

# Practical do-Shapley Explanations with Estimand-Agnostic Causal Inference

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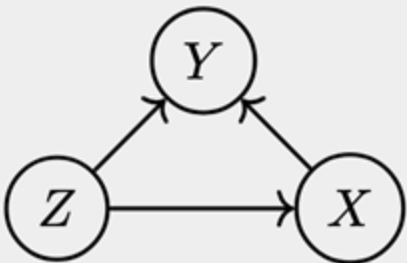


# Introduction

- **Goal**: make causal explanations practical.
- **do-SHAP** (Jung et al. 2022): Shapley Values with *causal interventional* effects.

$$\phi_{\mathbf{x}}(X_k) = \sum_{\mathbf{S} \subseteq \mathbf{X} \setminus \{X_k\}} w(|\mathbf{S}|) \cdot (\nu_{\mathbf{x}}(\mathbf{S} \cup \{X_k\}) - \nu_{\mathbf{x}}(\mathbf{S})), \quad \nu_{\mathbf{x}}(\mathbf{S}) = \mathbb{E}[Y \mid do(\mathbf{S} = \mathbf{x}_{\mathbf{S}})]$$

- Two main **contributions**:
  - 1) A method to estimate  $\nu_{\mathbf{x}}(\mathbf{S})$  in an automatable, practical way.
  - 2) A method to reduce the number of coalitions that need to be evaluated.

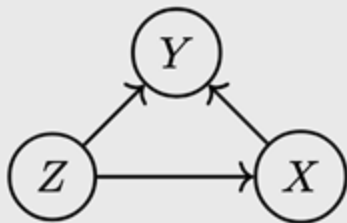


## 1) Why is do-SHAP impractical?

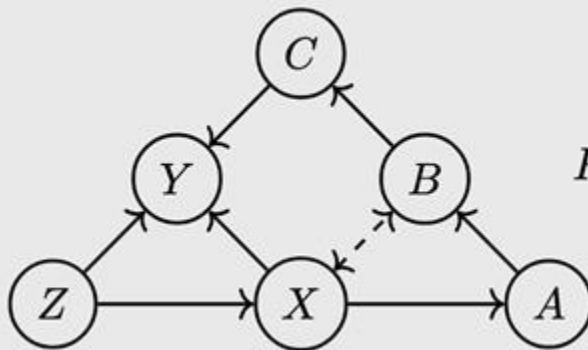
- By default, do-SHAP employs the **Estimand-Based approach**.
- Given a query (e.g., the effect of X on Y), compute an **estimand** of the query.

$$P_x(Y) = \mathbb{E}_Z [P(Y \mid x, Z)]$$

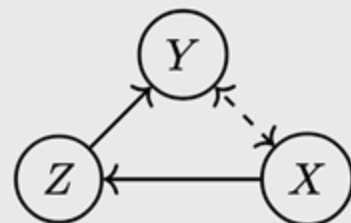
# Estimand-Based approach



$$P_x(Y) = \mathbb{E}_Z [P(Y \mid x, Z)]$$



$$P_x(Y) = \mathbb{E}_{Z,C} \left[ \frac{P(Y \mid x, Z, C)}{P(Z, C)} \cdot \mathbb{E}_{A|x} [\mathbb{E}_X [P(Z, C \mid X, A)]] \right]$$



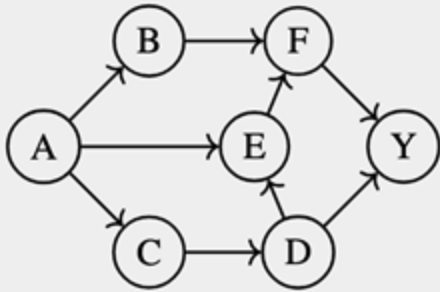
$$P_x(Y) = \mathbb{E}_{Z|x} [\mathbb{E}_X [P(Y \mid X, Z)]]$$

# Estimands are impractical for do-SHAP

- **Problem:** do-SHAP needs to evaluate multiple queries, one per coalition:

$$\nu_{\mathbf{x}}(\mathbf{S}) = \mathbb{E}[Y \mid do(\mathbf{S} = \mathbf{x}_{\mathbf{S}})]$$

- **Alternative:** Estimand-Agnostic Approach.
  - Train **a single Structural Causal Model**, following the known graph.
  - Use **general estimation procedures** to estimate any  $\nu_{\mathbf{x}}(\mathbf{S})$ .
- Guaranteed results as long as the queries are **identifiable**.



