University of Minnesota
Driven to Discover℠

imagine.umn.edu

Neural Information Processing Systems

Time-Embedded Algorithm Unrolling for Computational MRI

Junno Yun, Yaşar Utku Alçalar, and Mehmet Akçakaya
University of Minnesota, Minneapolis

Introduction

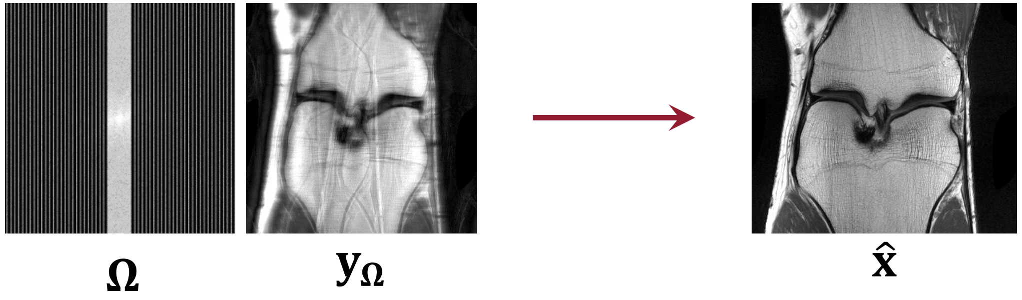
- **Algorithm unrolling** solving the regularized least squares problem in computational magnetic resonance imaging (MRI)

$$\mathbf{y}_\Omega = \mathbf{E}_\Omega \mathbf{x} + \mathbf{n}$$

$$\arg\min_{\mathbf{x}} \|\mathbf{y}_\Omega - \mathbf{E}_\Omega \mathbf{x}\|_2^2 + \mathcal{R}(\mathbf{x})$$

$\mathbf{y}_\Omega$: Sub-sampled measurements
$\mathbf{E}_\Omega$: Encoding matrix sampling Fourier locations $\mathbf{\Omega}$
$\mathbf{n}$: Gaussian measurement noise
$\mathcal{R}(\cdot)$: Regularizer

Inverse Problem in computational MRI



- Unrolling iterative algorithms (e.g., VSQP, ADMM) with fixed iterations, alternating between 1) **Data fidelity**, 2) **Regularization** $\mathcal{R}(\cdot)$, and 3) **Auxiliary updates**

Proximal operator implemented by **neural networks** (***Shared, Unshared*** across iterations)

Variable Splitting Quadratic Penalty (VSQP)

$$\mathbf{x}^t = \left(\mathbf{E}_\Omega^H \mathbf{E}_\Omega + \mu \mathbf{I}\right)^{-1} \left(\mathbf{E}_\Omega^H \mathbf{y}_\Omega + \mu \mathbf{z}^t\right),$$
$$\mathbf{z}^{t+1} = \arg\min_{\mathbf{z}} \frac{1}{2} \left\|\mathbf{x}^t - \mathbf{z}\right\|_2^2 + \mathcal{R}(\mathbf{z}) \triangleq \mathrm{Prox}_{\mathcal{R}}(\mathbf{x}^t),$$

Alternating-Direction Method of Multipliers (ADMM)

$$\mathbf{x}^{t+1} = \left(\mathbf{E}_\Omega^H \mathbf{E}_\Omega + \mu \mathbf{I}\right)^{-1} \left(\mathbf{E}_\Omega^H \mathbf{y}_\Omega + \mu \left(\mathbf{z}^t - \mathbf{u}^t\right)\right),$$
$$\mathbf{z}^{t+1} = \mathrm{Prox}_{\mathcal{R}}(\mathbf{x}^{t+1} + \mathbf{u}^t),$$
$$\mathbf{u}^{t+1} = \mathbf{u}^t + \lambda(\mathbf{x}^{t+1} - \mathbf{z}^{t+1}),$$

