MSE
$-f(x) + y|x = z \sim \mathcal{N}(0, \sigma^2)$

Apply T to y →

T-MSE
$-f(x) + \mathrm{T}(y)|x = z \sim \mathcal{N}(0, \sigma^2)$

Alter obj: $\max p_{y|x}(y|x) \rightarrow \max p_z(z)$

Specify $h(\cdot)$ and $\mathcal{P}_z$

NF
$h(y|x, x) = z \sim \mathcal{P}_z$

Alter obj: $\max p_{y|x}(y|x) \rightarrow \max \mathbb{P}_{y|x}(y < v|x)$

CTM
$h(y|x, x) = z \sim \mathcal{P}_z$

$K = 1,\ z_1 \sim \mathcal{N}(f(x), \sigma^2)$

Add $\mathcal{L}_{\mathrm{sun}}^{\mathrm{T}}$

Specify $h(\cdot)$ as linear, $\mathcal{P}_z$ as Gaussian

MDM
$p(y|x) = \sum_{k=1}^{K} \pi_k(x)\, p_{z_k}(y|x)$

$s.t. \sum_{k=1}^{K} \pi_k(x) = 1, \quad \pi_k(x) \geq 0$

TranSUN
$-f(x) + \dfrac{y|x}{\left| \mathrm{T}^{-1}\big(\mathbb{E}_{\mathrm{Y}|x}[\mathrm{T}(y)]\big)\right| + \epsilon} = z \sim \mathcal{N}(0, \sigma^2)$

$\mathbb{Q} = \mathbb{E},\ \kappa(x) = 1/(|\mathrm{T}^{-1}(x)| + \epsilon)$

GTS
$-f(x) + \kappa\big(\mathbb{Q}_{\mathrm{Y}|x}[\mathrm{T}(y)]\big) \cdot y|x = z \sim \mathcal{N}(0, \sigma^2)$

Alter obj: $\max \mathbb{P}_{y|x}(y < v|x) \rightarrow \max p_{y|x}(y|x)$

Replace $\alpha(x)$ with $\kappa(\mathbb{Q}_{\mathrm{Y}|x})$, and omit $\mathrm{T}(\cdot)$

CLTM
$-f(x) + \alpha(x) \cdot \mathrm{T}(y)|x = z \sim \mathcal{N}(0, 1)$

Unbiasedness
★ Holds all the time

⯪ Depends on some designs (e.g. $\mathcal{P}_z$ and $h(\cdot)$ in NF&CTM)

☆ Barely holds (e.g. T in T-MSE is barely linear)

Convergence
★ Fast and smooth on skewed data

⯪ Occasionally unstable due to complex design

☆ Unstable on skewed data

Pluggability
★ For any regression model

⯪ For specific models (e.g. TranSUN is for T-MSE)

☆ Not pluggable

Overhead
★ Minimal

⯪ Large in some cases (e.g. MDN's K grows too much)

☆ Often significant (e.g. Jacobian computation in NF)

Notes: A graphical comparison of the pros & cons & model assumptions among Mean Squared Error (MSE), transformed MSE (T-MSE), Normalizing Flow (NF), Conditional Transformation Models (CTM), Conditional Linear Transformation Models (CLTM), Mixture Density Model (MDM), TranSUN, and Generalized TranSUN (GTS).