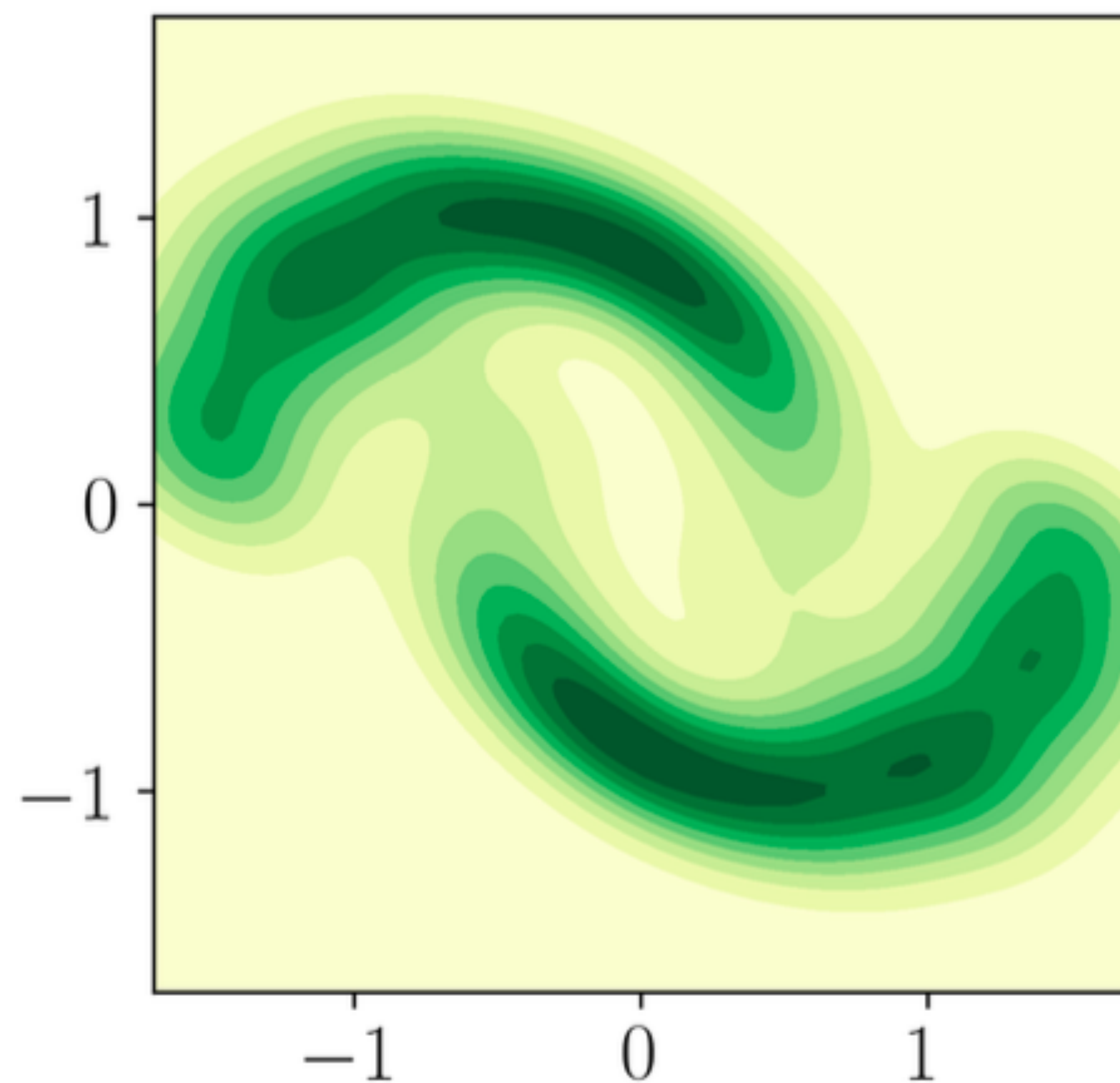