Introduction

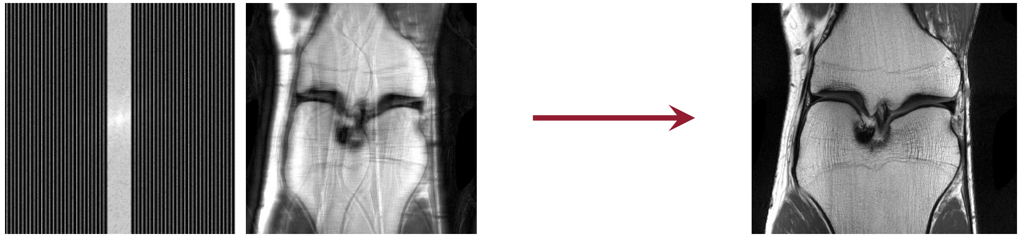
- **Algorithm unrolling** solving the regularized least squares problem in computational magnetic resonance imaging (MRI)

$$\mathbf{y}_\Omega = \mathbf{E}_\Omega \mathbf{x} + \mathbf{n}$$

$$\arg\min_{\mathbf{x}} \|\mathbf{y}_\Omega - \mathbf{E}_\Omega \mathbf{x}\|_2^2 + \mathcal{R}(\mathbf{x})$$

$\mathbf{y}_\Omega$: Sub-sampled measurements
$\mathbf{E}_\Omega$: Encoding matrix sampling Fourier locations $\boldsymbol{\Omega}$
$\mathbf{n}$: Gaussian measurement noise
$\mathcal{R}(\cdot)$: Regularizer

Inverse Problem in computational MRI

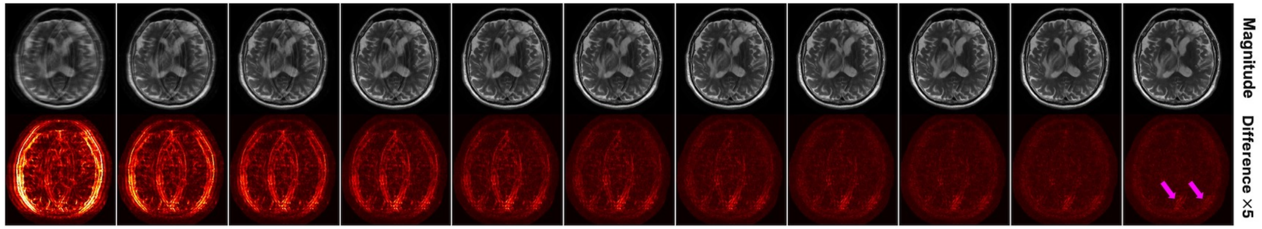


$\boldsymbol{\Omega}$ $\mathbf{y}_\Omega$ $\hat{\mathbf{x}}$

- Unrolling iterative algorithms (e.g., VSQP, ADMM) with fixed iterations, alternating between 1) **Data fidelity**, 2) **Regularization** $\mathcal{R}(\cdot)$, and 3) **Auxiliary updates**

Proximal operator implemented by **neural networks (*Shared, Unshared*** across iterations)

Shared regularizer may cause artifacts/blurring



Distinct (unshared) regularizers

- can improve performance

- However, greatly increases parameters and **overfitting risk**

- especially for applications ***with limited training data***

Introduction

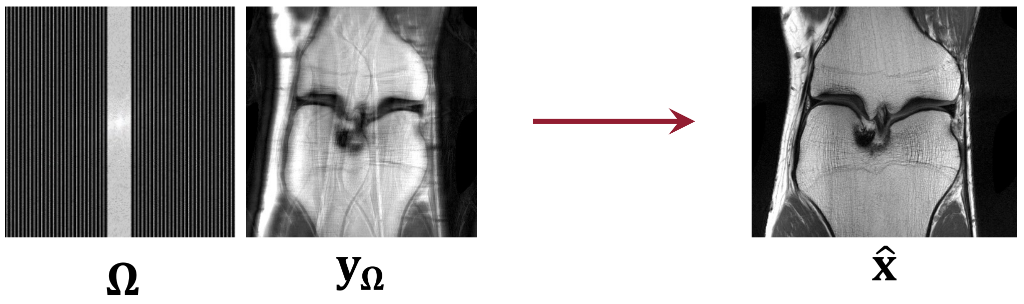
- **Algorithm unrolling** solving the regularized least squares problem in computational magnetic resonance imaging (MRI)

$$\mathbf{y}_\Omega = \mathbf{E}_\Omega \mathbf{x} + \mathbf{n}$$

$$\arg\min_{\mathbf{x}} \|\mathbf{y}_\Omega - \mathbf{E}_\Omega \mathbf{x}\|_2^2 + \mathcal{R}(\mathbf{x})$$

