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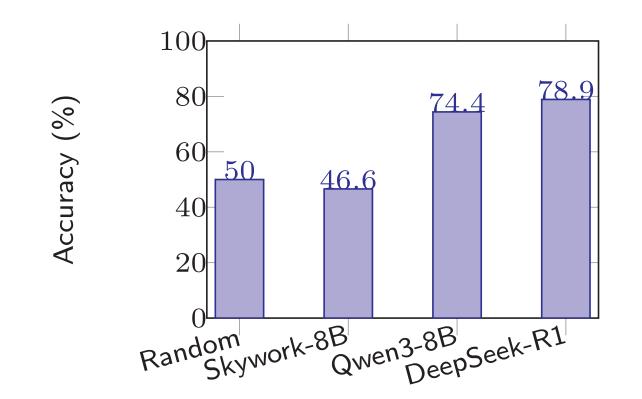




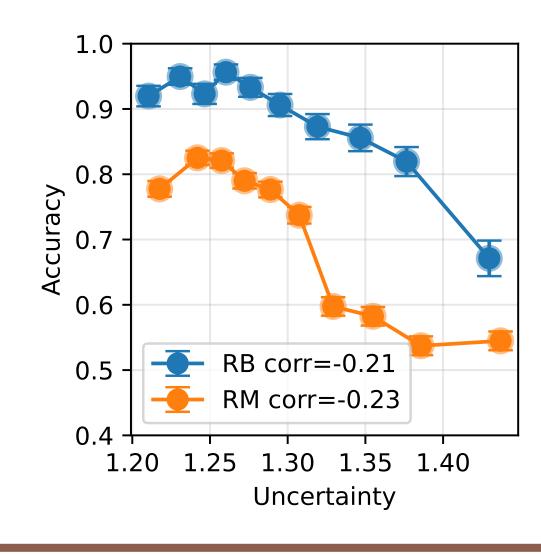
Background and Motivation

- ▶ Motivation: Reward Models (RM) are central to RLHF but suffer from reliability and efficiency issues.
- ▶ Bradley-Terry (BT)/classification RM: Fast but brittle.
 - Output: Scalar pointwise reward or pairwise preference score
 - **Pros:** Low inference cost (\sim generate 1 token)
 - Cons: Poor OOD generalization
 - Overfit to spurious patterns in limited training data, e.g., style, tense, format biases
 - Require large amount of data to fix/mitigate
- ▷ Strong LLM judges w/ reasoning: More reliable but slow.
 - Output: Long CoT and final verdict of better response
 - Pros: Strong performance gain from CoT
 - Can identify key attributes from superficial ones,
 e.g., correctness from format
 - Cons: High inference cost (often > 1k tokens) might block the online RLHF training loop

BT RM vs LLM judges on RM-Bench (Hard)

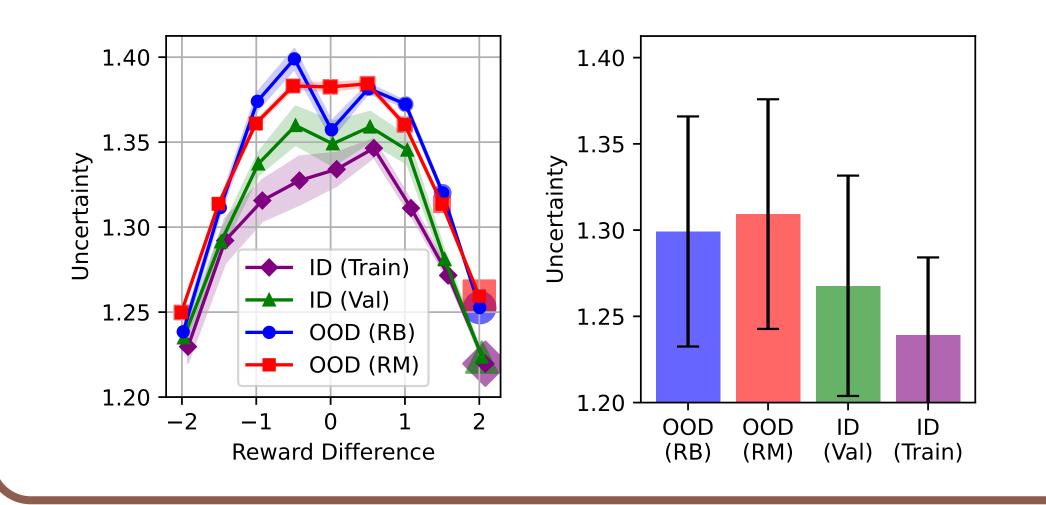


- ▶ Question: How to get LLM judge quality at low cost?
- > Insight: Not all queries are equally difficult for RM
 - High uncertainty \rightarrow Low accuracy \rightarrow Routing to strong LLM judge



