UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Abstract Rendering:

Certified Rendering Under 3D Semantic Uncertainty

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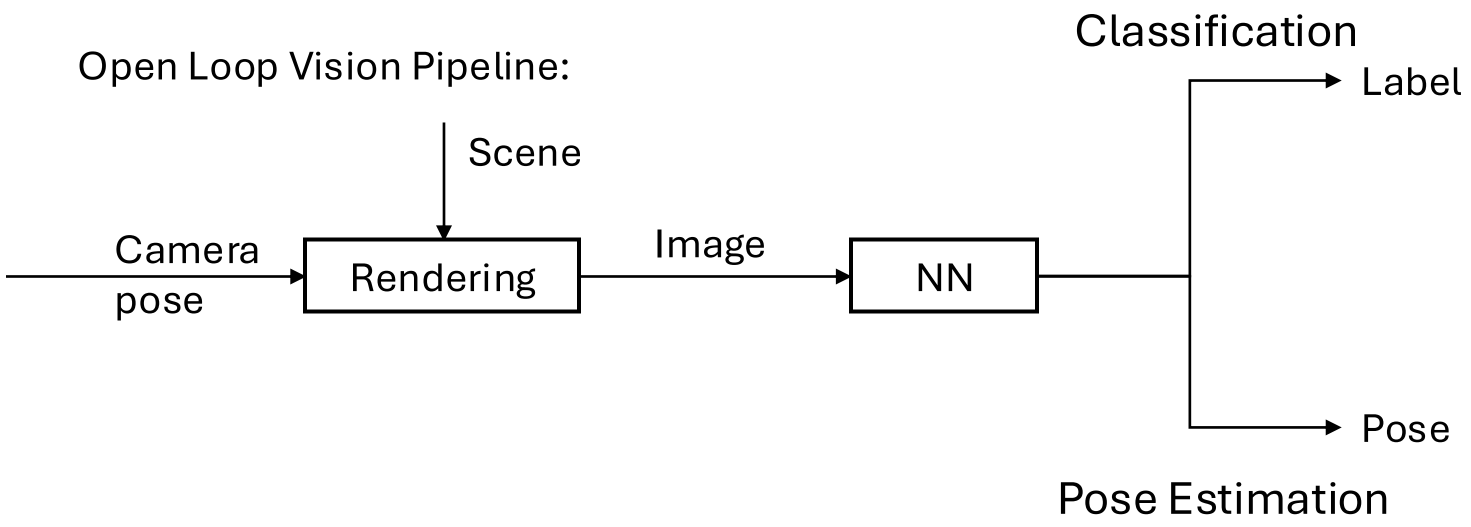
Motivation: Safety Assurance of Vision Pipeline



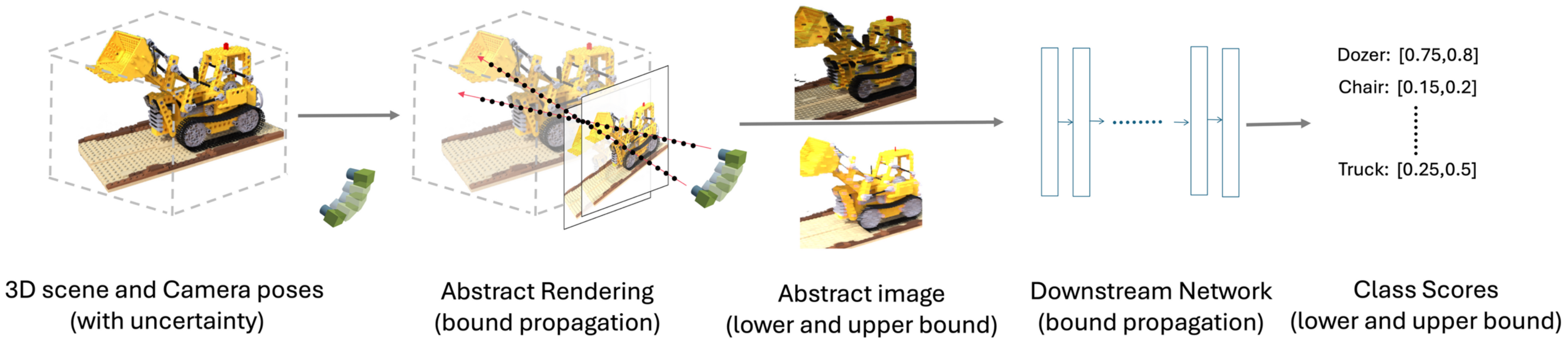
- **Safety/Verification Question:** Given continuous variations in camera poses and scene configurations, determine the regions where a trained image-based neural network reliably outputs correct predictions and where it does not.

