NEURAL INFORMATION PROCESSING SYSTEMS

Efficient Low Rank Attention for Long-Context Inference in Large Language Models

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- Find a lightweight proxy attention to approximate the original attention.

Method Overview

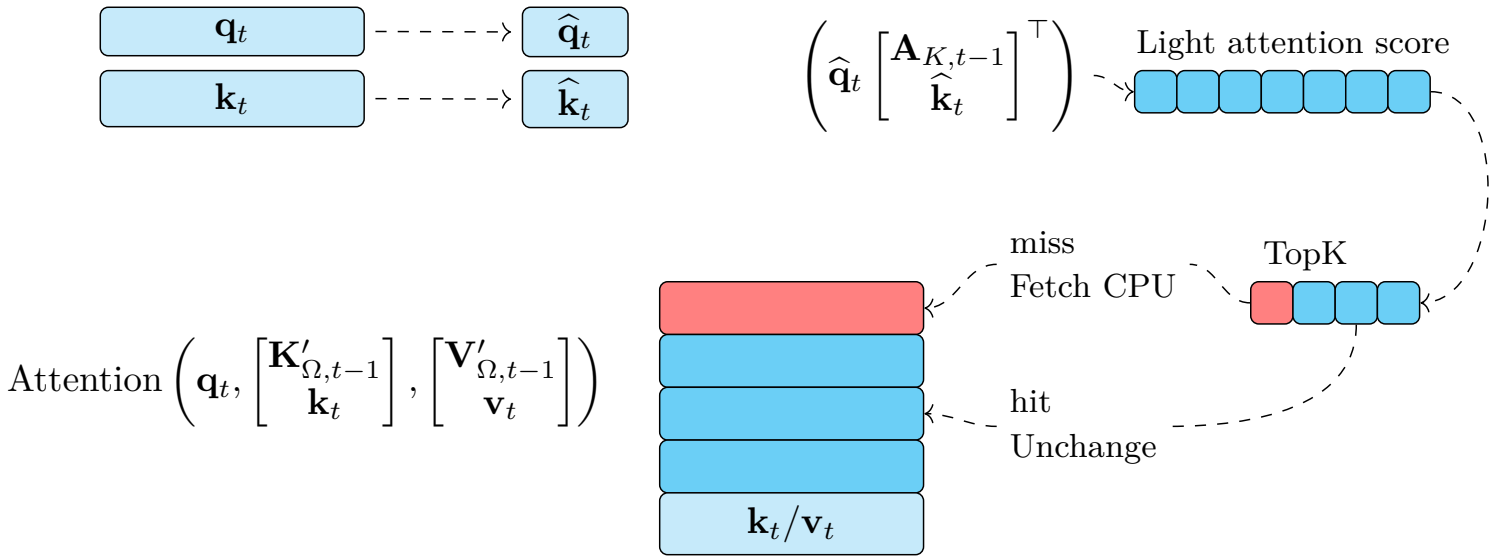
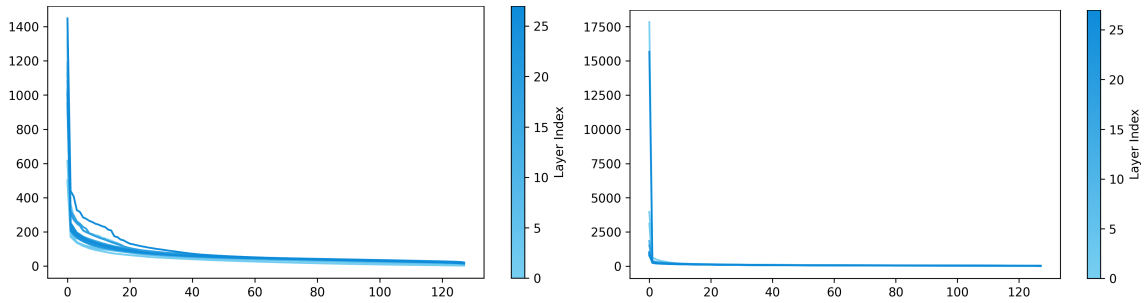