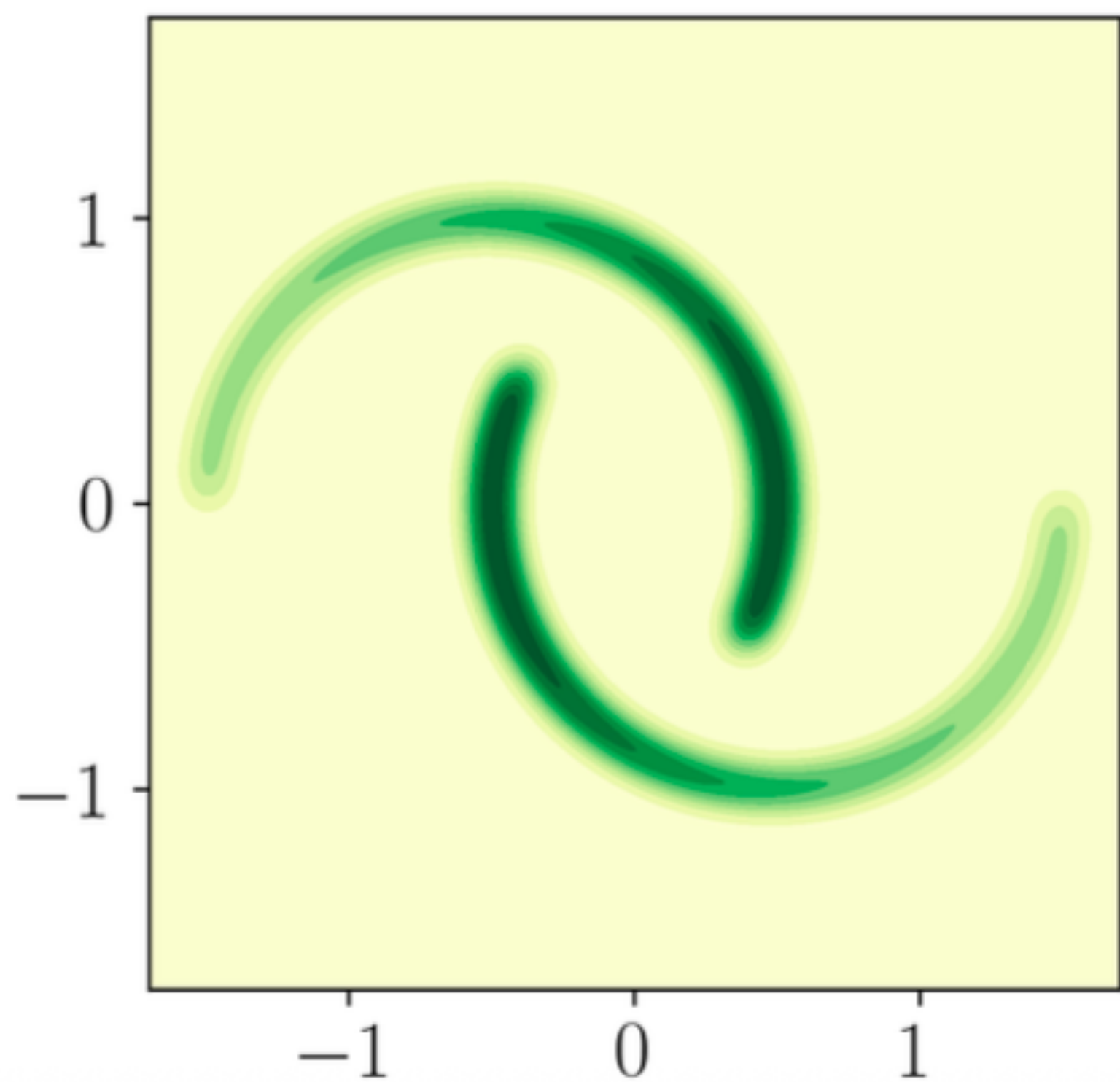


