

# DenseAM with Epanechnikov Energy

Unlocking simultaneous **creativity** and **memorization** in  
*Dense Associative Memories (DenseAMs)*



Benjamin  
Hoover



Zhaoyang  
Shi



Krishna  
Balasubramanian



Dmitry  
Krotov



Parikshit  
Ram



We want models that can

1. **Memorize** training data (near zero training loss)
2. **Generalize** to new data

**Models that memorize training data still generalize well**

**Dense Associative Memories** (DenseAMs) are great at memorizing...

*but **novel generation** sacrifices retrievability of **stored patterns***

We want DenseAMs that can **perfectly memorize**  
and **generate novelty** *at the same time*



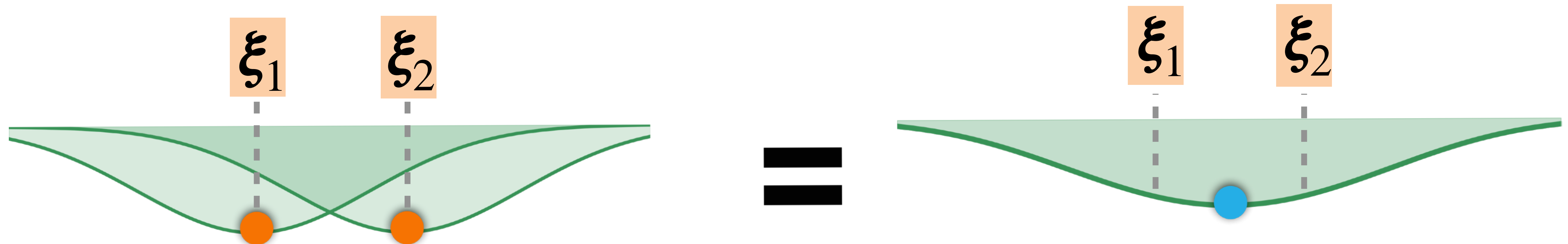
# DenseAM is unnormalized KDE

**LogSumExp** (LSE) energy sacrifices **memorization** for **generalization**

$$E^{\text{LSE}} = -\frac{1}{\beta} \log \sum_{\mu=1}^M \exp \left( -\frac{\beta}{2} \|\mathbf{x} - \xi_{\mu}\|^2 \right)$$

Gaussian Kernel

$M$  stored patterns  $\xi_{\mu}$ ,  
 $\mu = \{1 \dots M\}$



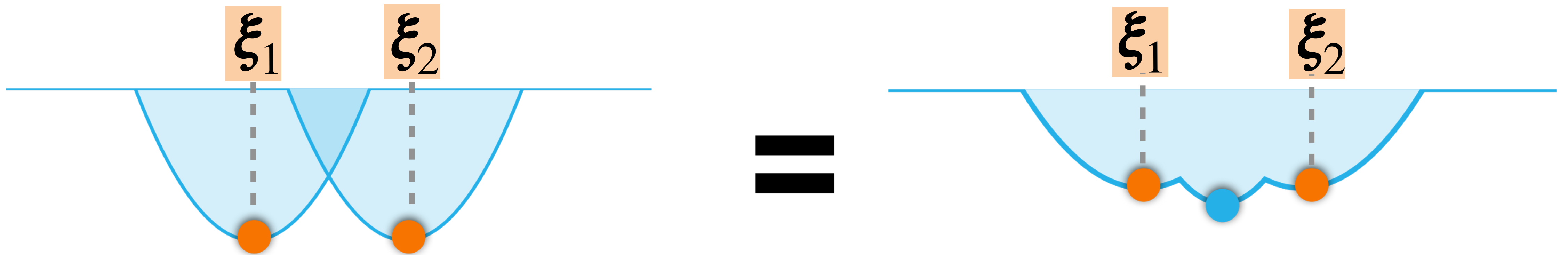
*Novel generation* sacrifices retrievability of **stored patterns**

# DenseAM is unnormalized KDE

**LogSumReLU** (LSR) energy both **memorizes** *and* **generalizes**

Epanechnikov Kernel

$$E^{\text{LSR}} = -\frac{1}{\beta} \log \sum_{\mu} \text{ReLU} \left( 1 - \frac{\beta}{2} \|\mathbf{x} - \xi_{\mu}\|^2 \right) \quad M \text{ stored patterns } \xi_{\mu}, \mu = \{1 \dots M\}$$