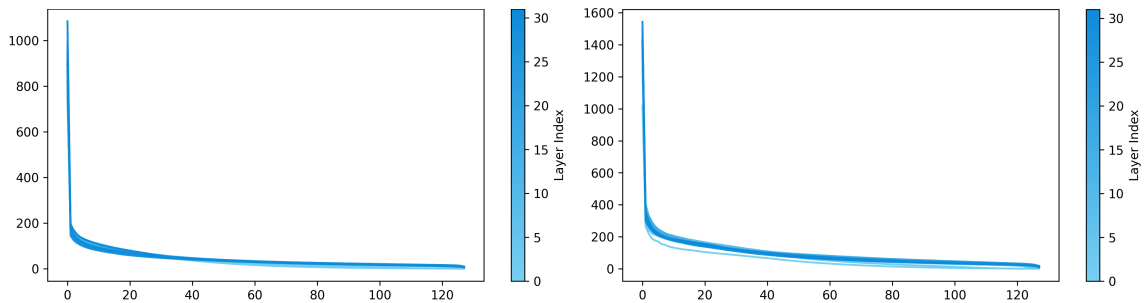
Figure: Overview of the LRQK method. Subscript Ω denotes selected tokens, t denotes current token. $\mathbf{q}_t, \mathbf{k}_t$ are original query and key, $\widehat{\mathbf{q}}_t, \widehat{\mathbf{k}}_t$ are approximated query and key.

Low Rank Structure of $\mathbf{QK}^\top$

$$\text{rank}(\mathbf{QK}^\top) \leq \min\left(\text{rank}(\mathbf{Q}), \text{rank}(\mathbf{K}^\top)\right) = \min\left(\text{rank}(\mathbf{Q}), \text{rank}(\mathbf{K})\right).$$



(a) Qwen2.5 **Q** (b) Qwen2.5 **K**



(c) LLaMA3 **Q** (d) LLaMA3 **K**

Figure: Examples of the mean of singular values of the query and key matrix over different layers on Qwen2.5-7B and LLaMA-3-8B-1M models. The singular values are summed over batches and attention heads.

Proposed

Identify relevant KV pairs $\rightarrow$ *Joint Low-Rank Approximation*: For $\mathbf{Q}, \mathbf{K} \in \mathbb{R}^{l \times d}$ (consider last two dimensions), approximate interaction matrix as:

$$\mathbf{Q}\mathbf{K}^\top \approx \mathbf{A}_Q \mathbf{A}_K^\top \quad \text{with} \quad \mathbf{A}_Q, \mathbf{A}_K \in \mathbb{R}^{l \times r}, \; r < d$$

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Avoid reconstruction errors $\rightarrow$ *Precision-Preserving Attention:* Maintain full-precision computation using original keys/values,

$$\text{Attention}(\mathbf{q}_t, \mathbf{K}_{\Omega,t}, \mathbf{V}_{\Omega,t})$$

Avoid reconstruction as,

$$\mathbf{K}_{\Omega,t} \leftarrow \text{rec}(\text{factors } \mathbf{K}, \Omega_t) \text{ or } \mathbf{V}_{\Omega,t} \leftarrow \text{rec}(\text{factors } \mathbf{V}, \Omega_t).$$

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Memory management $\rightarrow$ *Hybrid Memory Architecture:* CPU-GPU coordination with asynchronous transfers and hierarchical caching.

Algorithm Prefill

Within the standard LLM inference framework, prompt processing (prefill) followed by autoregressive generation (decode). This approach introduces a low-rank attention and a dynamic CPU-GPU KV management.

Prefill (prompt processing): To derive a low rank representation $\mathbf{A}_Q, \mathbf{A}_K \in \mathbb{R}^{l\times r}$, and $\mathbf{B}_Q, \mathbf{B}_K \in \mathbb{R}^{r\times d}$ from the original query and key matrices $\mathbf{Q}, \mathbf{K} \in \mathbb{R}^{l\times d}$,

$$\underset{\mathbf{A}_Q,\mathbf{B}_Q,\mathbf{A}_K,\mathbf{B}_K}{\arg\min} \frac{1}{2}\left\|\mathbf{Q}\mathbf{K}^\top - \mathbf{A}_Q\mathbf{A}_K^\top\right\|_{\mathrm{F}}^2, \text{s.t. } \mathbf{Q} = \mathbf{A}_Q\mathbf{B}_Q, \mathbf{K} = \mathbf{A}_K\mathbf{B}_K.$$