## 2) Reduce coalition evaluations

- Remove all **superfluous** nodes from any coalition.

$$\left. \begin{array}{l} \{A, B, C, E, F\}, \{A, B, C, F\}, \\ \{A, E, C, F\}, \{B, E, C, F\}, \\ \{A, C, F\}, \{B, C, F\}, \{E, C, F\} \end{array} \right\} = \{C, F\}$$

- Cache:** only compute and store values for irreducible coalitions.

# How to find the irreducible coalition?

*Theorem.* Given a topological order  $<_{\mathcal{G}}$  in  $\mathcal{G}$  and  $\mathbf{S} \subseteq \mathbf{X}$ , let  $\mathbf{Z} := \{X \in \mathbf{S} \mid \mathbf{S}_{>_{\mathcal{G}}X} \in Fr_{\mathcal{G}}(X, Y)\}$ , with  $\mathbf{S}_{>_{\mathcal{G}}X} := \{Z \in \mathbf{S} \mid Z >_{\mathcal{G}} X\}$ . Then  $\nu(\mathbf{S}) = \nu(\mathbf{S} \setminus \mathbf{Z})$ , and  $\mathbf{S} \setminus \mathbf{Z}$  is irreducible.

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**Algorithm 1** Frontier-Reducibility Algorithm (FRA) – set version

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<b>Require:</b> $\mathbf{S} \subseteq \mathbf{X}$ , coalition.	13:	<b>while</b> $\mathbf{C} \neq \emptyset$ and $Y \notin \mathbf{C}$ <b>do</b>
<b>Require:</b> $Fr$ , a map: $\text{tuple}[\text{int}] \rightarrow \text{bool}$ .	14:	$\mathbf{P}' \leftarrow \mathbf{P}' \cup \mathbf{C}$
<b>Require:</b> $\mathcal{G}$ , causal graph.	15:	$\mathbf{C} \leftarrow \bigcup_{C \in \mathbf{C}} Ch_{\mathcal{G}}(C) \setminus \mathbf{P}'$
1: <b>procedure</b> FRA( $\mathbf{S}, Fr; \mathcal{G}$ )	16:	<b>end while</b>
2: $\text{SORT}(\mathbf{S}, <_{\mathcal{G}})$	17:	$Fr[\mathbf{T}] \leftarrow (C = \emptyset)$
3: $\mathbf{P} \leftarrow \emptyset$	18:	<b>end if</b>
4: $\mathbf{Z} \leftarrow \emptyset$	19:	<b>if</b> $Fr[\mathbf{T}]$ <b>then</b>
5: $k \leftarrow  \mathbf{S} $	20:	$\mathbf{Z} \leftarrow \mathbf{Z} \cup \{X\}$
6: <b>while</b> $k > 0$ <b>do</b>	21:	<b>end if</b>
7: $X \leftarrow \mathbf{S}[k]$	22:	<b>end if</b>
8: <b>if</b> $X \notin Pa_{\mathcal{G}}(Y)$ <b>then</b>	23:	$\mathbf{P} \leftarrow \mathbf{P} \cup \{X\}$
9: $\mathbf{P}' \leftarrow \mathbf{P} \cap Deg_{\mathcal{G}}(X)$	24:	$k \leftarrow k - 1$
10: $\mathbf{T} \leftarrow (\mathbf{P}' \setminus \mathbf{Z}) \cup \{X\}$	25:	<b>end while</b>
11: <b>if</b> $\mathbf{T} \notin Fr$ <b>then</b>	26:	<b>return</b> $\mathbf{S} \setminus \mathbf{Z}$
12: $\mathbf{C} \leftarrow \{X\}$	27:	<b>end procedure</b>

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# Conclusions

- **Estimand-Agnostic Causal Inference** as a **practical approach** for do-SHAP.
- **FRA** as an efficient algorithm to significantly **speed up do-SHAP**.

# Questions?

- **Poster:** Wed 3 Dec, 11 a.m. – 2 p.m. PST
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