Method: Compute Over-Approximation of Output via Bound Propagation

- **Approach:** Compute an over-approximation of the pipeline's output over each input partition. If this over-approximation fully lies within the safe region, correctness is guaranteed; otherwise, unsafe or mispredictions may occur.

- **Abstract Rendering:** A framework that computes provable bounds on all images rendered under continuously varying camera poses and scenes. The set of such images, called an **Abstract Image**, is compactly represented using interval or linear constraints on pixel values.

Our framework: Abstract Rendering + Downstream Certification



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Rendering Algorithm – Gaussian Splatting

Algorithm 1 GaussianSplat(ScG, C, u)

$$
\begin{aligned}
&\textbf{for all } i \in I \textbf{ do}\\
&\quad \mu_c[i] \leftarrow R \times (\mu_w[i] - T)\\
&\quad \Sigma_c[i] \leftarrow R \times \Sigma_w[i]\\
&\quad K[i] \leftarrow \begin{bmatrix} \frac{f_x}{\mu_c[i,2]} & 0 & \frac{c_x}{\mu_c[i,2]} \\ 0 & \frac{f_y}{\mu_c[i,2]} & \frac{c_y}{\mu_c[i,2]} \end{bmatrix}\\
&\quad J[i] \leftarrow \begin{bmatrix} \frac{f_x}{\mu_c[i,2]} & 0 & -\frac{f_x \cdot \mu_c[i,0]}{{\mu_c}^2[i,2]} \\ 0 & \frac{f_y}{\mu_c[i,2]} & -\frac{f_y \cdot \mu_c[i,1]}{{\mu_c}^2[i,2]} \end{bmatrix}\\
&\quad \mu_p[i] \leftarrow K \times \mu_c[i]\\
&\quad \Sigma_p[i] \leftarrow J[i] \times \Sigma_c[i] \times J[i]^\top\\
&\quad \mathrm{Conic}[i] \leftarrow \mathrm{Inv}(\Sigma_p[i])\\
&\quad q[i] \leftarrow (u - \mu_p[i])^\top \times \mathrm{Conic}[i] \times (u - \mu_p[i])\\
&\quad a[i] \leftarrow o[i] \cdot \mathrm{Exp}(-\tfrac{1}{2} \cdot q[i])\\
&\quad d[i] \leftarrow \mu_c[i,2]\\
&\textbf{end for}\\
&as \leftarrow \mathrm{Sort}(a, d)\\
&cs \leftarrow \mathrm{Sort}(c, d)\\
&\textbf{for all } i \in I \textbf{ do}\\
&\quad oc[i] \leftarrow \textstyle\prod_{j=1}^{i-1}(1 - as[j])\\
&\textbf{end for}\\
&pc \leftarrow \textstyle\sum_{i=1}^{N}(oc[i] \cdot as[i] \cdot cs[i])\\
&\textbf{return } pc
\end{aligned}
$$