When and how can inexact generative models still sample from the data manifold?

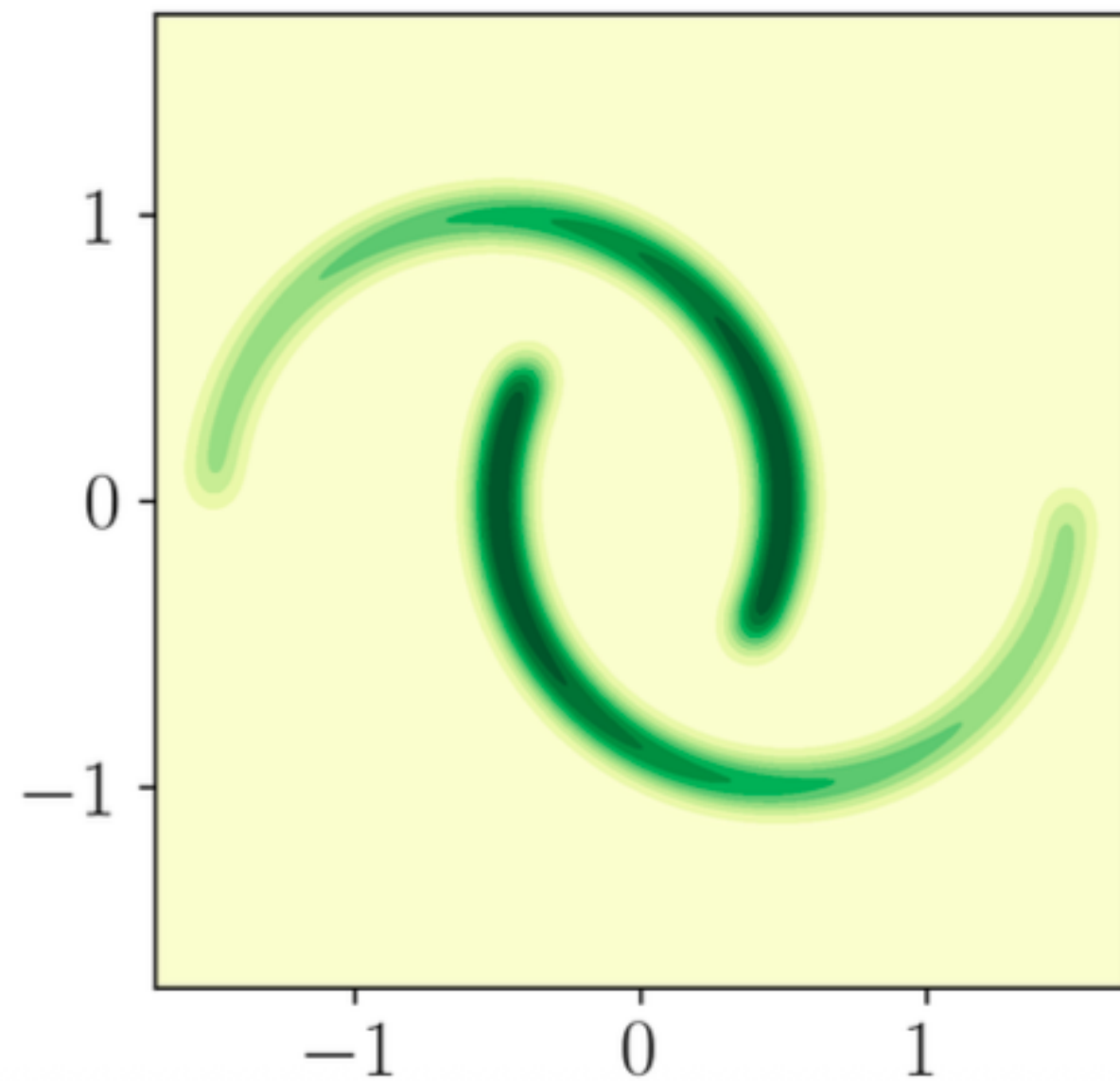
Nisha Chandramoorthy and Adriaan de Clercq