$\mathbf{y}_\Omega$: Sub-sampled measurements
$\mathbf{E}_\Omega$: Encoding matrix sampling Fourier locations $\boldsymbol{\Omega}$
$\mathbf{n}$: Gaussian measurement noise
$\mathcal{R}(\cdot)$: Regularizer

Inverse Problem in computational MRI



- Unrolling iterative algorithms (e.g., VSQP, ADMM) with fixed iterations, alternating between 1) **Data fidelity**, 2) **Regularization** $\mathcal{R}(\cdot)$, and 3) **Auxiliary updates**

Proximal operator implemented by **neural networks (*Shared, Unshared*** across iterations)

Both shared and unshared regularizer is NOT time (iteration)-dependent!

Introduction

In this study, we propose

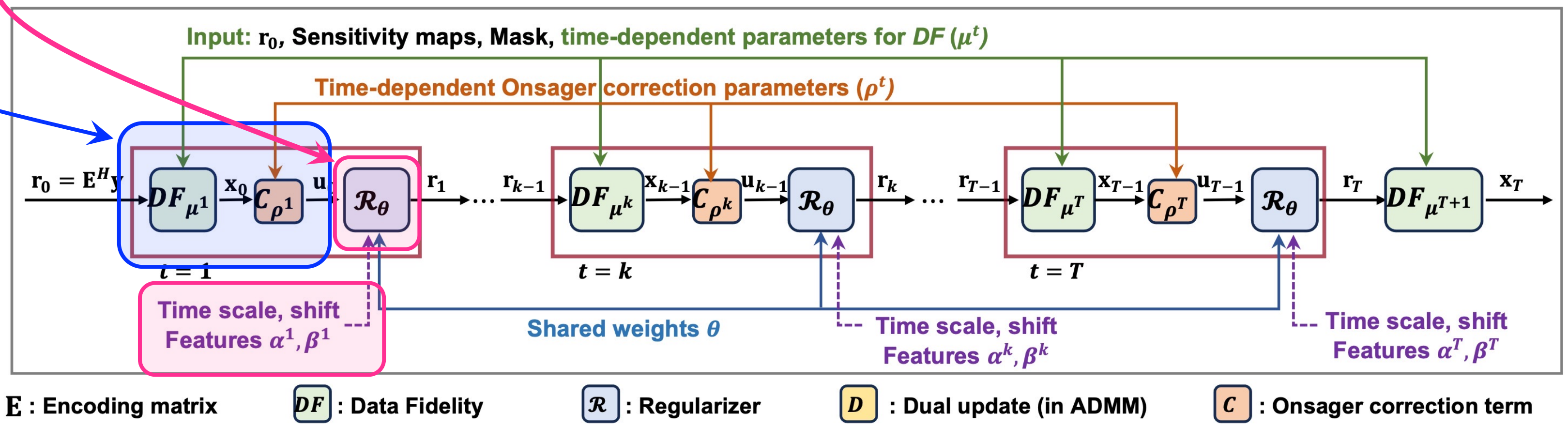
a **time-embedded algorithm unrolling** scheme for inverse problems

→ **Iteration-dependent proximal operator** in vector approximate message passing (VAMP)

→ Subsequent **Onsager correction**, framing them as a **time-embedded neural network**

Method

- Approximate message passing (**AMP**) algorithm uses an ***iteration-dependent proximal operator*** and an **Onsager correction term**
- **Vector AMP** algorithm extends the AMP framework to **vector-valued nodes**, while preserving the desirable properties of AMP.
  - **Data fidelity** operation based on linear **MMSE estimation**, and its associated Onsager correction
  - **Proximal operator/denoising** step with its Onsager correction



