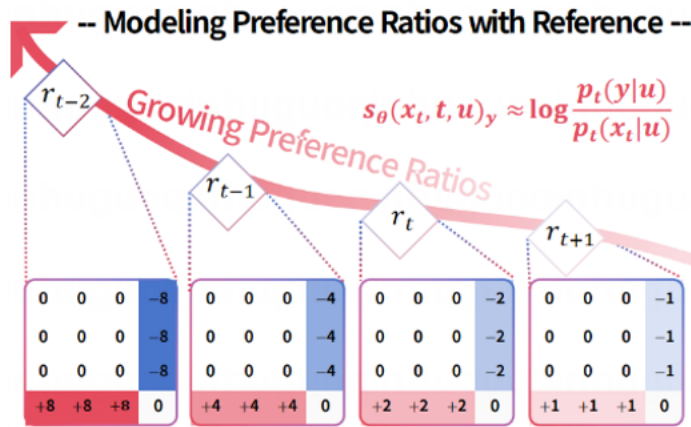
Connection to Ranking Loss:

$$\pi_{y \succ x_t | x_0} = p(y \succ x_t | x_0) = \sigma(r_t(x_0, x_t, y))$$

$$\mathcal{L}_{sBCE}(x_0, x_t, y) = -\pi_{y \succ x_t | x_0} \log \sigma(s_{\Theta}(x_t, t, u)_y) - (1 - \pi_{y \succ x_t | x_0}) \log(1 - \sigma(s_{\Theta}(x_t, t, u)_y)).$$

$$\nabla_{s_{\Theta}} \mathcal{L}_{SE} = (1 + e^{s_{\Theta}(x_t, t, u)_y})(1 + e^{r_t(x_0, x_t, y)}) \nabla_{s_{\Theta}} \mathcal{L}_{sBCE}.$$

Output: score entropy loss $\mathcal{L}_{SE}$



♡ Backward Generation:

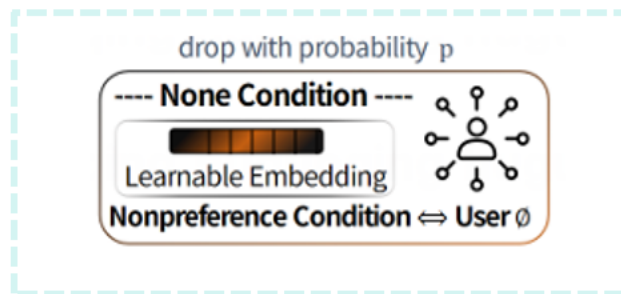
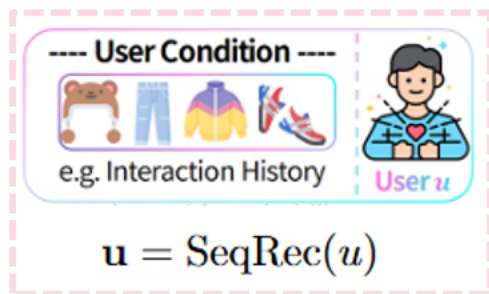
$$\mathbf{E} = \frac{\vec{p}_T \vec{1}^\top}{\vec{1}^\top \vec{p}_T}.$$


Output: preference score $p(x_0|u), x \in \mathcal{X}$

Input: user information u

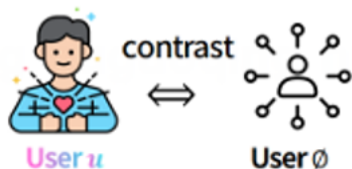
-- Non-preference User Modeling --

Output:



pos user condition u
neg user condition ϕ

Personalized Contrast
-- against Non-preference User --



$$w \cdot (s_{\theta}(x_t, t, u) - s_{\theta}(x_t, t, \emptyset))$$

Personalization Enhancement

Table 1: Performance comparison across different datasets and baselines.