The University of Chicago

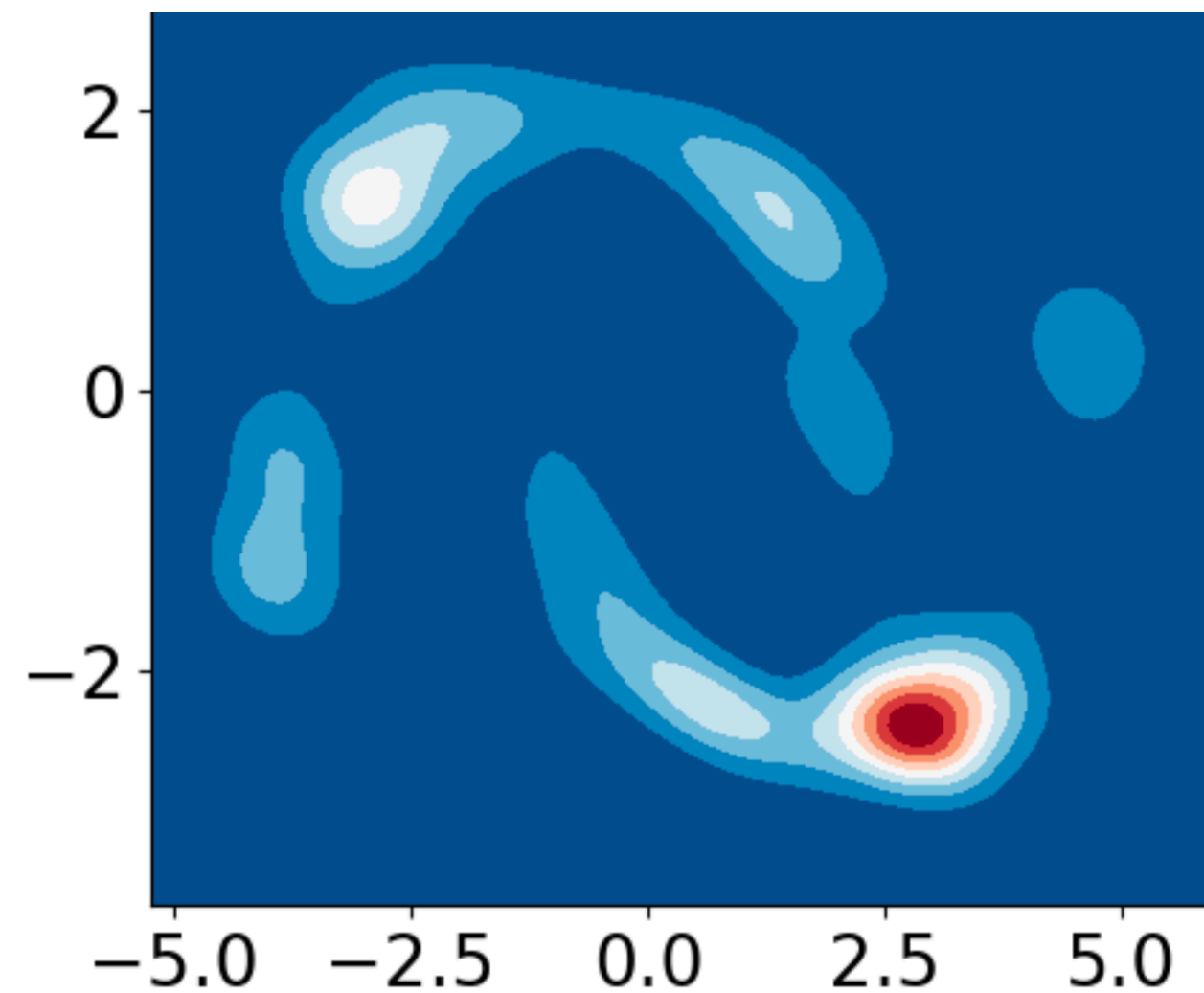


The support appears robust

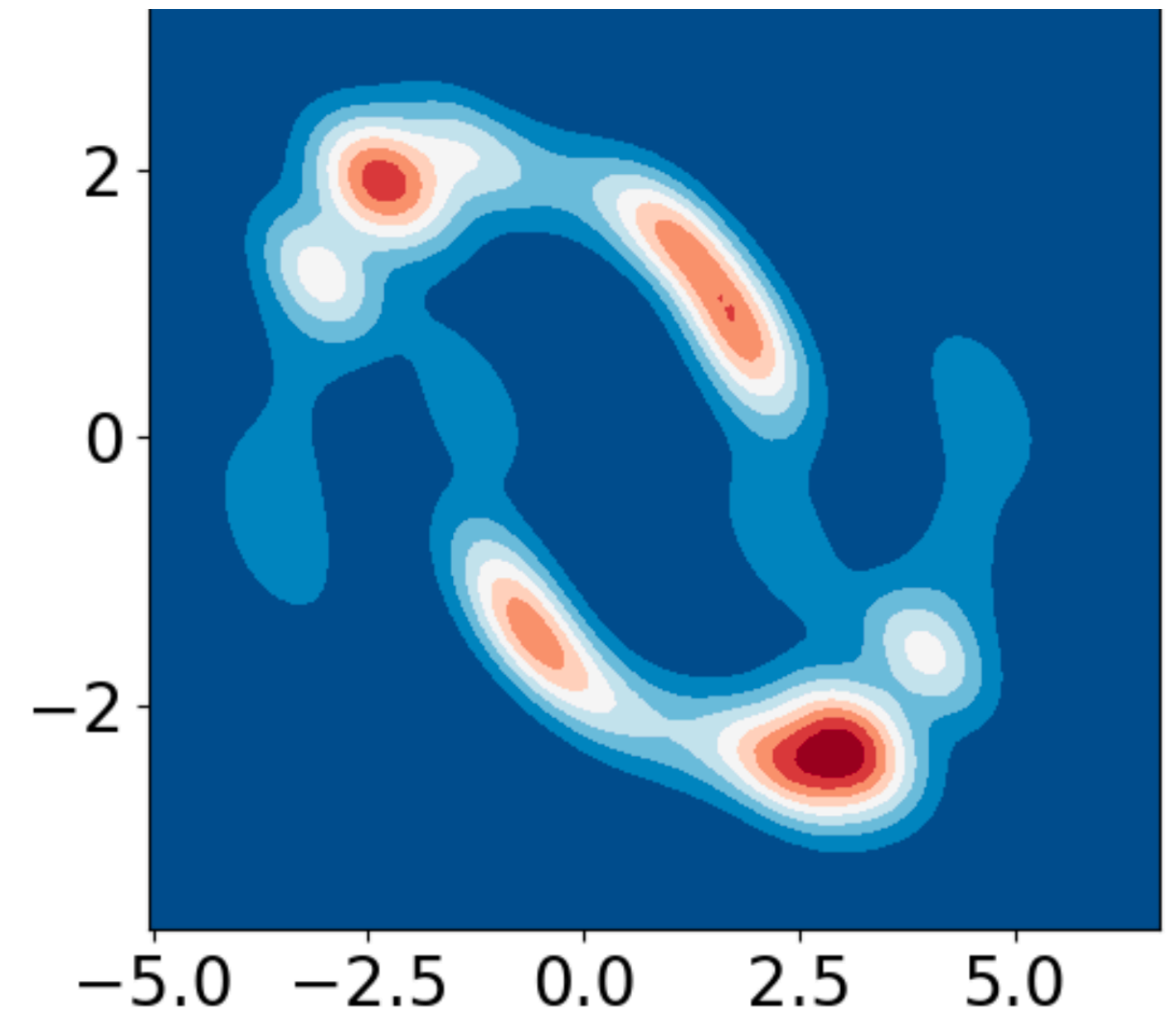
Robustness of the support



Target