**Novel generation** coexists with **exact memorization**

# DenseAM is unnormalized KDE

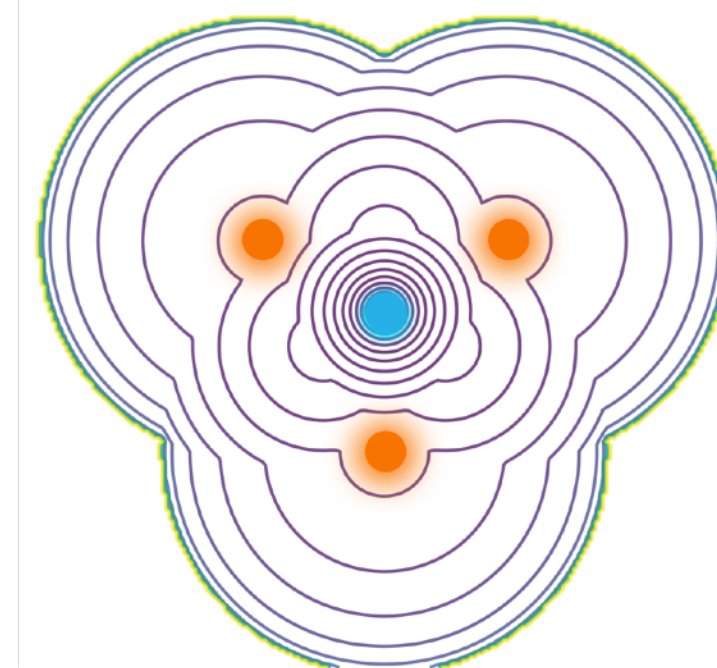
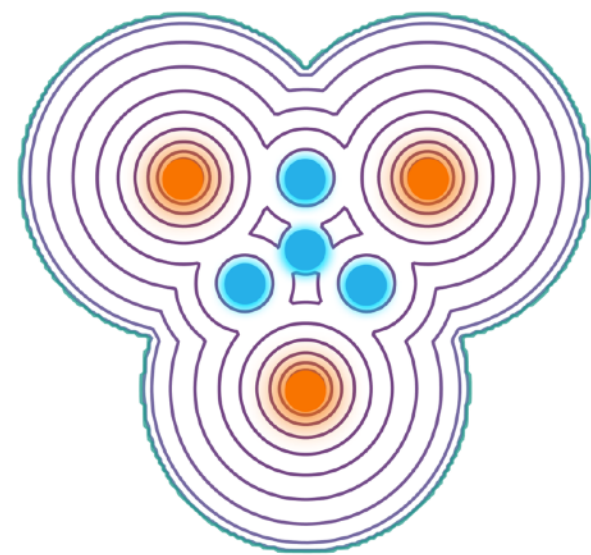
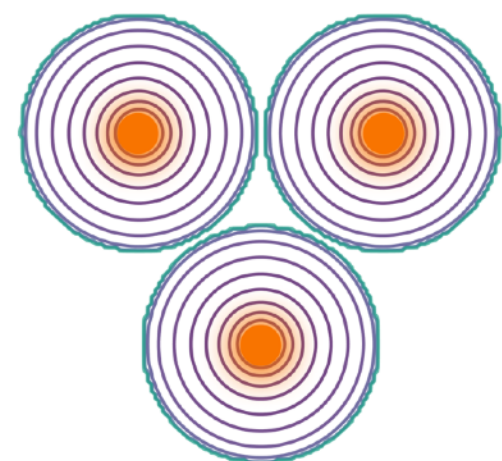
**LogSumReLU** (LSR) energy both **memorizes** *and* **generalizes**

Epanechnikov Kernel

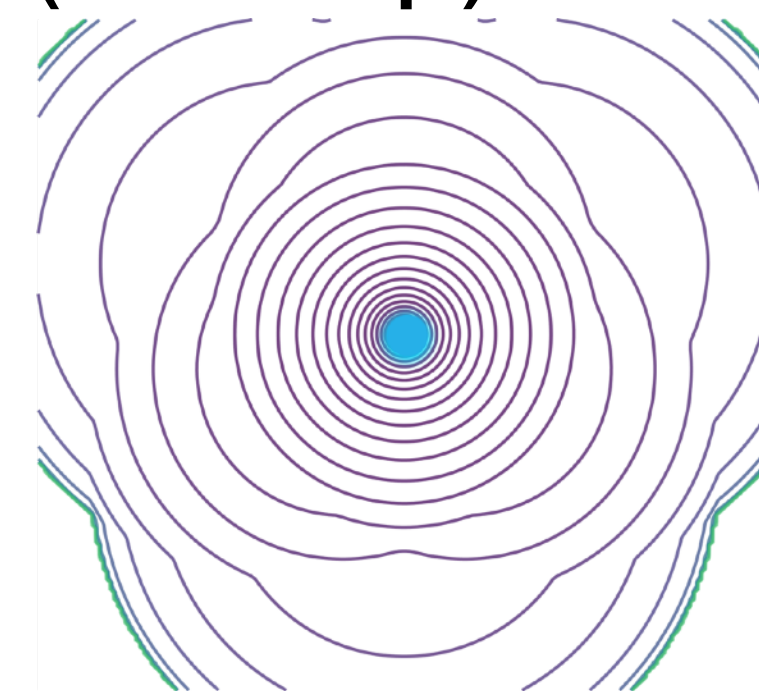
$$E^{\text{LSR}} = -\frac{1}{\beta} \log \sum_{\mu} \text{ReLU} \left( 1 - \frac{\beta}{2} \|\mathbf{x} - \xi_{\mu}\|^2 \right)$$

