Solving Partial Differential Equations via Radon Neural Operator

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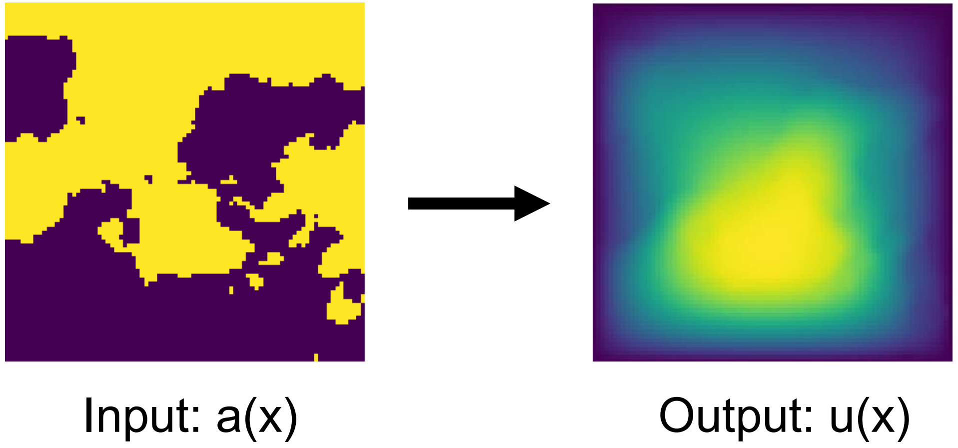
Neural Operator

Neural Operators(NOs) learn mappings between continuous function spaces, enabling strong generalization across resolutions and domains with minimal retraining.

Existing work: FNO, LNO, WNO, Transolver, etc.

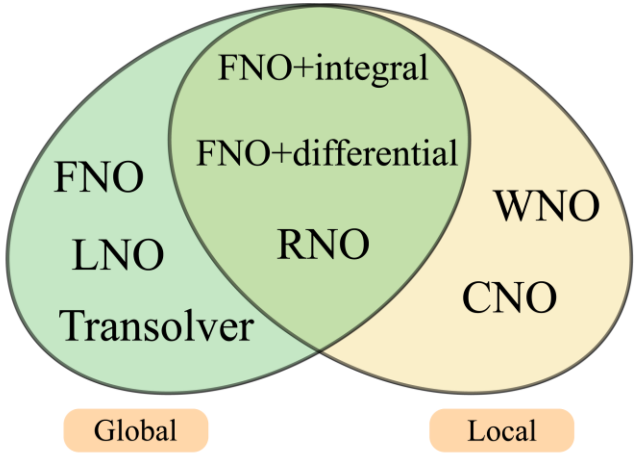


A key challenge in neural operators is balancing **global** and **local** feature learning.

Global learning: miss local details (e.g., shocks)

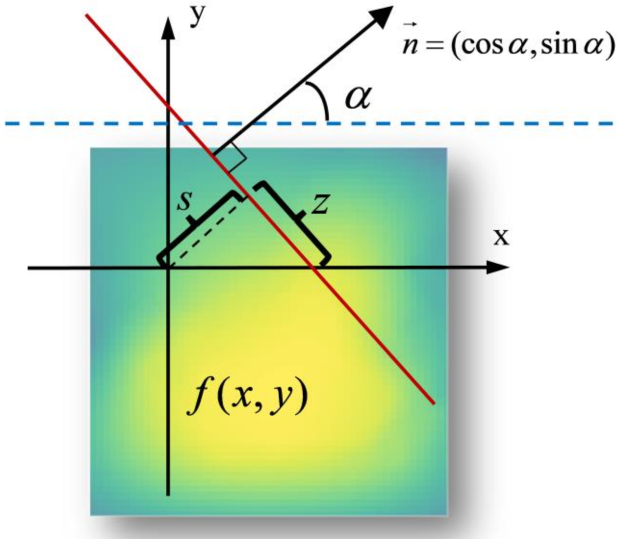
Local learning: lacks global awareness

holistic feature learning



Radon Transform

The **Radon Transform** projects a function into a space of its line integrals, capturing its internal structure from all angles.