World coord -> Camera coord

Camera coord -> Pixel coord

Matrix Inverse

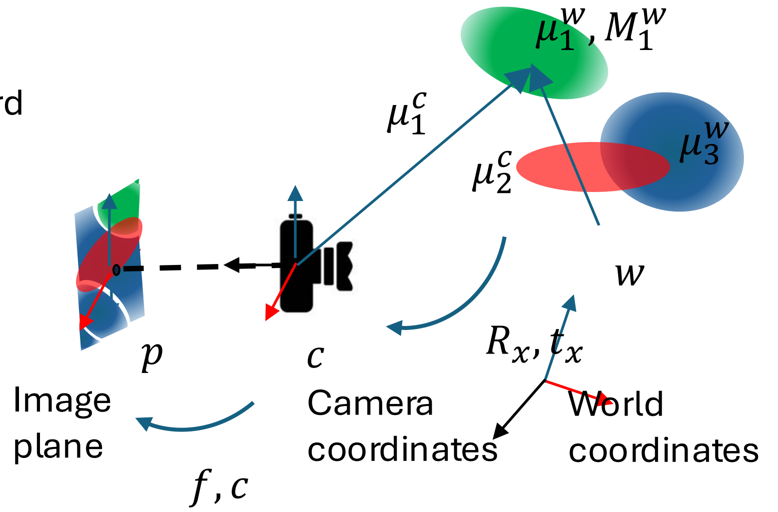
Evaluate GS at pixel

Sorting-based Aggregation

Sum of Cumulative Product

Depth-Sorted Color Blending

Gaussian splat scene



Operation	Inputs	Output
Element-wise add (Add)	$x, y \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}^{n \times m}$
Element-wise multiply (Mul)	$x, y \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}^{n \times m}$
Division (Div)	$x \in \mathbb{R}_{>0}$	$z \in \mathbb{R}_{>0}$
Matrix multiplication (Mmul)	$x \in \mathbb{R}^{n \times m}, y \in \mathbb{R}^{m \times k}$	$z \in \mathbb{R}^{n \times k}$
Matrix inverse (Inv)	$x \in \mathbb{R}^{n \times n}$	$z \in \mathbb{R}^{n \times n}$
Matrix power (Pow)	$x \in \mathbb{R}^{n \times n}, k \in \mathbb{N}$	$z \in \mathbb{R}^{n \times n}$
Summation (Sum)	$x \in \mathbb{R}^{n}, k \in \mathbb{N}$	$z \in \mathbb{R}$
Product (Prod)	$x \in \mathbb{R}^{n}, k \in \mathbb{N}$	$z \in \mathbb{R}$
Matrix transpose ($\top$)	$x \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}^{m \times n}$
Element-wise exponential (Exp)	$x \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}^{m \times n}$
Frobenius norm (Norm)	$x \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}$
Element-wise indicator (Ind)	$x \in \mathbb{R}^{n \times m}$	$z \in \mathbb{R}^{n \times m}$
Sorting (Sort)	$z \in \mathbb{R}^{n}$	$x \in \mathbb{R}^{n}, y \in \mathbb{R}^{n}$

Kerbl, Bernhard, et al. "3D Gaussian splatting for real-time radiance field rendering." *ACM Trans. Graph.* 42.4 (2023): 139-1.

Abstracting three Rendering-Specific functions

Matrix Inverse

Algorithm 3 $\textsc{MatrixInv}(\mathsf{X}, \mathsf{X}_{\mathsf{ref}}, \mathsf{k})$

$\Delta \mathsf{X} \leftarrow -(\mathsf{X} - \mathsf{X}_{\mathsf{ref}}) \times \mathsf{X}_{\mathsf{ref}}^{-1}$

assert $||\Delta \mathsf{X}|| < 1$

$\mathsf{Xp} \leftarrow \sum_{i=0}^{k} \mathsf{X}_{\mathsf{ref}}^{-1} \times \mathrm{Pow}(\Delta \mathsf{X}, \mathsf{i})$

$\mathsf{X}_{\mathsf{R}} \leftarrow ||\mathsf{X}_{\mathsf{ref}}^{-1}|| \cdot \frac{||\Delta \mathsf{X}||^{k+1}}{1 - ||\Delta \mathsf{X}||}$