$M$  stored patterns  $\xi_{\mu}$ ,  
 $\mu = \{1 \dots M\}$

**Narrow basins**  
(high  $\beta$ )



**Wide basins**  
(small  $\beta$ )

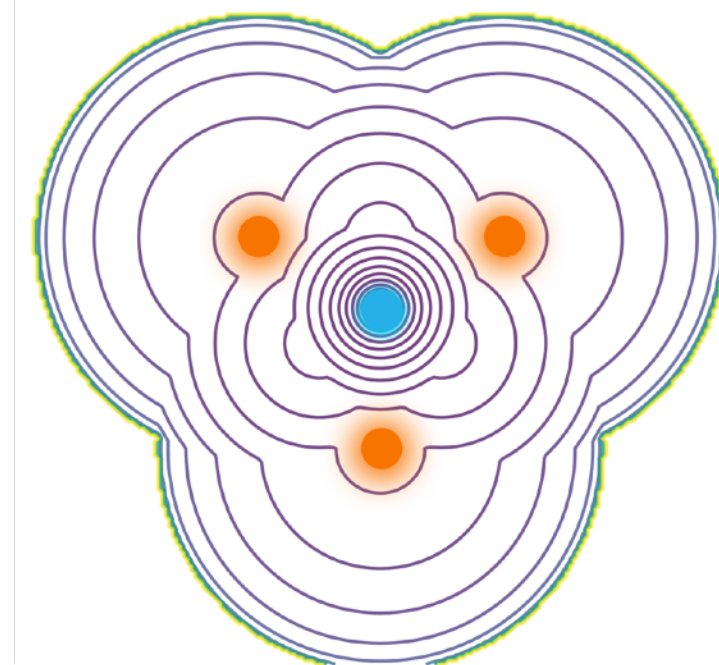
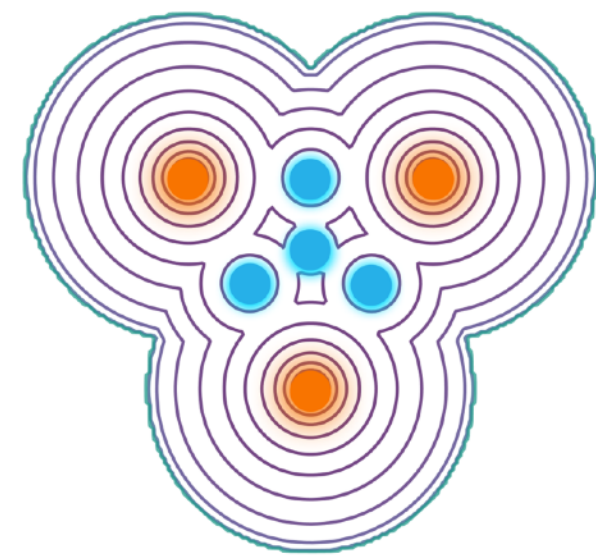
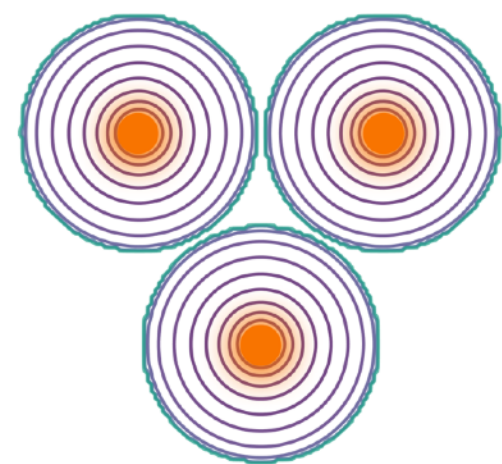


