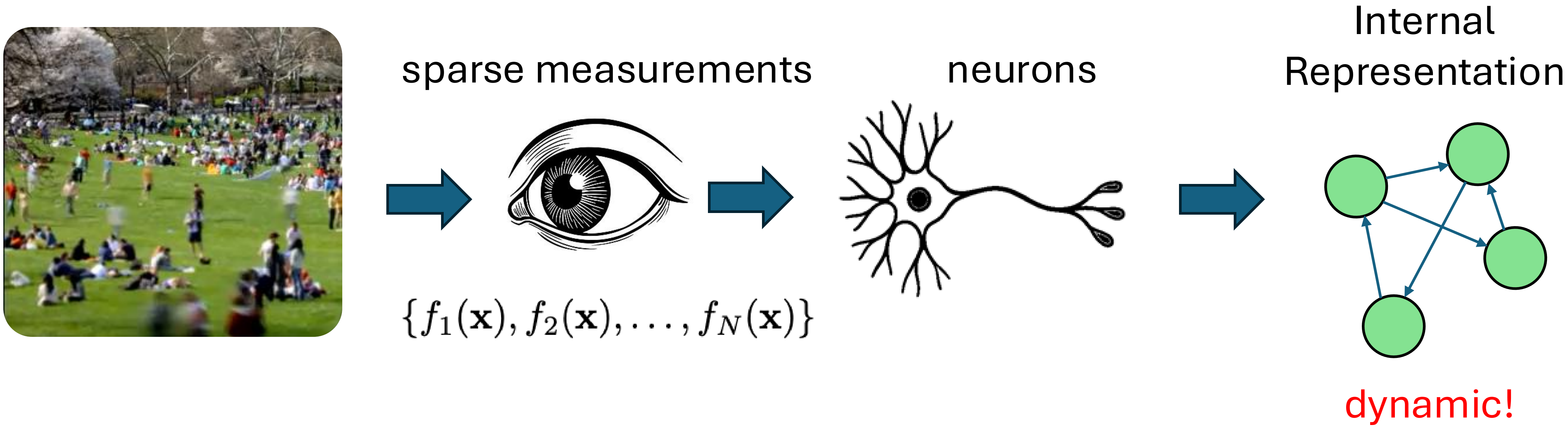
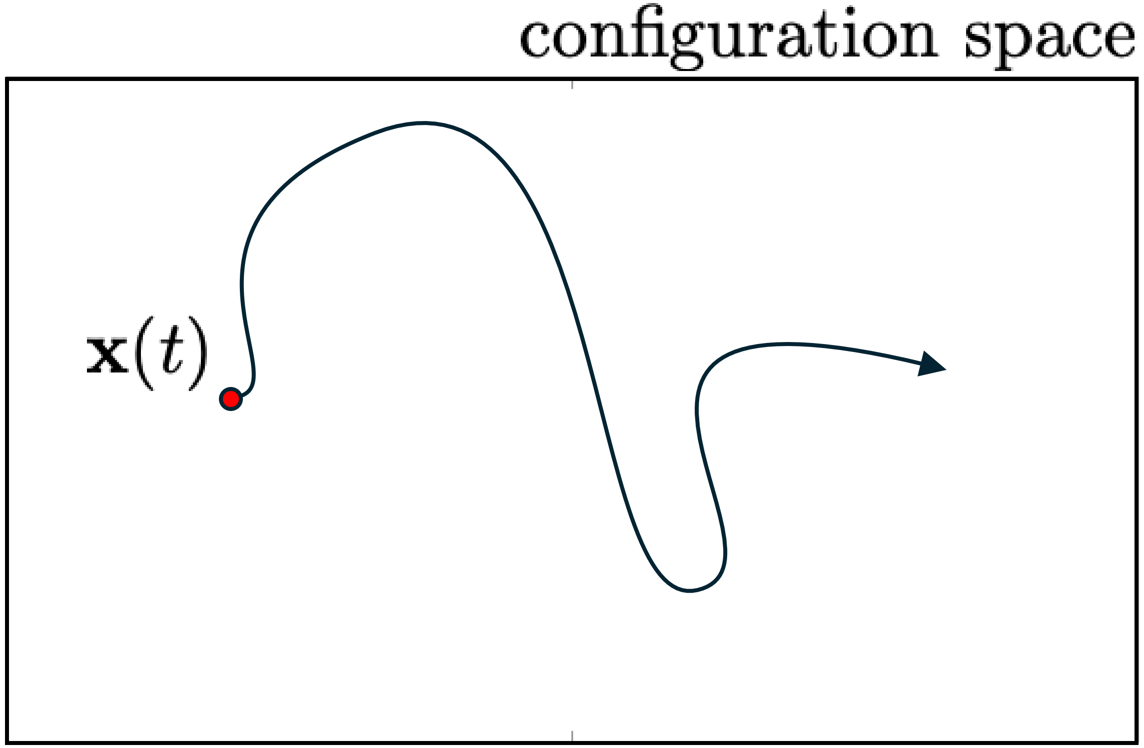
Neurons as Detectors of Coherent Sets in Sensory Dynamics