Two-dimensional Radon transform

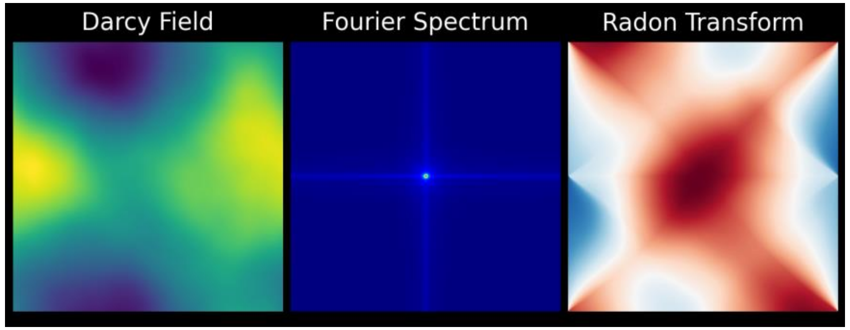
$$Rf(\varphi, s) = \int\int_{\mathbb{R}^2} f(x, y)\delta(x\cos\varphi + y\sin\varphi - s)dxdy$$

S is the perpendicular distance from the origin to the line,

φ is the angle of projection.

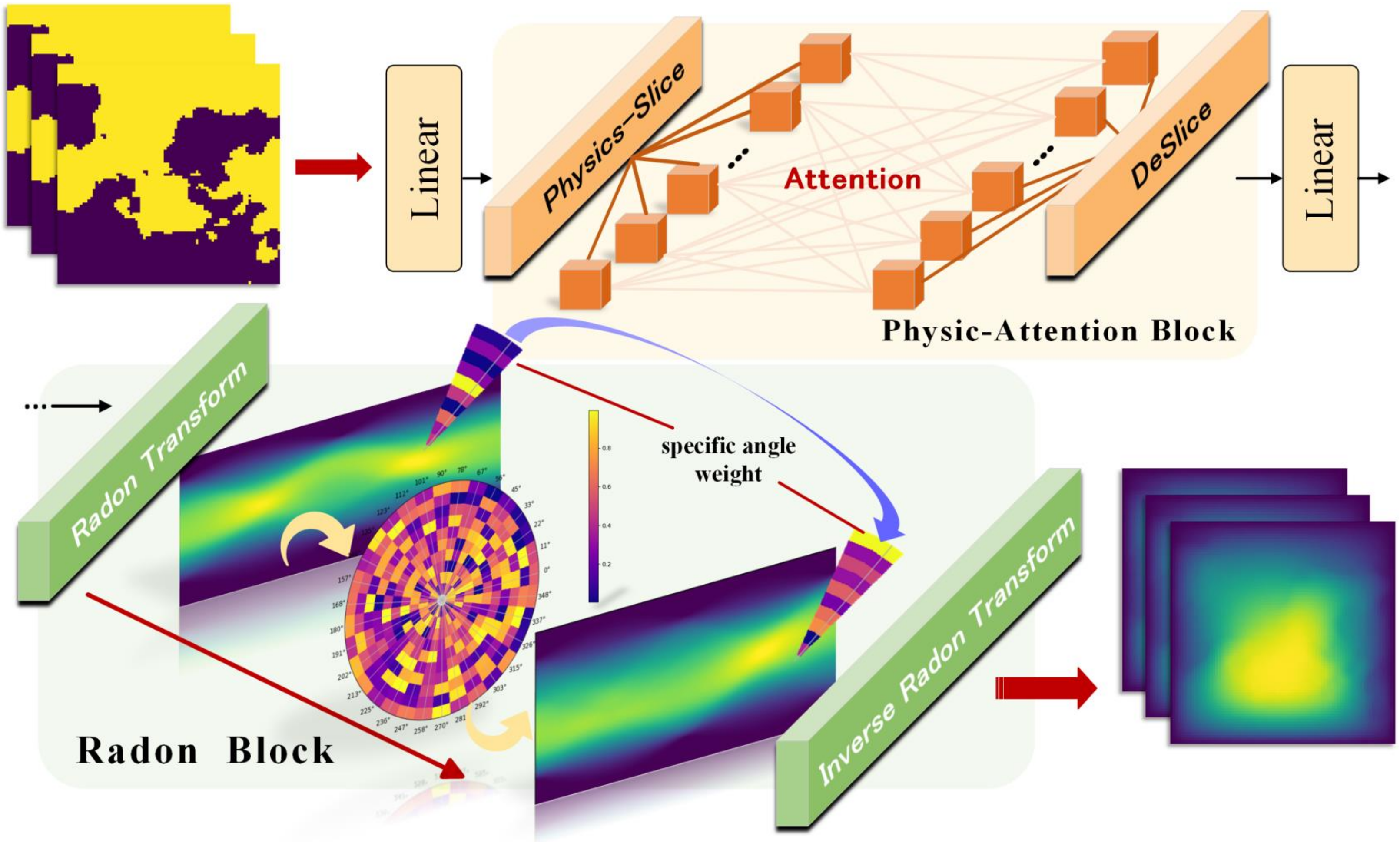
Fourier (FNO), wavelet (WNO), or Laplace (LNO) transforms → input data to the frequency domain, spectral domain, or scale space