**Emergence** happens *only* at critical  $\beta$  values

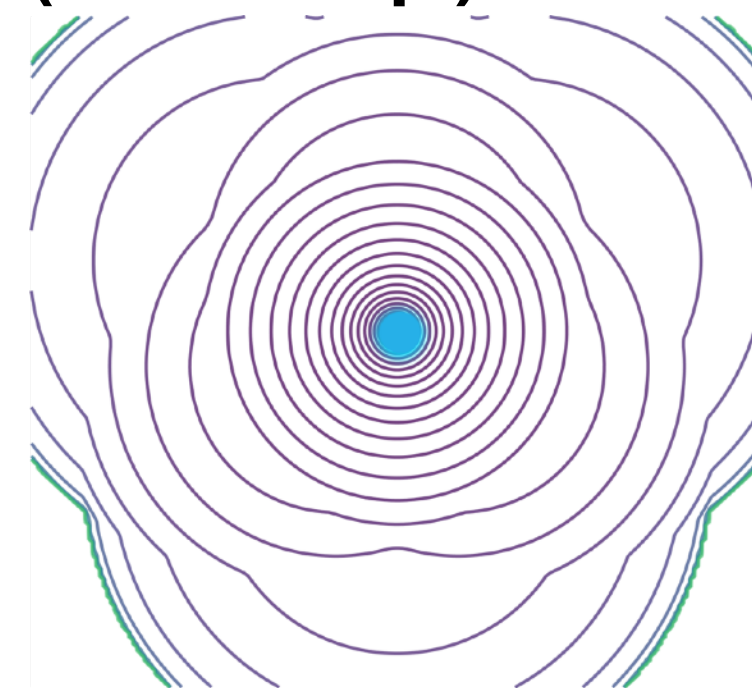


**Emergent memories** form when stored patterns interact to form a new memory, and those interacting patterns remain memories themselves

**Narrow basins**  
(high  $\beta$ )



**Wide basins**  
(small  $\beta$ )



**Emergence** happens *only* at critical  $\beta$  values

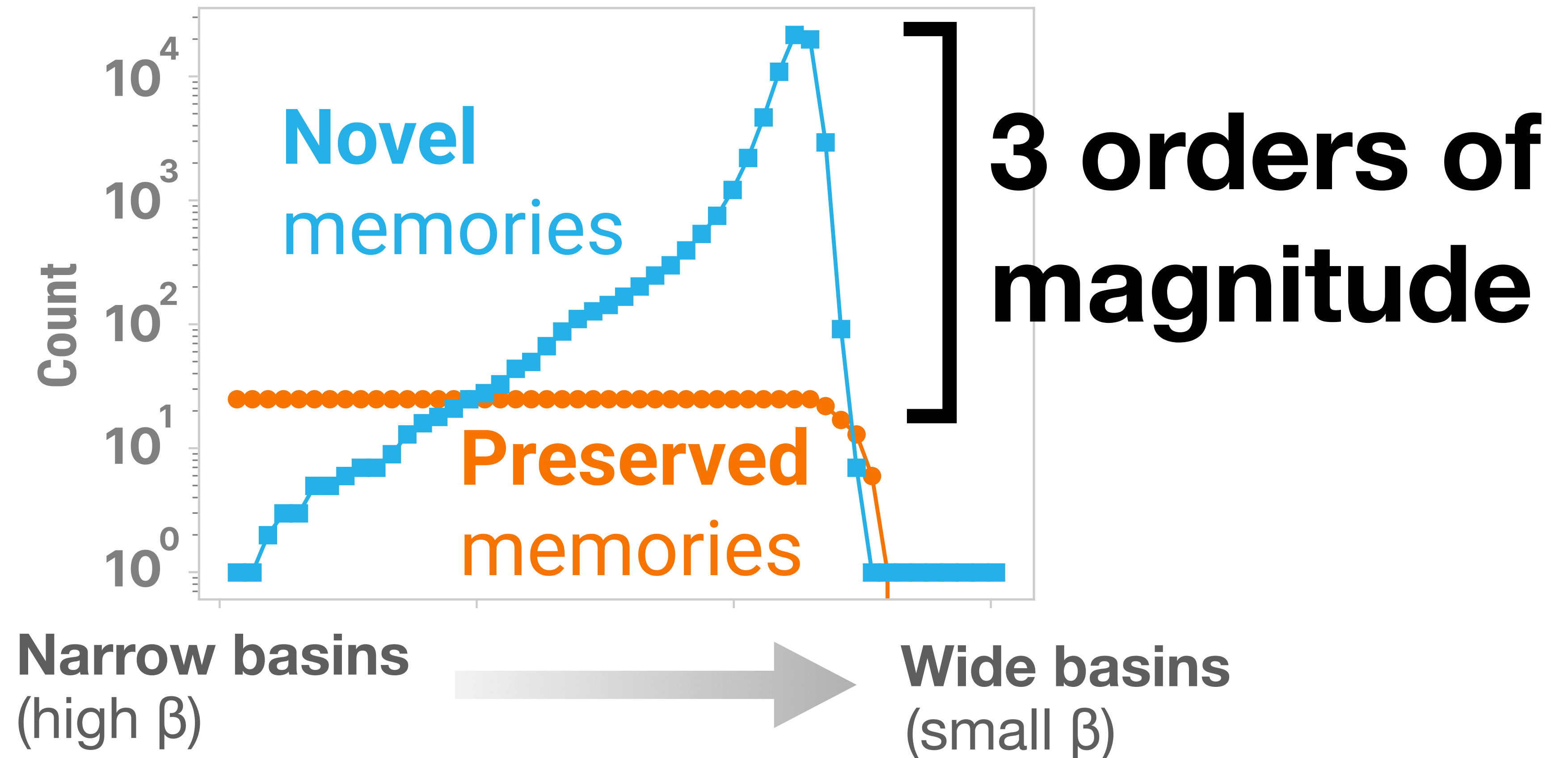
# Three research questions

1. How **abundant** are emergent memories?
2. How **meaningful** are emergent memories?
3. Can emergent memories **scale** to real data?