$\mathsf{lXinv} \leftarrow \mathsf{Xp} - \mathsf{X}_{\mathsf{R}}$

$\mathsf{uXinv} \leftarrow \mathsf{Xp} + \mathsf{X}_{\mathsf{R}}$

return $\langle \mathsf{lXinv}, \mathsf{uXinv} \rangle$

Sorting-based Aggregation

Algorithm 4 $\textsc{VR-Ind}(\mathsf{a}, \mathsf{c}, \mathsf{d})$

for all $\mathsf{i} \in \mathsf{I}$ **do**

  $\mathsf{oc}[\mathsf{i}] \leftarrow \prod_{j=1}^{N} \left(1 - \mathsf{a}[\mathsf{j}] \cdot \mathrm{Ind}(\mathsf{d}[\mathsf{i}] - \mathsf{d}[\mathsf{j}])\right)$

end for

$\mathsf{pc} \leftarrow \sum_{i=1}^{N} (\mathsf{oc}[\mathsf{i}] \cdot \mathsf{a}[\mathsf{i}] \cdot \mathsf{c}[\mathsf{i}])$

return pc

Sum of Cumulative Product

Algorithm 5 $\textsc{SumCumProd}(\mathsf{a}, \mathsf{c})$

$\mathsf{pc} \leftarrow [0, 0, 0]$

for all $\mathsf{i} \in \mathrm{reverse}(\mathsf{I})$ **do**

  $\mathsf{pc} \leftarrow \mathsf{a}[\mathsf{i}] \cdot \mathsf{c}[\mathsf{i}] + (1 - \mathsf{a}[\mathsf{i}]) \cdot \mathsf{pc}$

end for

return pc

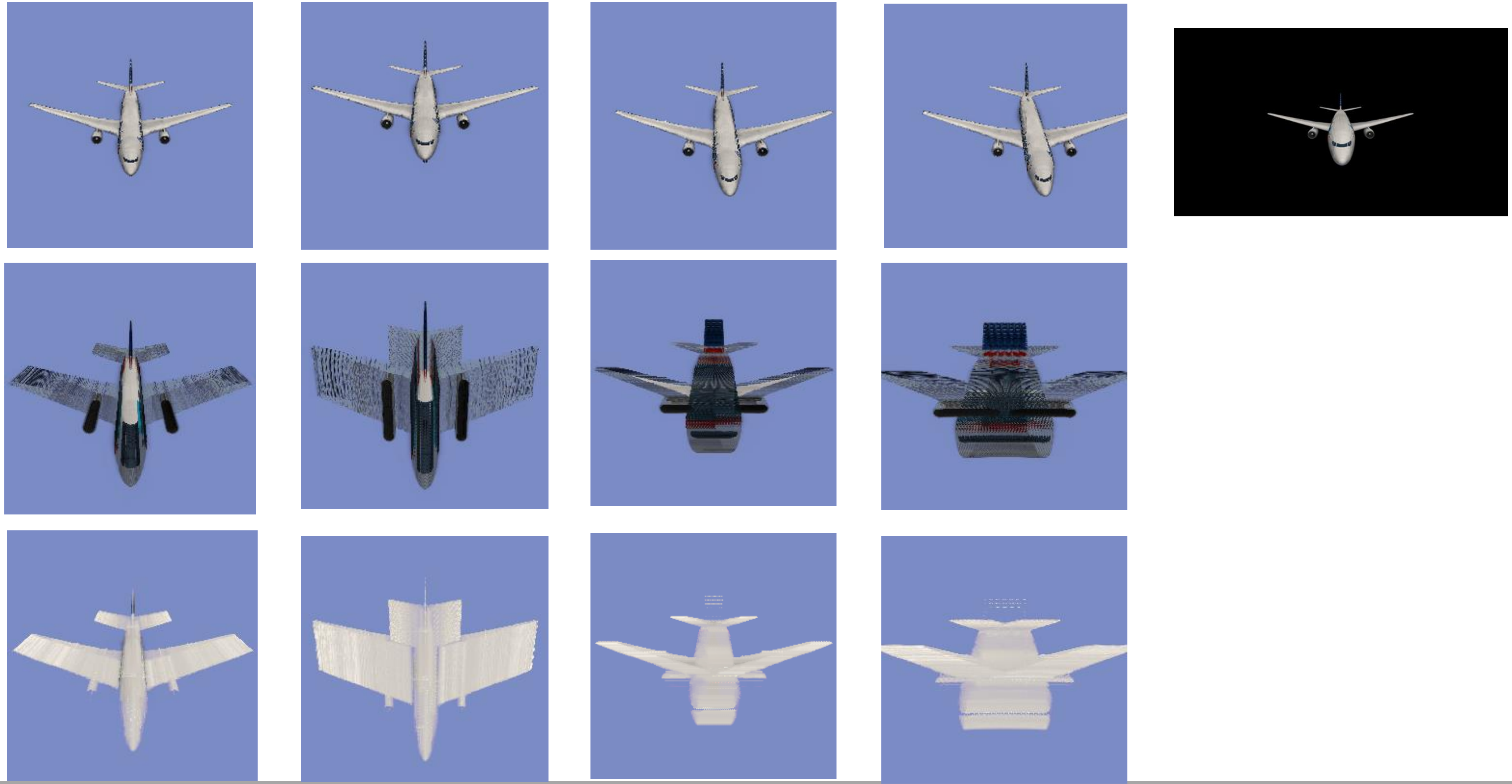
Experiment Results --- Abstract Image under Camera Pose Uncertainty