The Radon transform reduces PDE dimensionality via line integrals, preserves solution space geometry, and allows stable inversion through Filtered Back Projection (FBP).

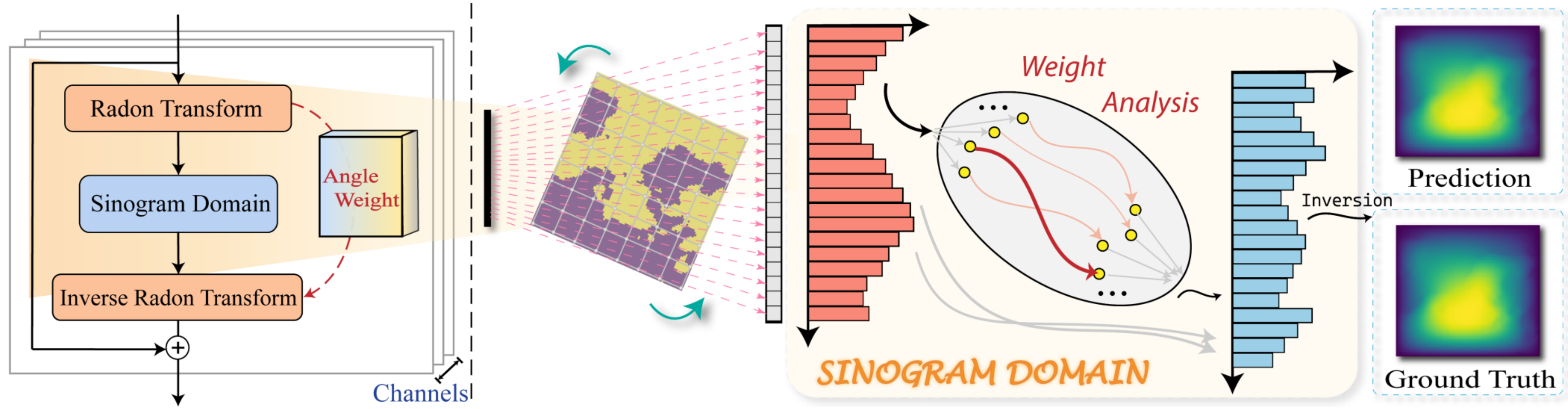


Radon Neural Operator

Given the input data, non-local features are derived through physics-attention, followed by the operations of **weight analysis** and **sinogram domain convolution** with the aid of forward/inverse Radon transform. thereby enriching the more holistic features.