# How many emergent memories can LSRDAM create?

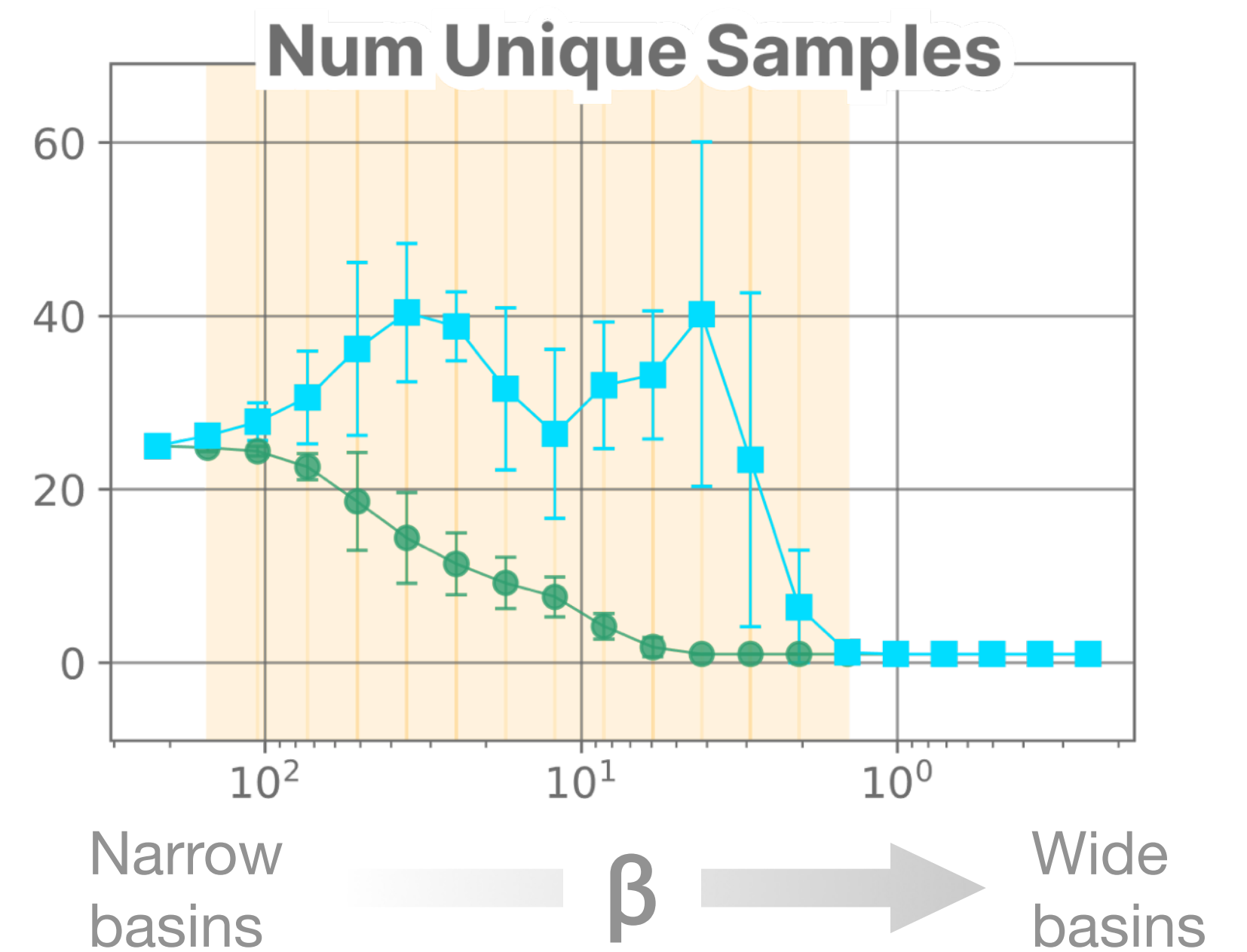
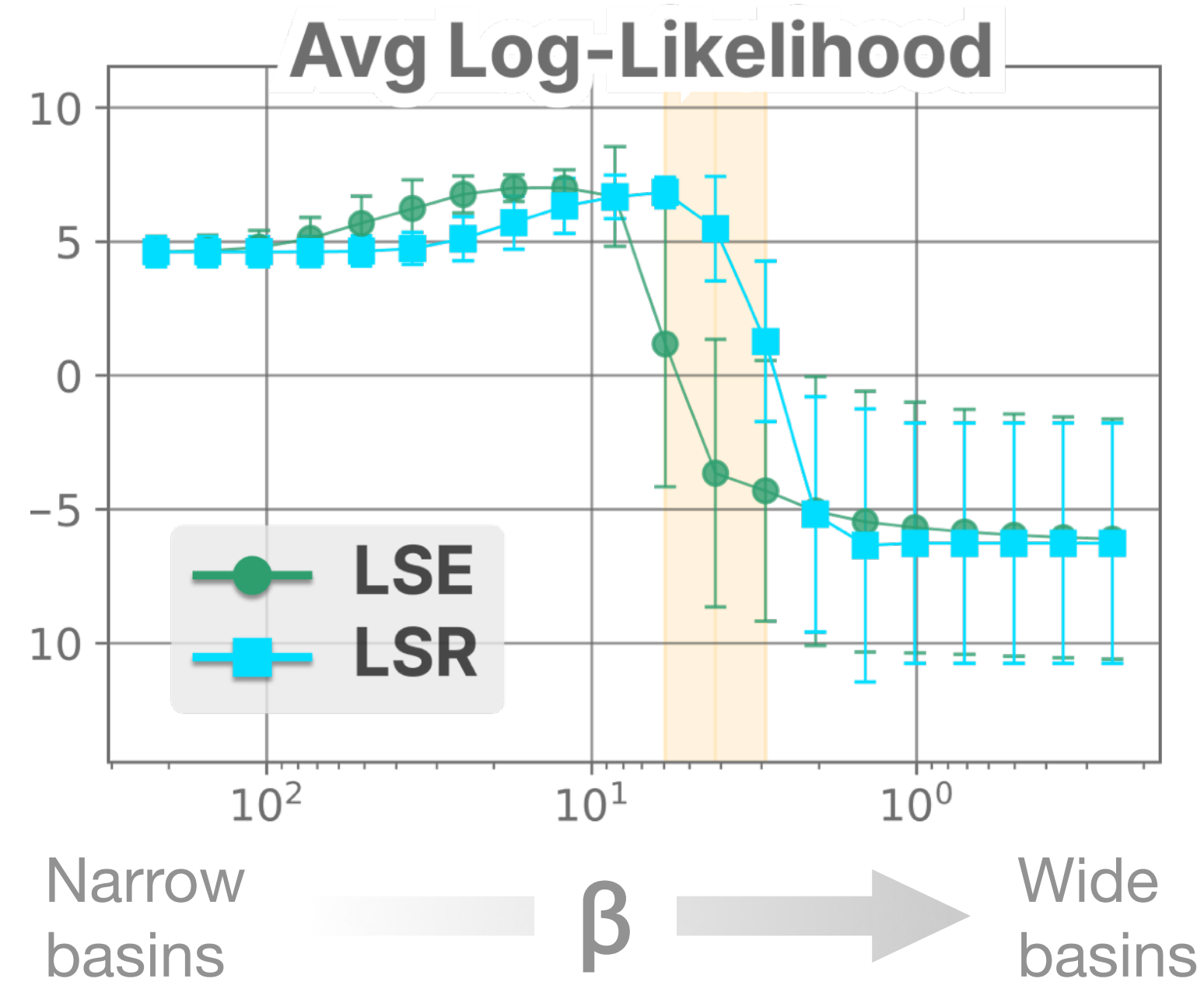