Perturb x (distance) | Perturb y (elevation) | Perturb z | Perturb yaw | 3D GS scene

Pose Range

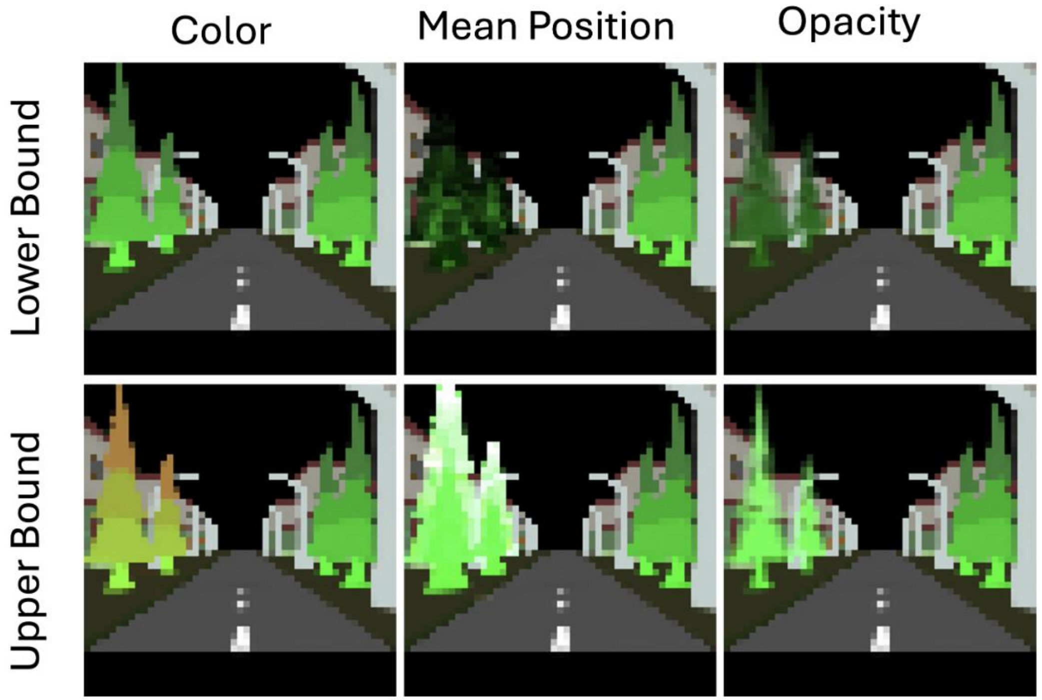
Lower Bound

Upper Bound

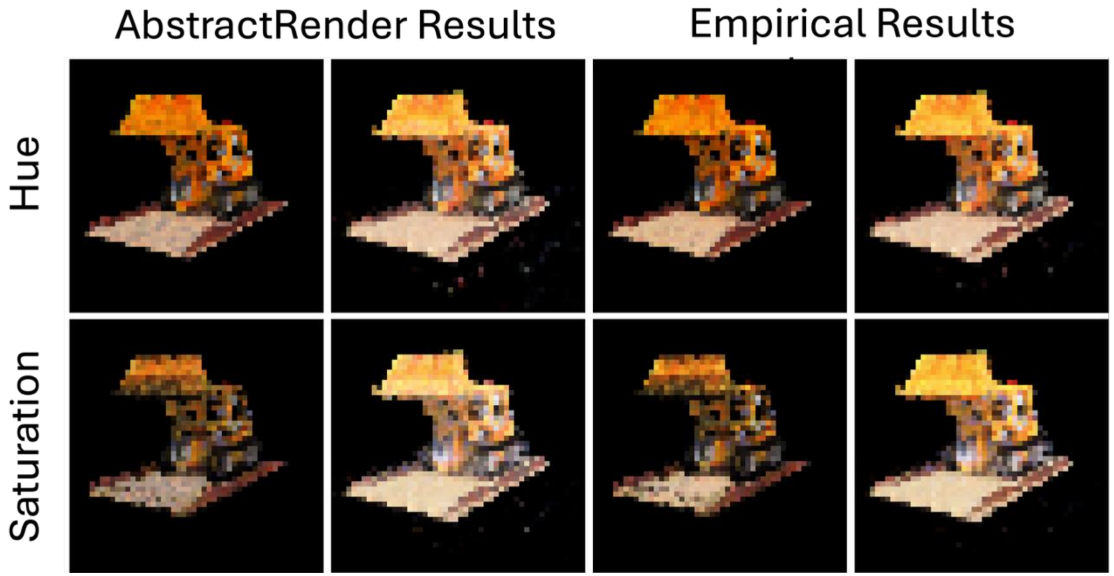