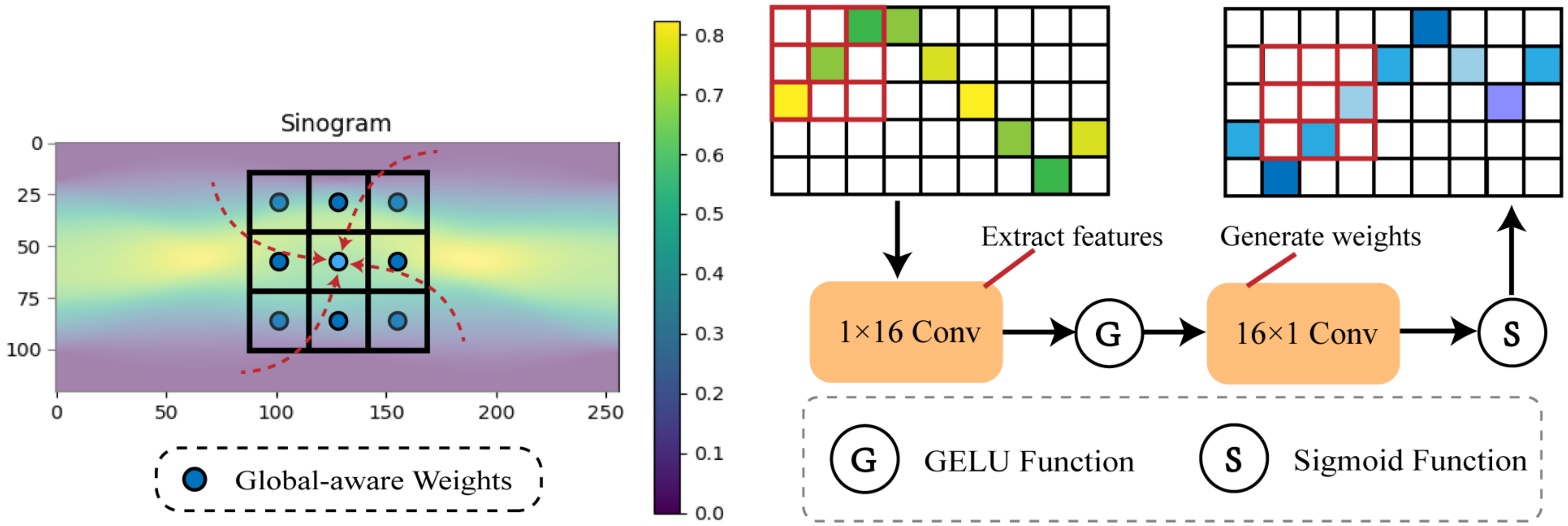
Angle-Reweighting in Sinogram Domain



Key Insight: Different projection angles contribute unequally to feature representation.

Solution: Introduce learnable weights to dynamically adjust the importance of each angle

Sinogram-Domain Convolution



Key Feature: Learns local patterns and dependencies across different projection angles.

Benefit: Enhances feature learning by capturing inter-angle relationships while maintaining consistency across different grid resolutions.

Theoretical Guarantee: Bilipschitz Strong Monotonicity

Core Property: RNO is proven to be a **bilipschitz operator** under diffeomorphism.

- Introduces "no-go theorem" explaining infinite vs. finite dimension obstacles
- Develops discretization invariance under diffeomorphism in category theory
- Proves bilipschitz NO layers can be expressed as strongly monotone layers with isometry

Can neural operators always be continuously discretized?

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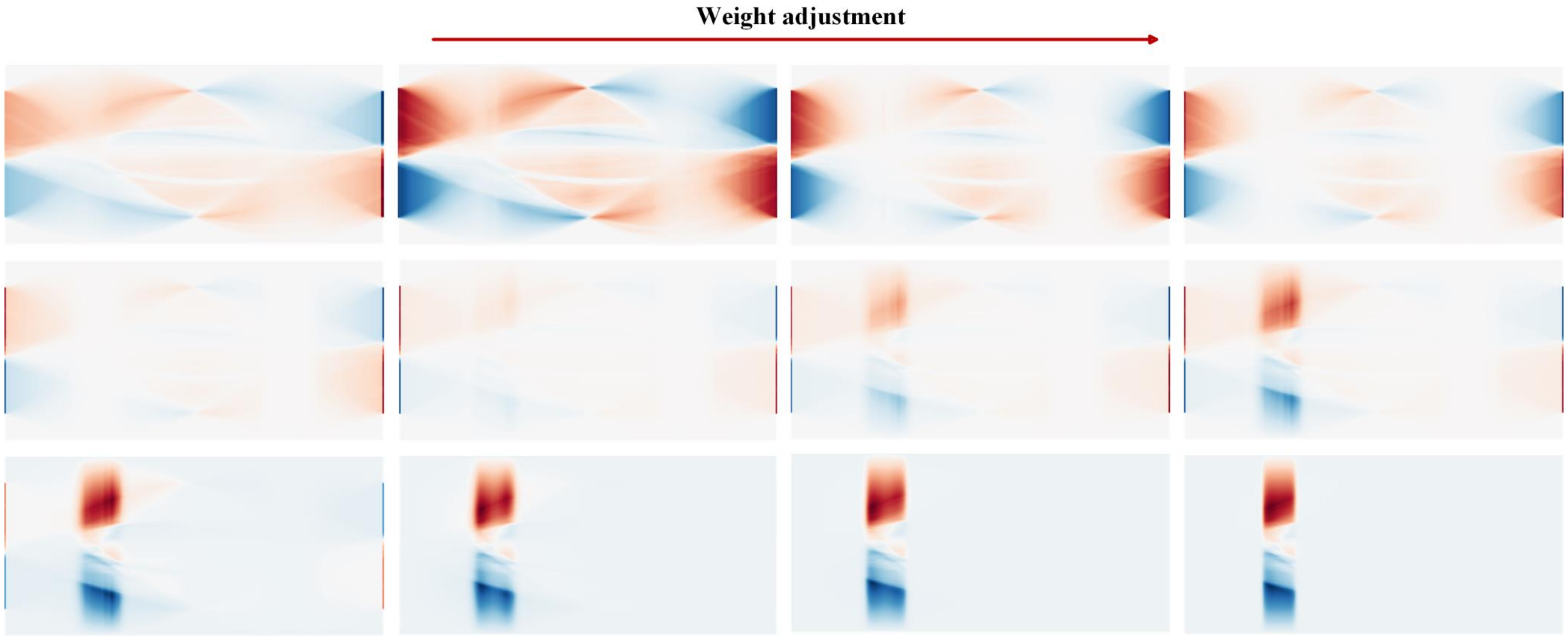
Reference paper： Can neural operators always be continuously discretized? （NeurIPS 2024）

