Permutation Equivariant Neural Controlled Differential Equations for Dynamic Graph Representation Learning

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Main Contribution:

Project Neural Controlled Differential Equations on Graphs onto Equivariant function spaces.

Significantly reduces parameter count without compromising representational power, resulting in more efficient training and improved generalisation.

Graph Neural Controlled Differential Equations (GNCDEs)[1]

Given graph snapshots $\mathcal{G} = \{G_1, \ldots, G_n\}$ with a dynamic graph topology, GNCDEs interpolate into continuous edge data $A : [0, T] \to \mathbb{R}^{n\times n}$, and learn paths of the form

$$Z_t = Z_{t_0} + \int_{t_0}^{t} Z_s^{(L)}\, ds, \qquad Z_s^{(l)} = \sigma\Big(\tilde{A}_s\, Z_s^{(l-1)}\, W^{(l-1)}\Big)$$

with a GCN vector field and *fusion*

$$\tilde{A}_s = W_1\, A_s + W_2\, \frac{dA_s}{ds}$$

[1] Learning dynamic graph embeddings with neural controlled differential equations, Qin et al. (2025)

Permutation Equivariant Graph Neural Controlled Differential Equations (PENG-NCDEs)

Problem: The *fusion* is not equivariant under permutation of the node-set!

Fix: Expand linear maps L_i in the basis of linear permutation equivariant functions[2] and define the *equivariant fusion*

$$\bar{A}_s = L_1(A_s) + L_2(\frac{dA_s}{ds})$$

Theorem: This is the most general permutation equivariant GNCDE we can consider!

[2]Invariant and Equivariant Graph Networks, Maron et al. (2018)

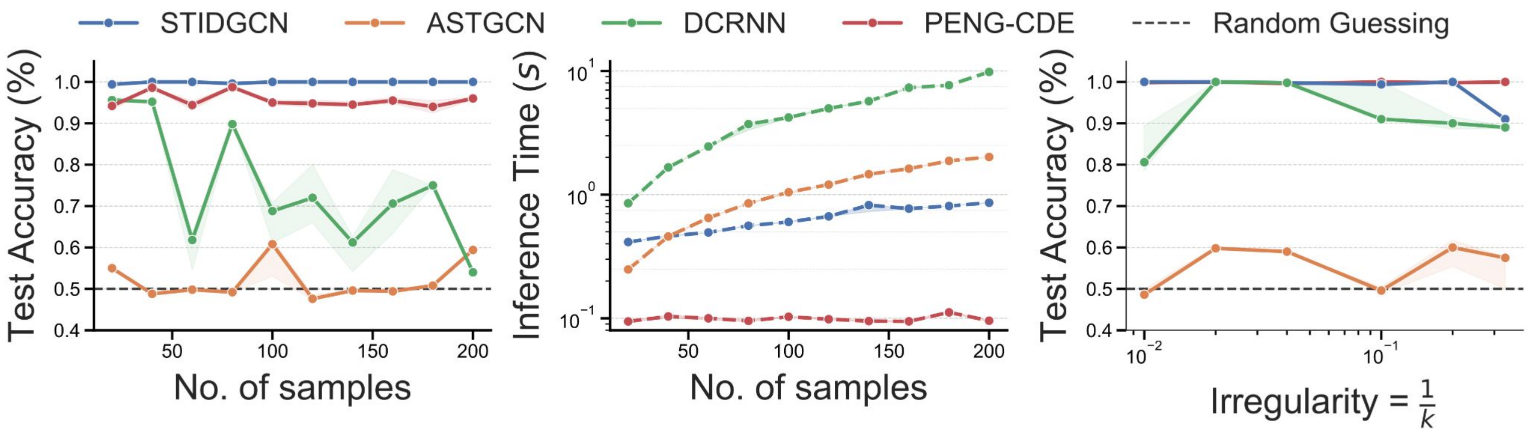
Results

Model	trade NDCG@10 ↑	genre NDCG@10 ↑
Persistent Forecast (L)†	**0.855**	0.357
Moving Avg (L)†	0.823	**0.509**
Moving Avg (M)	0.777	0.472
JODIE‡ [39]	0.374±0.09	0.350±0.04
TGAT‡ [15]	0.375±0.07	0.352±0.03
CAWN‡ [66]	0.374±0.09	–
TCL‡ [65]	0.375±0.09	0.354±0.02
GraphMixer‡ [14]	0.375±0.11	0.352±0.03
DyGFormer‡ [68]	0.388±0.64	0.365±0.20
DyRep† [60]	0.374±0.001	0.351±0.001
TGN† [53]	0.374±0.001	0.367±0.058
TGNv2* [59]	**0.735**±0.006	0.469±0.002
STG-NCDE [10]	0.618±0.024	0.438±0.038
GN-CDE [48]	0.713±0.026	0.460±0.016
PENG-CDE	0.716±0.029	**0.523**±0.017
+ Source/Target Id	**0.734**±0.024	–

PENG-CDEs are state-of-the-art on popular Temporal Graph benchmark[3]!

[3]TGB 2.0, Gastinger et al. (2023)

Results

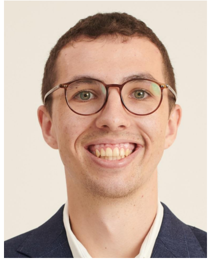


PENG-CDEs are robust to oversampling and irregular sampling!

Summary

1. Introduce geometrically-informed approach to employing CDEs on graphs
2. Set new state-of-the-art result in the TGB dataset
3. Inherit robustness properties of Neural CDEs

Authors



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