Pairwise Uncertainty Quantification

- **Pointwise BT RM:** prompt x, response $y \rightarrow r(x,y)$
 - $\max_{r} \mathbb{E}_{(x,y_w,y_l)\sim\mathcal{D}_{pref}}[\log(\sigma(r(x,y_w)-r(x,y_l)))]$
 - Issue: Solution $r(x,y) \approx r^{\star}(x,y) + s(x)$ not unique, hard to assign prior, UQ ill-posed
- \triangleright Pairwise RM: prompt x, response $y_1, y_2 \rightarrow p(x, y_1, y_2)$
 - $\max_{r} \mathbb{E}_{(x,y_w,y_l) \sim \mathcal{D}_{pref}}[\log(\sigma(p(x,y_w,y_l)))]$
 - Benefit: Well-defined binary classification, yields generalized preference model, principled UQ possible
- - Architecture: SN layer and GP head
 - Logit g(h): Measures preference strength (Aleatoric).
 - Posterior covariance $u(x, y_1, y_2)$: Measures distance to training data (Epistemic).
 - Prediction: $p(x, y_1, y_2) = g(h)/u(x, y_1, y_2)$
 - Benefits: Single model (vs ensemble), single inference (vs MC dropout), distance awareness



Uncertainty-Based Routing

⊳ Routing Strategy: Define threshold \bar{u} , route highly uncertain pairs to the strong LLM judge, get $\tilde{p}(x,y_i,y_j) \approx r^*(x,y_i) - r^*(x,y_j)$

$$\widetilde{p}(x, y_i, y_j) = \begin{cases} p(x, y_i, y_j), & u \leq \overline{u} \quad \text{(Cheap)} \\ J(x, y_i, y_j), & u > \overline{u} \quad \text{(Accurate)} \end{cases}$$

Assume LLM judge's verdict is reliable

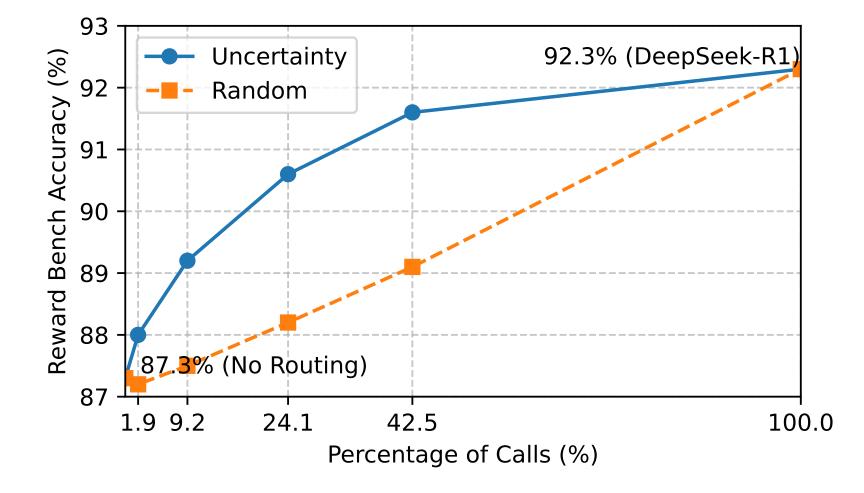
$$J(oldsymbol{x},oldsymbol{y}_i,oldsymbol{y}_j) = egin{cases} \sigma^{-1}(1-\epsilon), & oldsymbol{y}_i \stackrel{J}{\succ} oldsymbol{y}_j \ \sigma^{-1}(\epsilon), & oldsymbol{y}_i \stackrel{J}{\sim} oldsymbol{y}_j \ \sigma^{-1}(1/2), & oldsymbol{y}_i \stackrel{J}{\sim} oldsymbol{y}_j \end{cases}$$