Self-learning in sinogram space

Self-learning adjustment of the sinogram domain weight. From top to bottom, from left to right, it represents the adaptive change of the sinogram domain weight as the training time increases.



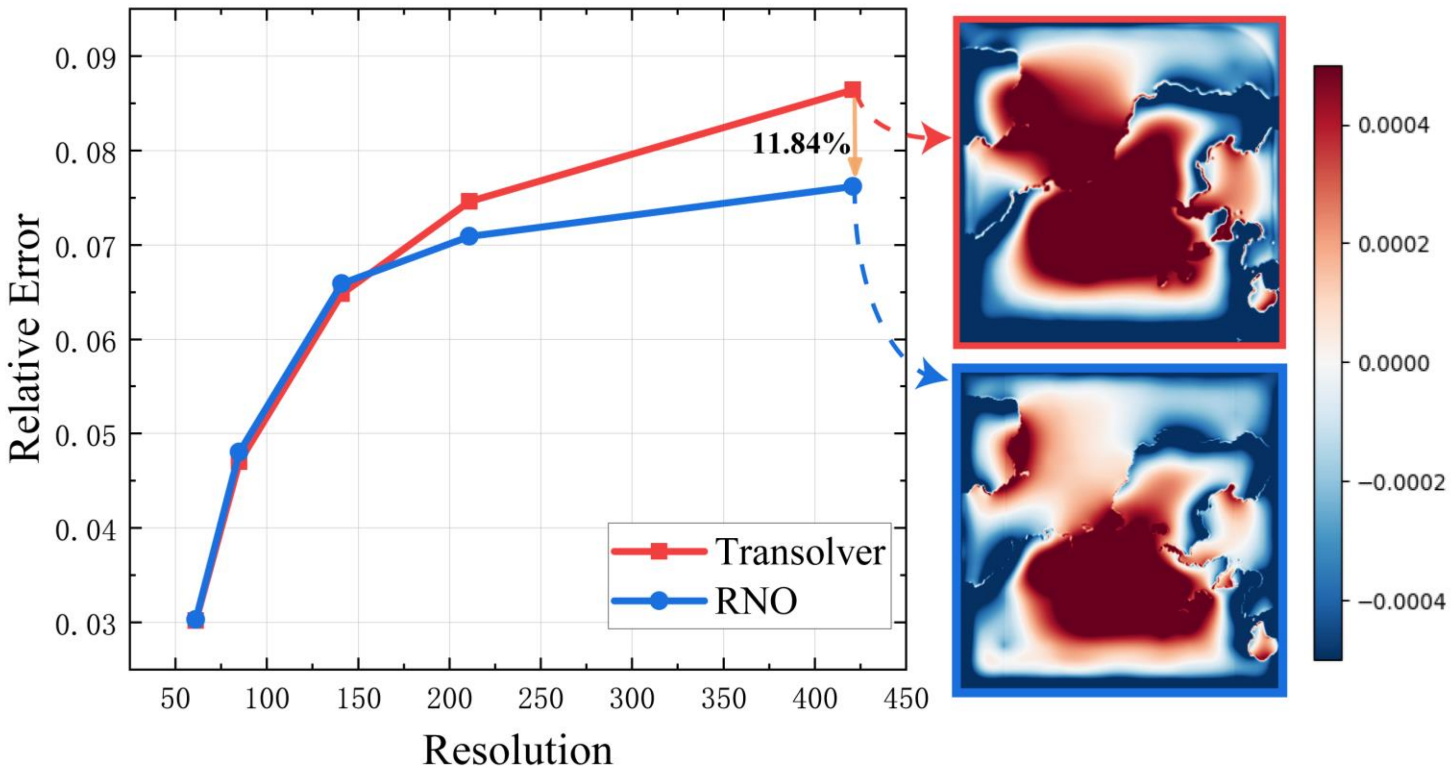
Experimental results

Achieves new **state-of-the-art(SOTA)** accuracy across multiple PDE benchmarks (Darcy, Navier-Stokes, Allen-Cahn, etc.).

MODEL	MECHANISM	REGULAR GRID			STRUCTURED MESH		
		Darcy	Navier-Stokes	Allen-Cahn	Airfoil	Plasticity	Pipe
DEEPONET	/	5.88×10^{-2}	2.97×10^{-1}	/	3.85×10^{-2}	1.35×10^{-2}	9.70×10^{-3}
FNO	Fourier Transform	1.08×10^{-2}	1.56×10^{-1}	7.52×10^{-3}	/	/	/
WMT	Wavelet Transform	8.20×10^{-3}	1.54×10^{-1}	1.12×10^{-2}	7.50×10^{-3}	7.60×10^{-3}	7.70×10^{-3}
GALERKIN	Galerkin Attention	8.40×10^{-3}	1.40×10^{-1}	/	1.18×10^{-2}	1.20×10^{-2}	9.80×10^{-3}
GNOT	Transformer	1.05×10^{-2}	1.38×10^{-1}	/	7.60×10^{-3}	3.36×10^{-2}	4.70×10^{-3}
U-NO	U-Net	1.13×10^{-2}	1.71×10^{-1}	4.31×10^{-2}	7.80×10^{-3}	3.40×10^{-3}	1.00×10^{-2}
ONO	Orthogonal Attention	7.60×10^{-3}	1.20×10^{-1}	1.71×10^{-2}	6.10×10^{-3}	4.80×10^{-3}	5.20×10^{-3}