Dataset		MoviesLens		Steam		Beauty		Toys		Sports	
Method		HR	NDCG	HR	NDCG	HR	NDCG	HR	NDCG	HR	NDCG
SASRec	@5	0.0905	0.0502	<u>0.0321</u>	<u>0.0192</u>	0.0326	0.0221	0.0340	0.0258	0.0154	0.0101
	@10	0.1709	0.0760	0.0572	<u>0.0272</u>	<u>0.0438</u>	0.0257	0.0438	0.0289	<u>0.0230</u>	0.0126
	@20	<u>0.2899</u>	<u>0.1059</u>	0.0957	0.0368	0.0595	0.0297	0.0499	0.0305	<u>0.0298</u>	0.0143
Caser	@5	<u>0.0925</u>	<u>0.0576</u>	0.0258	0.0163	0.0170	0.0106	0.0093	0.0072	0.0051	0.0030
	@10	0.1605	<u>0.0794</u>	0.0446	0.0223	0.0219	0.0122	0.0129	0.0083	0.0093	0.0043
	@20	0.2592	0.1042	<u>0.0736</u>	0.0295	<u>0.0295</u>	0.0141	<u>0.0185</u>	0.0098	0.0149	0.0057
GRURec	@5	0.0892	0.0551	0.0255	0.0156	0.0130	0.0082	0.0124	0.0078	0.0070	0.0048
	@10	0.1534	0.0757	0.0441	0.0216	0.0188	0.0101	0.0180	0.0092	0.0107	0.0059
	@20	0.2501	0.1000	0.0769	0.0298	0.0313	0.0132	0.0304	0.0117	0.0171	0.0076
DreamRec	@5	0.0676	0.0437	0.0109	0.0073	0.0300	0.0247	0.0381	0.0304	0.0095	0.0084
	@10	0.1083	0.0568	0.0157	0.0088	0.0353	0.0265	0.0412	0.0314	0.0112	0.0089
	@20	0.1610	0.0701	0.0218	0.0104	<u>0.0402</u>	<u>0.0277</u>	<u>0.0463</u>	<u>0.0327</u>	0.0149	0.0098
PreferDiff	@5	0.0538	0.0349	0.0167	0.0105	<u>0.0335</u>	<u>0.0272</u>	<u>0.0386</u>	0.0308	<u>0.0168</u>	<u>0.0130</u>
	@10	0.0852	0.0450	0.0297	0.0145	<u>0.0434</u>	<u>0.0304</u>	<u>0.0494</u>	<u>0.0343</u>	0.0211	<u>0.0144</u>
	@20	0.1270	0.0555	0.0526	0.0203	0.0577	<u>0.0340</u>	<u>0.0644</u>	<u>0.0380</u>	0.0256	<u>0.0155</u>
DiffRec	@5	0.0266	0.0121	0.0268	0.0143	0.0095	0.0055	0.0053	0.0028	0.0063	0.0035
	@10	0.0889	0.0320	<u>0.0657</u>	0.0268	0.0218	0.0094	0.0153	0.0059	0.0117	0.0052
	@20	0.1768	0.0541	<u>0.1159</u>	<u>0.0394</u>	0.0355	0.0128	0.0226	0.0078	0.0194	0.0072
DDSR	@5	0.0871	0.0533	0.0252	0.0157	0.0291	0.0217	<u>0.0386</u>	0.0302	0.0146	0.0109
	@10	0.1523	0.0742	0.0437	0.0216	0.0434	0.0262	0.0479	0.0332	0.0213	0.0130
	@20	0.2450	0.0975	0.0736	0.0291	<u>0.0608</u>	0.0306	0.0618	0.0367	<u>0.0298</u>	0.0151
PreferGrow	Impr.	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hybrid	@5	0.1409	0.0911	0.0615	0.0406	0.0420	0.0322	0.0419	0.0311	0.0207	0.0142
	@10	0.2229	0.1174	0.0935	0.0508	0.0532	0.0358	0.0480	0.0331	0.0267	0.0162
	@20	0.3355	0.1458	0.1413	0.0628	0.0708	0.0402	0.0625	0.0367	0.0343	0.0181
Adaptive	@5	0.1413	0.0912	0.0583	0.0375	0.0396	0.0310	0.0413	0.0304	0.0168	0.0127
	@10	0.2240	0.1177	0.0914	0.0481	0.0508	0.0347	0.0513	0.0337	0.0207	0.0140
	@20	0.3362	0.1460	0.1387	0.0600	0.0610	0.0373	0.0642	0.0370	0.0267	0.0155

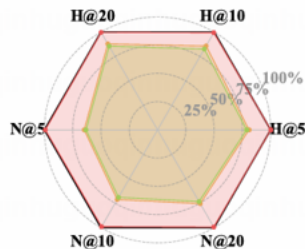
PreferGrow

(discrete diffusion-based recommender)

works well and outperform

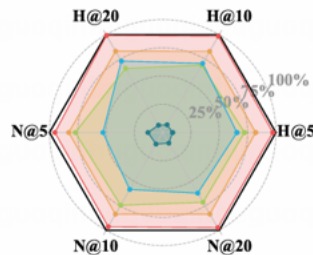
other diffusion-based recommenders.

— PreferGrow-Adaptive — w/o Nonpreference
 — w/o PointWise — w/o Nonpreference + PointWise



(a) Abalation of Adaptive on Steam.

— PreferGrow-Hybrid — w/o PairWise
 — w/o PointWise — w/o Nonpreference + PointWise
 — w/o Nonpreference — w/o Nonpreference + PairWise



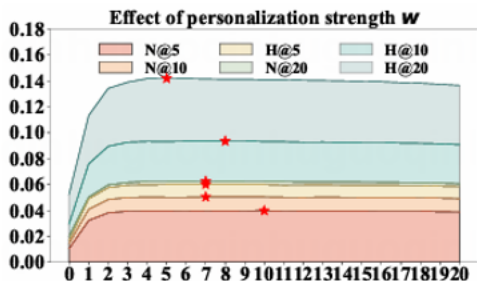
(b) Ablation of Hybrid on Steam.

Pairwise ranking

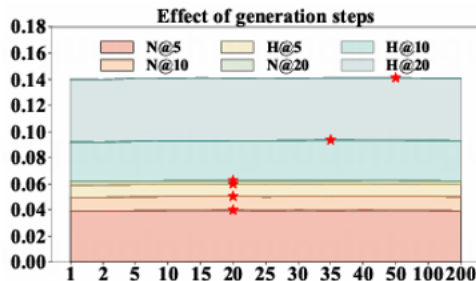
(via Uniform Diffusion)

combined with **personalized guidance**
(via Classifier-Free Guidance)

is crucial for PreferGrow.



(e) w of Hybrid on Steam.



(f) Steps of Hybrid on Steam.

♡ Personalization strength

♡ Multi-step v.s. Single-step

♡ **Theoretical Foundation:** a well-defined matrix-based formulation, flexible and interpretable closed-form solution of this preference fading matrix and theoretically prove that its idempotent property is critical for ensuring both the Markov property and reversibility of the diffusion process.

♡ **Empirical validation:** extensive experiments on five benchmark datasets demonstrate that PreferGrow consistently outperforms existing diffusion-based recommenders, validating the practical effectiveness of its theoretical foundations.