Proposed Time-Embedded Unrolled Framework

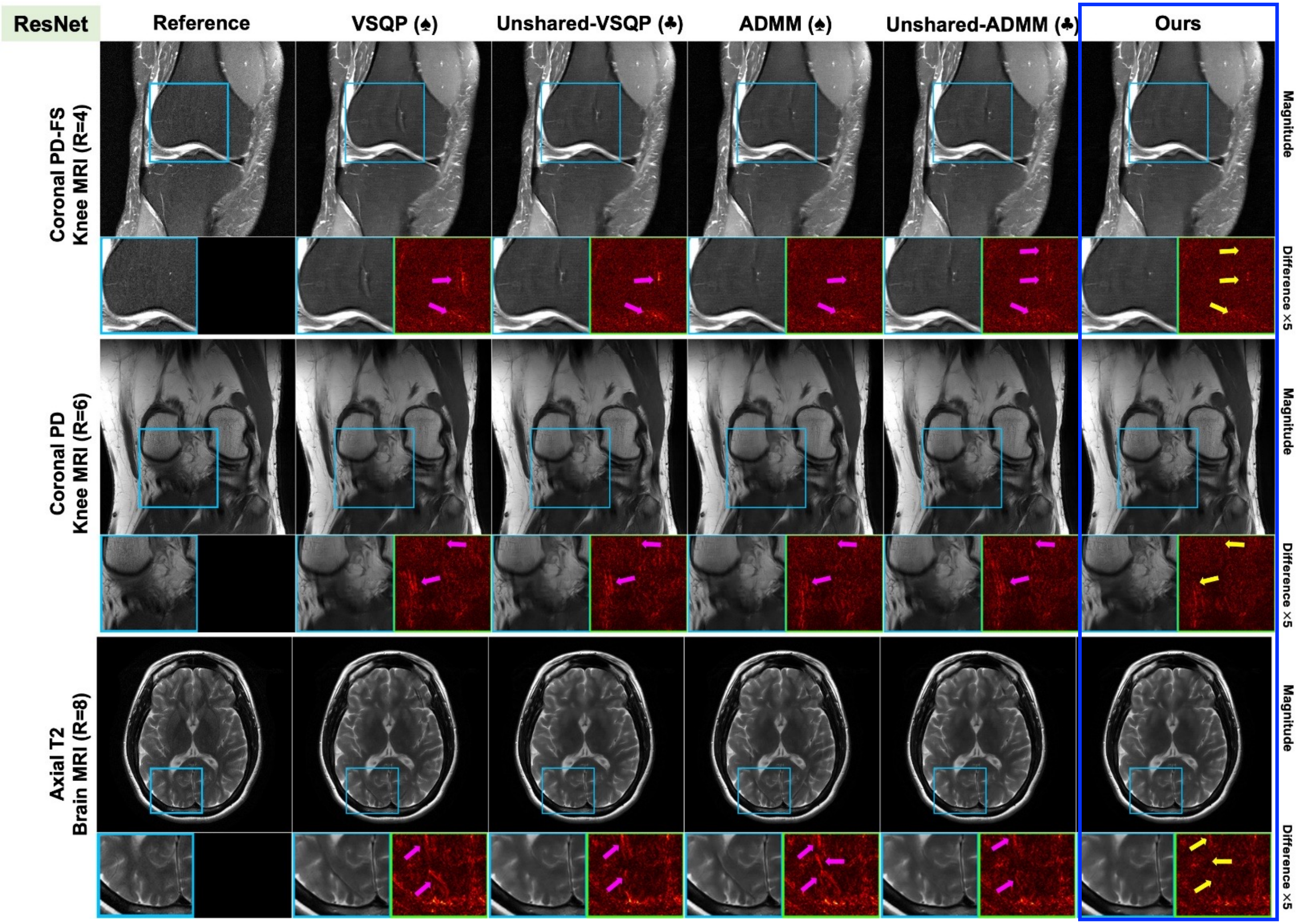
- Adapting the regularizer over time **without significantly increasing network size.**

Number of Parameters

Networks	VSQP (♠)	VSQP (♣)	ADMM (♠)	ADMM (♣)	Ours
U-Net	1,724,035	17,240,341	1,724,036	17,240,342	1,963,479
ResNet	592,129	5,921,281	592,130	5,921,282	866,581

* Shared(♠), unshared (♣) $\mathcal{R}(\cdot)$

Experiments



* Shared(♠), unshared (♣) $\mathcal{R}(\cdot)$

- The proposed method **reduces artifacts** (yellow arrow) that the shared and unshared methods fail to eliminate (pink arrow)

Links & Contact Information

Please refer to the paper and code for additional results and details.

Paper

Code

IMAGINE Lab's homepage

Poster Session Information

Date : Thu 4 Dec

Time : 11 AM - 2 PM

Place : Exhibit Hall C,D,E (San Diego)

Email : {yun00049, alcal029, akcakaya}@umn.edu

Thank you !