TRANSOLVER	Physic-Attention	$\underline{5.70} \times 10^{-3}$	$\underline{9.00} \times 10^{-2}$	$\underline{5.32} \times 10^{-3}$	$\underline{5.77} \times 10^{-3}$	$\underline{1.20} \times 10^{-3}$	$\mathbf{3.30} \times 10^{-3}$
RNO (Ours)	Radon Transform	$\mathbf{5.10} \times 10^{-3}$ ▲	$\mathbf{8.94} \times 10^{-2}$ ▲	$\mathbf{4.61} \times 10^{-3}$ ▲	$\mathbf{4.90} \times 10^{-3}$ ▲	$\mathbf{1.15} \times 10^{-3}$ ▲	$\underline{4.20} \times 10^{-3}$ ▼

Generalization results

Shows **significant improvements** (~70% error reduction) in zero-shot super-resolution tasks vs. FNO.



Future Work

- **Enhanced Sinogram Analysis**: Develop more advanced architectures for deeper feature extraction in the sinogram domain.
- **Complex Problem Scaling**: Extend RNO's application to higher-dimensional and more complex industrial PDE problems.
- **Efficiency Optimization**: Optimize computational performance and explore model compression for real-time applications.
- **Theoretical Expansion**: Further investigate the theoretical foundations of operator learning in transformed domains.

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NEURAL INFORMATION PROCESSING SYSTEMS

Thanks for listening