Conditional Flow
matching

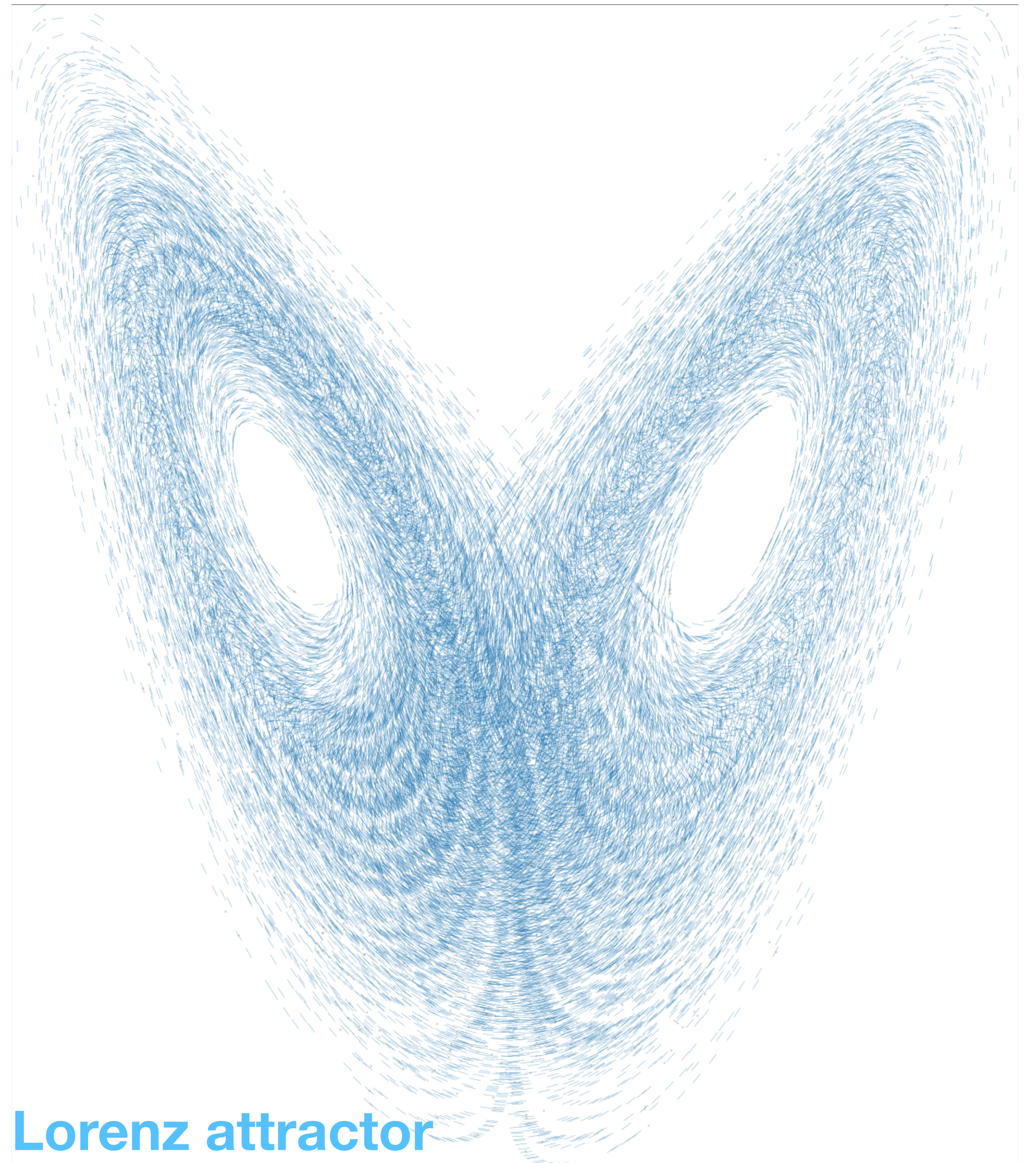


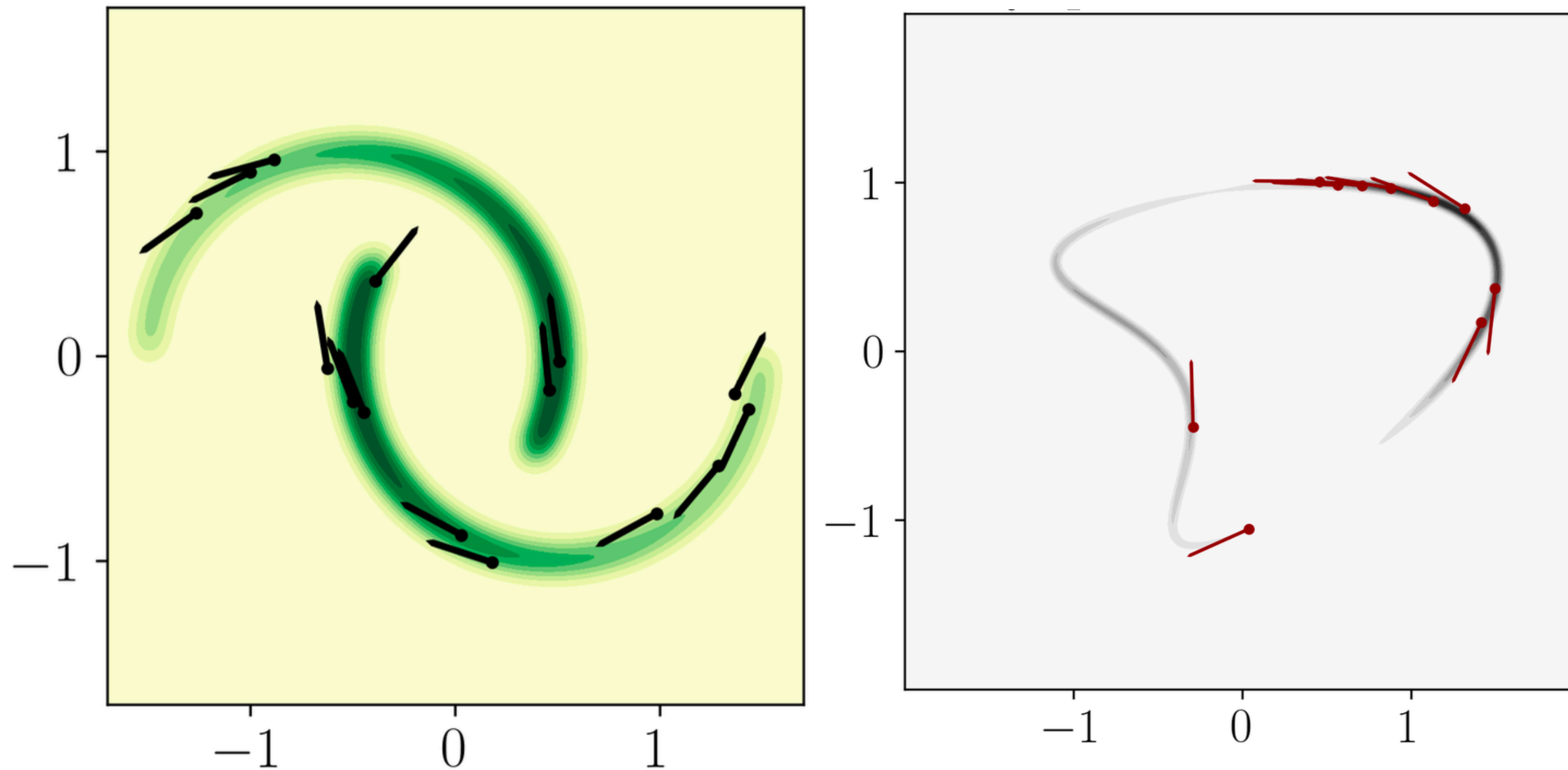
OT - Conditional Flow
matching

All models are incorrect, but some are usefully incorrect