Experiment Results --- Abstract Image under Scene Configuration Uncertainty

Gaussian Scene: Street
Variation: Two roadside trees



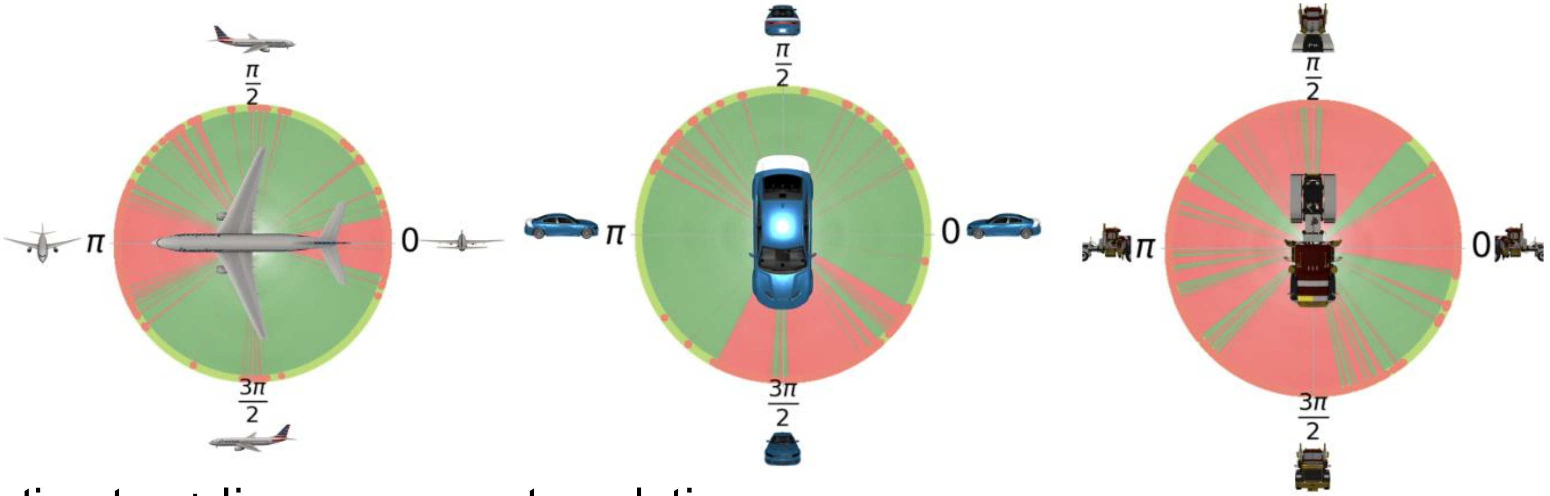
NeRF Scene: Dozer
Variation: Hue or Saturation