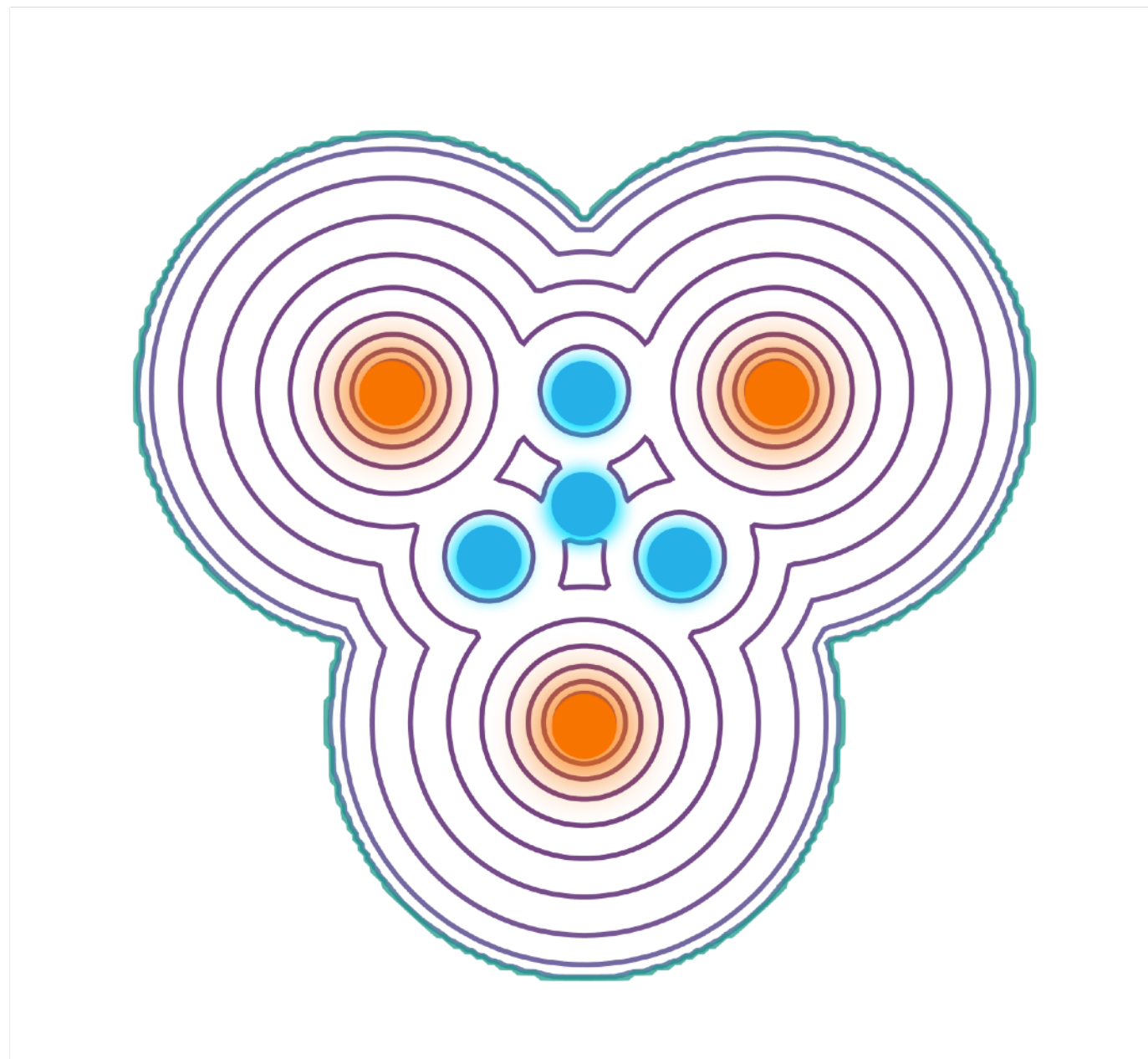
Create *orders-of-magnitude* more emergent memories than stored patterns