Low-dimensional absolutely continuous structure — in the physical sciences

The unstable subspaces on the Lorenz attractor



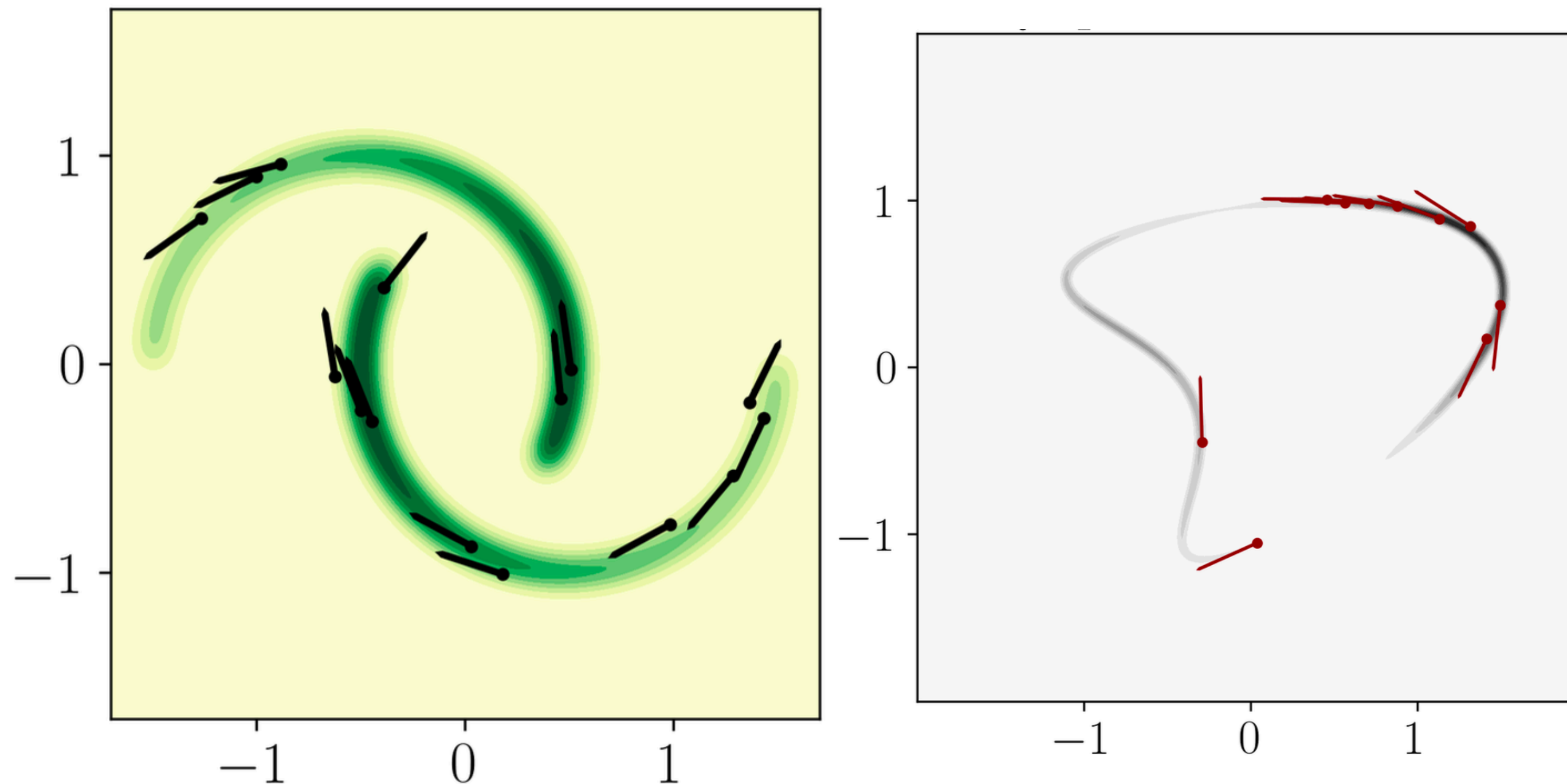


$E \approx$ top d Lyapunov Vectors of GM (F)

- d : intrinsic dimension.
- \approx top d eigenvectors of $\nabla F (\nabla F)^\top$: where F takes noise samples to target samples