Joshua L. Pughe-Sanford, Xuehao Ding, Jason J. Moore, Anirvan M. Sengupta, Charles Epstein, Philip Greengard, Dmitri B. Chklovskii

Problem Statement



Stochastic Processes



$$d\mathbf{X}(t) = \phi(\mathbf{X}(t))dt + d\mathbf{W}(t)$$

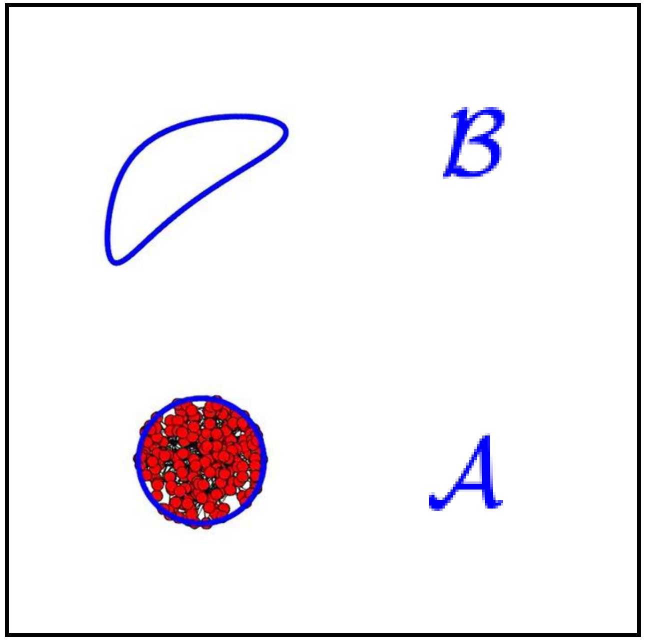
deterministic

stochastic

$$\{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_N(\mathbf{x})\}$$

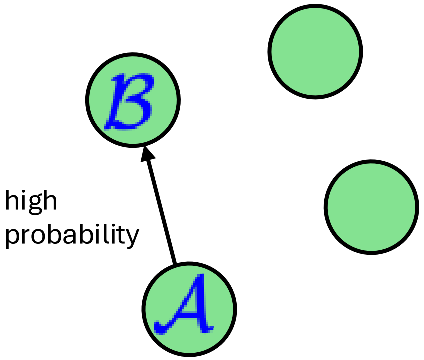
Coherent Set

$$C_\tau(\mathcal{A}, \mathcal{B}) = \mathbb{P}[\mathbf{X}(t+\tau) \in \mathcal{B} \mid \mathbf{X}(t) \in \mathcal{A}] \approx 1$$



configuration space

Internal Representation



Coherent Set Identification

$$C_\tau(\mathcal{A},\mathcal{B}) = \frac{\langle 1_\mathcal{A}, \mathsf{K}_\tau 1_\mathcal{B}\rangle}{\langle 1_\mathcal{A}, 1_\mathcal{A}\rangle}$$

Rayleigh Quotient = easy to solve maximization problem

$$[\mathsf{K}_\tau f](\mathbf{x}) = \mathbb{E}[f(\mathbf{X}(t+\tau))|\mathbf{X}(t) = \mathbf{x}]$$

$$\mathbb{E}[f_i(\mathbf{X})f_j(\mathbf{X})] \longrightarrow K$$

computed from covariances, utility well established

Coherent Set Identification

$$K_\tau^\dagger K_\tau \vec{a} = \lambda \vec{a}$$

Neural output

Neural input

$$1_{\mathcal{A}}(\mathbf{x}) \approx \begin{cases} 1 & \sum_i a_i f_i(\mathbf{x}) > 0 \\ 0 & \text{else} \end{cases}$$

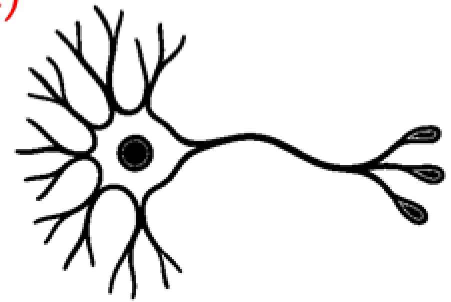
Problem Solution

Coherent set Identifier

$f_1(\mathbf{X})$

$f_2(\mathbf{X})$

$f_i(\mathbf{X})$



$$= \begin{cases} 1 & \sum_i a_i f_i(\mathbf{X}) > 0 \\ 0 & \text{else} \end{cases}$$