Algorithm Decode

Decode (autoregressive generation): Update the low rank factors during inference. At step t, continually compute smaller $\widehat{\mathbf{q}}_t, \widehat{\mathbf{k}}_t \in \mathbb{R}^{1\times r}$ from current input $\mathbf{q}_t, \mathbf{k}_t \in \mathbb{R}^{1\times d}$ and update the low rank factors $\mathbf{B}_{Q,t}, \mathbf{B}_{K,t} \in \mathbb{R}^{r\times d}$,

$$\arg\min_{\widehat{\mathbf{q}}_t, \widehat{\mathbf{k}}_t} \frac{1}{2} \left\| \widehat{\mathbf{q}}_t \mathbf{B}_{Q,t-1} - \mathbf{q}_t \right\|_{\mathrm{F}}^2 + \frac{1}{2} \left\| \widehat{\mathbf{k}}_t \mathbf{B}_{K,t-1} - \mathbf{k}_t \right\|_{\mathrm{F}}^2,$$

$$\text{s.t. } \widehat{\mathbf{q}}_t \widehat{\mathbf{k}}_t^\top = \mathbf{q}_t \mathbf{k}_t^\top, \widehat{\mathbf{q}}_t \mathbf{A}_{K,\Omega,t-1}^\top = \mathbf{q}_t \mathbf{K}_{\Omega,t-1}^\top.$$

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$$\Omega_k = \mathrm{topk}\left(\widehat{\mathbf{q}}_t \mathbf{A}_{K,\Omega,t-1}^\top,\ k\right)$$

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- Update basis matrices $\mathbf{B}_{Q,t}, \mathbf{B}_{K,t}$ and asynchronously offload $\mathbf{k}_t, \mathbf{v}_t$ to CPU.

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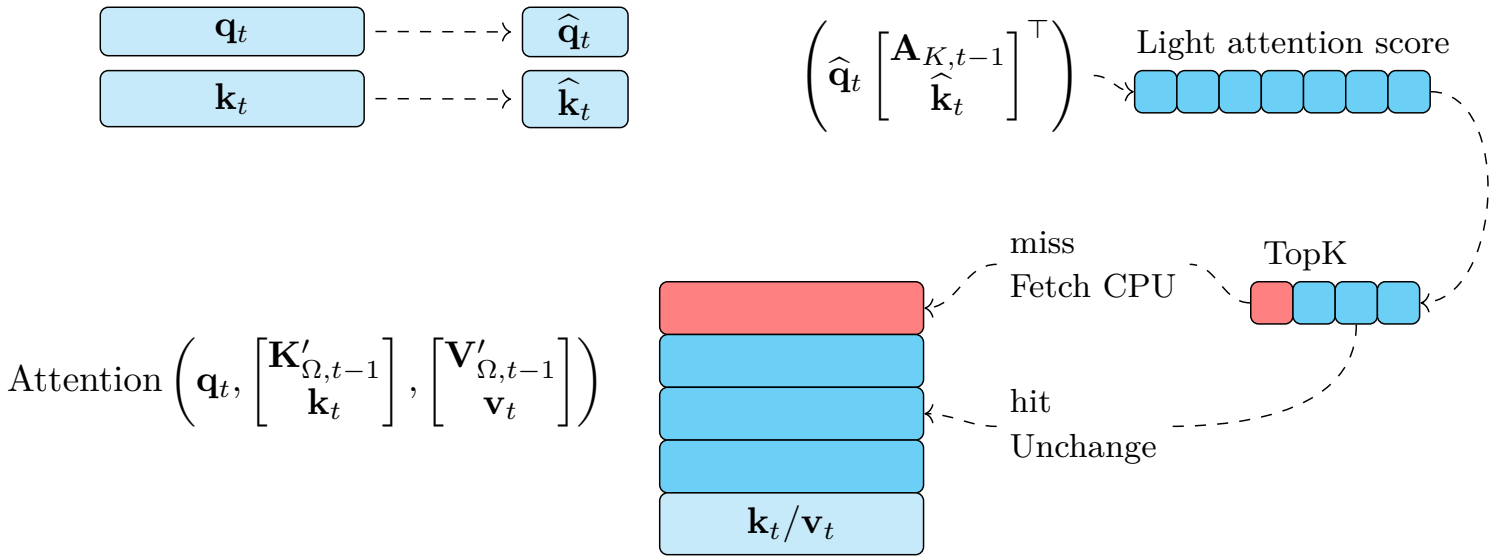

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