Storing **25** random patterns on the **32**-dim unit hypercube



# How meaningful can they be?

Emergent memories must be *centroids of subsets of patterns*



Stored patterns are 100 I.I.D. samples from a mixture of five 8-dim Gaussians, avged across 5 seeds



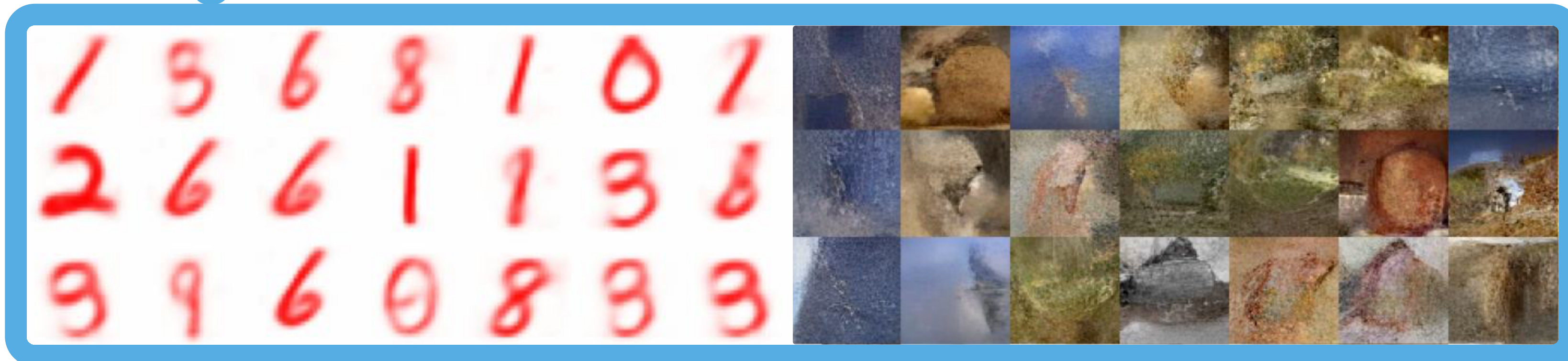
# Can emergent memories scale?