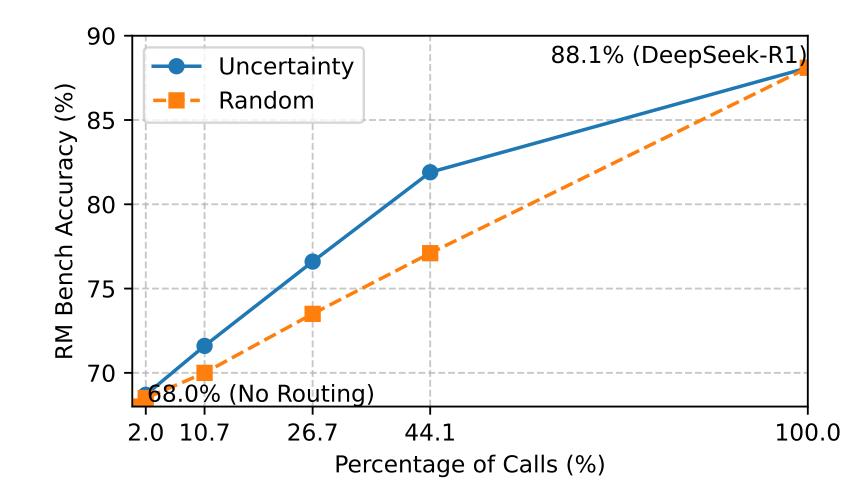
Reward Evaluation Results

- - Baseline: random routing with the same number of judge calls

Num of Calls	Reward Bench (%)						
	chat	chat hard	safety	reasoning	avg. (vs rand)		
0	95.8	73.8	89.4	90.0	87.3		
58 (1.9%)	96.1	74.8	89.5	91.7	88.0 (+0.8)		
274 (9.2%)	96.4	76.8	89.8	93.7	89.2 (+1.7)		
719 (24.1%)	96.9	80.3	89.8	95.4	90.6 (+2.4)		
1270 (42.5%)	98.3	81.2	90.0	97.0	91.6 (+2.5)		
58 (1.9%)	95.5	74.0	89.4	89.9	87.2		
274 (9.2%)	96.4	73.7	89.5	90.4	87.5		
719 (24.1%)	95.0	75.9	90.2	91.5	88.2		
1270 (42.5%)	95.5	77.5	91.6	91.9	89.1		
100%	95.5	85.8	91.1	96.9	92.3		
	0 58 (1.9%) 274 (9.2%) 719 (24.1%) 1270 (42.5%) 58 (1.9%) 274 (9.2%) 719 (24.1%) 1270 (42.5%)	chat 0 95.8 58 (1.9%) 96.1 274 (9.2%) 96.4 719 (24.1%) 96.9 1270 (42.5%) 98.3 58 (1.9%) 95.5 274 (9.2%) 96.4 719 (24.1%) 95.0 1270 (42.5%) 95.5	Num of Calls chat chat hard 0 95.8 73.8 58 (1.9%) 96.1 74.8 274 (9.2%) 96.4 76.8 719 (24.1%) 96.9 80.3 1270 (42.5%) 98.3 81.2 58 (1.9%) 95.5 74.0 274 (9.2%) 96.4 73.7 719 (24.1%) 95.0 75.9 1270 (42.5%) 95.5 77.5	Num of Calls chat chat hard safety 0 95.8 73.8 89.4 58 (1.9%) 96.1 74.8 89.5 274 (9.2%) 96.4 76.8 89.8 719 (24.1%) 96.9 80.3 89.8 1270 (42.5%) 98.3 81.2 90.0 58 (1.9%) 95.5 74.0 89.4 274 (9.2%) 96.4 73.7 89.5 719 (24.1%) 95.0 75.9 90.2 1270 (42.5%) 95.5 77.5 91.6	Num of Calls chat chat hard safety reasoning 0 95.8 73.8 89.4 90.0 58 (1.9%) 96.1 74.8 89.5 91.7 274 (9.2%) 96.4 76.8 89.8 93.7 719 (24.1%) 96.9 80.3 89.8 95.4 1270 (42.5%) 98.3 81.2 90.0 97.0 58 (1.9%) 95.5 74.0 89.4 89.9 274 (9.2%) 96.4 73.7 89.5 90.4 719 (24.1%) 95.0 75.9 90.2 91.5 1270 (42.5%) 95.5 77.5 91.6 91.9		

Routing	Num of Calls	RM Bench (%)							
-	rum or cums	chat	math	code	safety	easy	normal	hard	avg. (vs rand)
No routing	0	67.1	59.5	54.2	91.2	87.2	72.0	44.9	68.0
Uncertainty	242 (2.0%) 1285 (10.7%) 3188 (26.7%) 5270 (44.1%)	68.7 69.6 71.3 73.2	60.0 64.9 73.8 83.5	54.7 59.7 68.7 78.3	91.4 92.0 92.6 92.7	87.4 89.1 91.4 93.6	73.3 76.3 81.5 86.9	45.5 49.2 56.9 65.3	68.7 (+0.2) 71.6 (+1.6) 76.6 (+3.1) 81.9 (+4.8)
Random	242 (2.0%) 1285 (10.7%) 3188 (26.7%) 5270 (44.1%)	67.5 68.0 69.6 70.1	60.2 63.6 69.3 76.8	54.9 57.2 63.6 69.9	91.2 91.3 91.3 91.6	87.4 88.0 89.4 91.0	72.4 74.1 77.0 81.3	45.6 48.0 54.0 59.1	68.5 70.0 73.5 77.1
DeepSeek-R1	100%	76.8	95.7	87.8	92.0	94.0	91.3	78.9	88.1