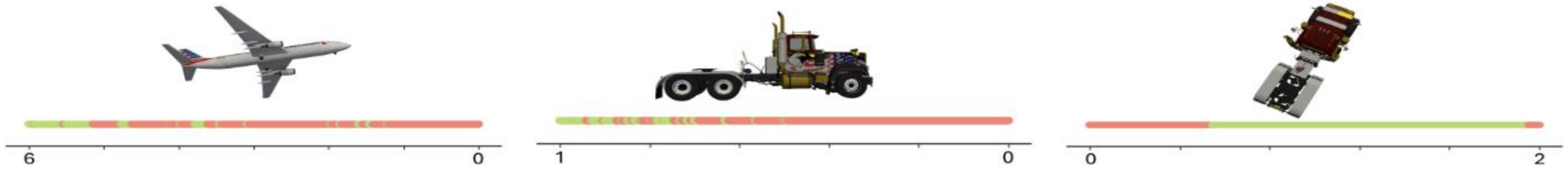
Experiment Results --- Verifying Robustness of Downstream NN

For the target network, green regions denote certified camera poses where all rendered images are correctly classified, while red regions indicate uncertified poses where misclassification may occur.

Classifier + $360°$ camera rotation:



Pose estimator + linear camera translation:

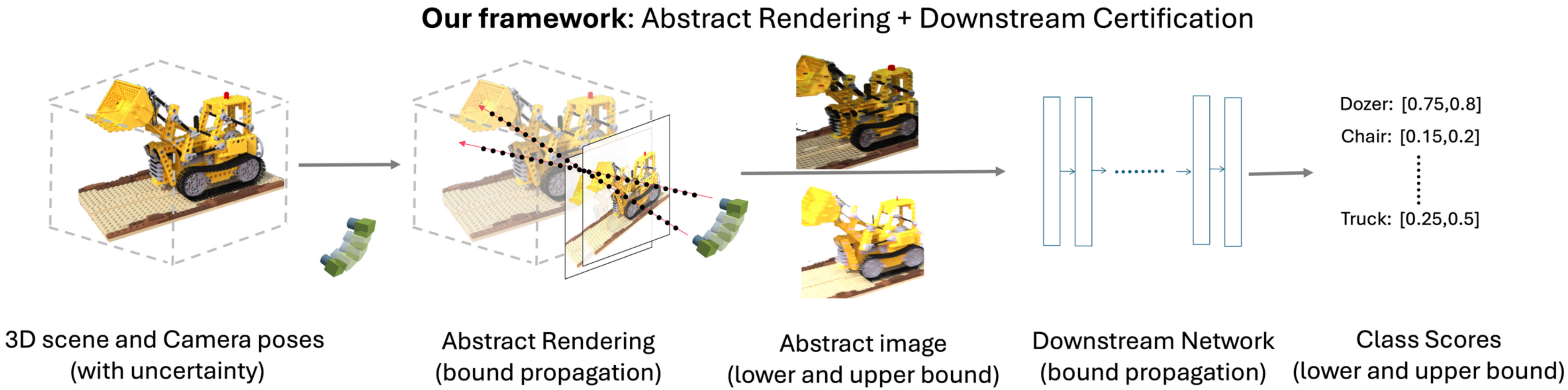


Conclusion

We propose the **first** framework for computing abstract images of scenes represented by Gaussian Splats and NeRF under camera pose or scene uncertainty.

We design **novel linear relational approximations** of three rendering-specific operations.

By integrating Abstract Rendering with CROWN, we have enabled certification of visual tasks with respect to semantic variations in 3D environments



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