Emergent memories capture novel regions of **latent space**

## Stored patterns



## Emergent memories

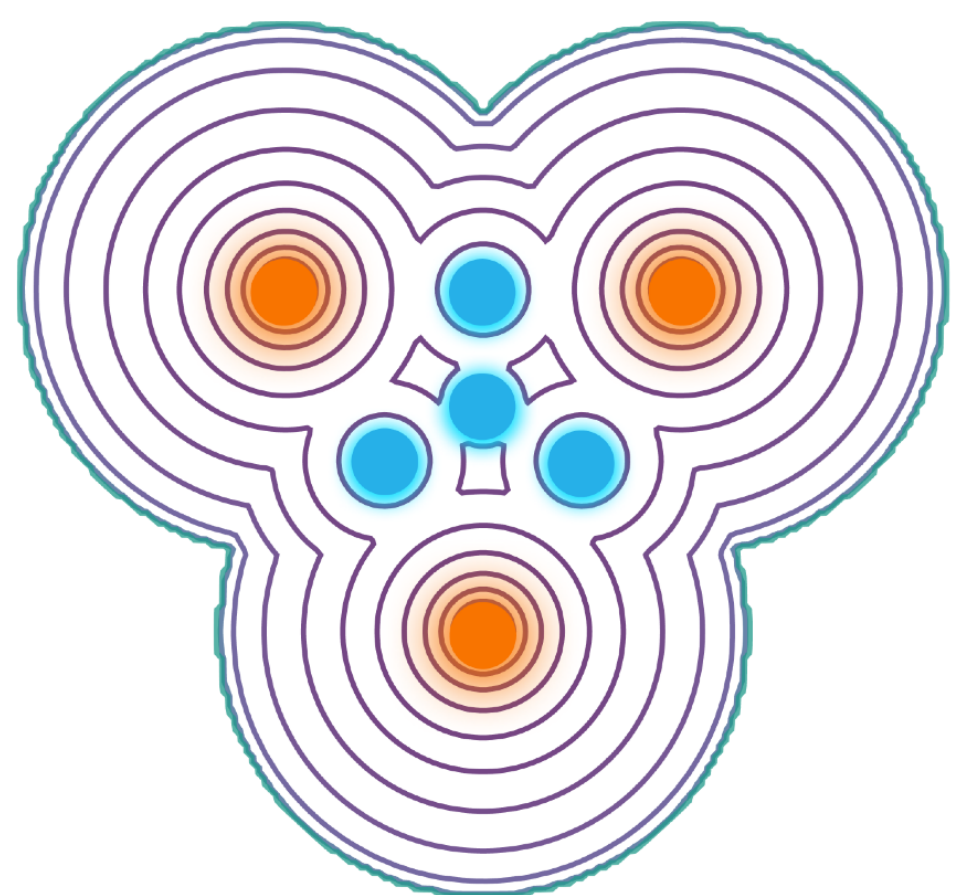
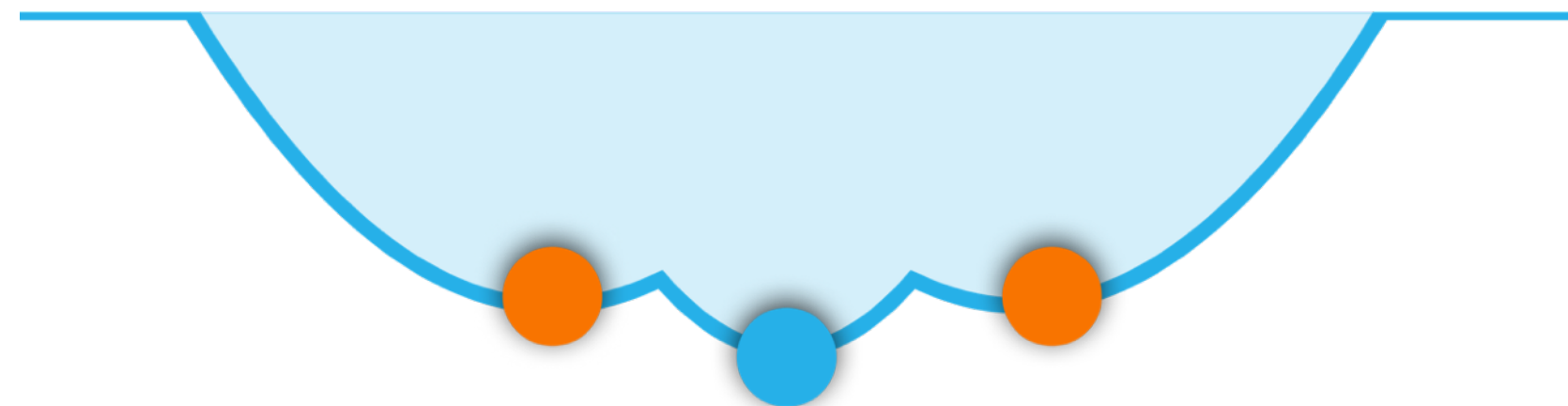




Thanks!

# DenseAM with Epanechnikov Energy

Unlocking simultaneous **creativity** and **memorization** in  
*DenseAMs* via **emergent memories**



HARVARD  
UNIVERSITY

UCDAVIS

IBM  
Research

Get in touch!