Alignment with support of p_{data}

- M : support of p_{data}
- A GM is **aligned** if TM is parallel to E



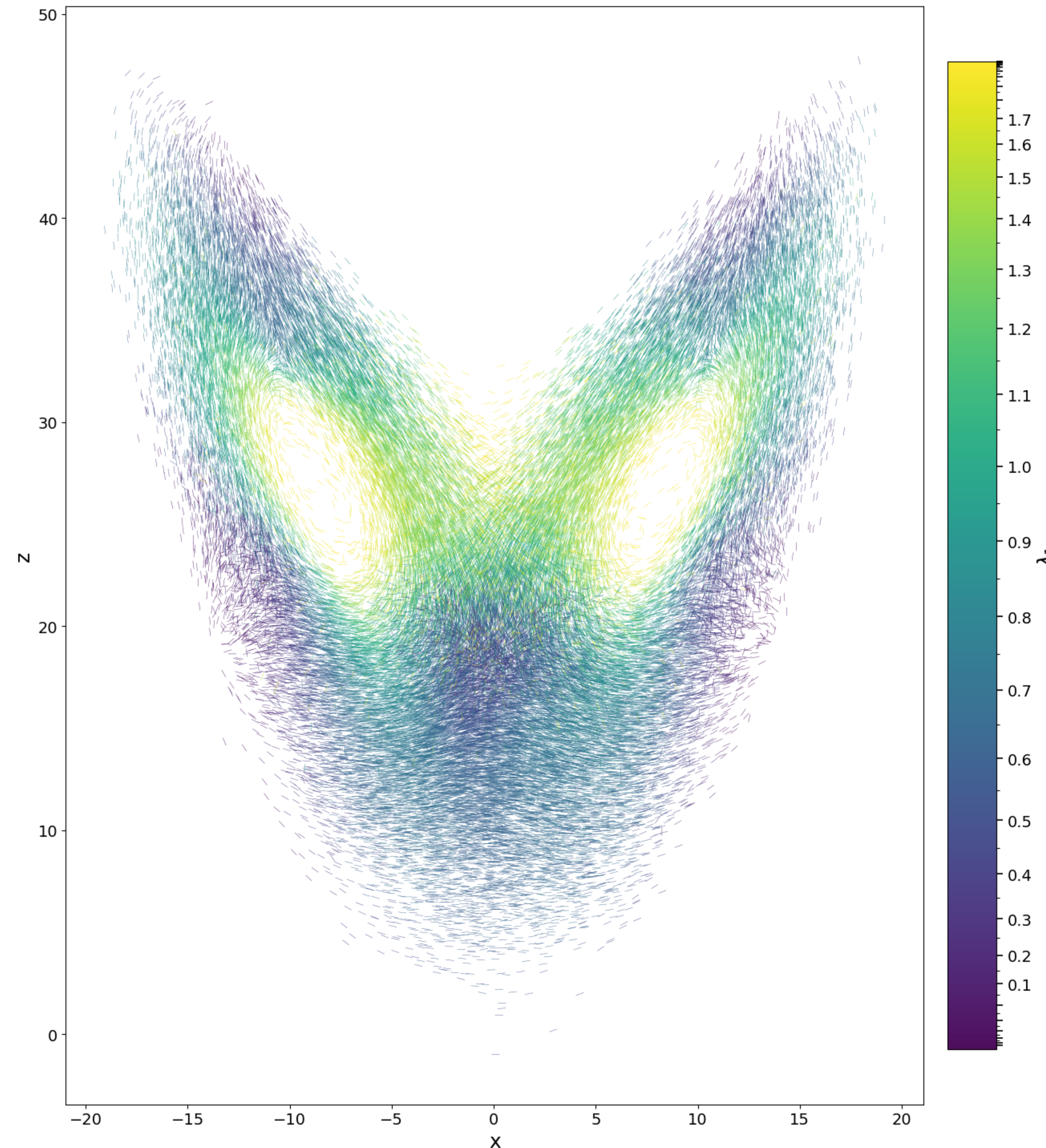
Learning the support

Informal: A convergent generative model learns the support of the target if **aligned**

- Convergent: optimal transport map of small norm exists between generated samples and the target samples; $\|x_i - y_i\| \sim \mathcal{O}(\epsilon^c)$ (e.g., Lee, Lu, Tan 2023)
- Support estimation \equiv Learning one-class classifier
- Margin does not change under alignment, hence has same generalization error

Alignment leads to learning the data manifold

- The most **sensitive subspaces** E_τ are constructively defined; scalable computation
- Is **alignment** preserved under perturbations? Yes, under smooth perturbations.
- What kind of dynamics leads to **alignment**?
- How to ensure **alignment**?



Summary

- GMs can be viewed as random dynamical systems
- This perspective explains their behavior under learning errors
- **Alignment** of the d most **sensitive subspaces** with the tangent space of the d -dimensional data manifold leads to robustness of the support
- Aligned generative models can learn the data manifold