- Inference time comparison: Uncertain pairs take more reasoning time, reflecting their difficulty
 - Still higher accuracy than random in same time

	Trigger Threshold	10	1.45	1.4	1.35	1.3	<1
ı	Num of Calls (Ratio)	0%	2.0%	10.7%	26.7%	44.1%	100%
	Inference Time (s) - Uncertainty	518	632	1113	2200	3007	5642
	Inference Time (s) - Random	518	625	1093	1979	2615	5642

Downstream Alignment

ightharpoonup Policy Gradient from Preferences: For each prompt x, sample $y_1, \ldots, y_K \sim \pi_{\theta}(\cdot \mid x)$, minimize the PG loss

$$\mathcal{L}_{\text{policy}}(\theta) = -\frac{1}{K} \sum_{i=1}^{K} A_i \log \pi_{\theta}(y_i | x_i)$$

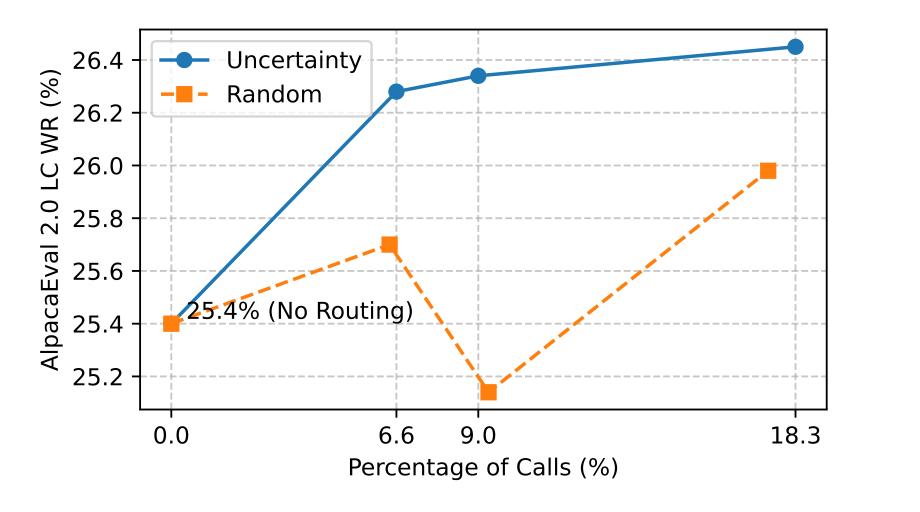
We use RLOO advantage estimator, which can be recovered from pairwise preference \widetilde{p} :

$$A_{i} = r(x, y_{i}) - \frac{1}{K - 1} \sum_{j \neq i} r(x, y_{j})$$

$$= \frac{1}{K - 1} \sum_{j \neq i} \underbrace{[r(x, y_{i}) - r(x, y_{j})]}_{\widetilde{p}(x, y_{i}, y_{j})}$$

- Also applies to other PG methods using MC samples and mean baseline, e.g., GRPO
- Downstream Alignment Results

Model	Num of Calls	Arena-Hard (%)	AlpacaEv	al 2.0 (%)	MT-Bench		
		v0.1 WR	LC WR	WR	Turn 1	Turn 2	Avg
Base model	-	24.5	22.31	23.63	7.98	6.80	7.47
No routing (10.0)	0	28.1	25.40	27.35	8.19	6.98	<u>7.65</u>
Uncertainty (1.35)	7668 (6.6%)	28.9	26.28	28.97	8.05	7.19	7.65
Uncertainty (1.30)	10522 (9.0%)	28.9	26.34	28.53	8.03	7.13	7.63
Uncertainty (1.20)	21363 (18.3%)	29.8	26.45	28.91	7.95	7.40	7.71
Random (1.35)	7523 (6.4%)	26.5	25.70	28.55	8.09	7.20	7.71
Random (1.30)	10854 (9.3%)	27.7	25.14	28.29	7.93	6.74	7.41
Random (1.20)	20474 (17.5%)	28.5	25.98	28.51	8.00	6.62	7.45



Takeaways

We propose an Uncertainty-Based Routing Framework

- Efficiency: Call costly judge only for uncertain samples
- Effectiveness: SNGP identifies OOD samples
- Impact: More reliable online RLHF with less computational overhead
- Future directions:
 - Active RM learning, filter samples for further annotations
 - Combine with specialized reasoning RMs